BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY
W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY
KARL PEARSON

VOLUME VIII

JULY 1911 TO JANUARY 1912

CAMBRIDGE
AT THE UNIVERSITY PRESS

LONDON: FETTER LANE, E.C.
(C. F. CLAY, MANAGER)
AND H. K. LEWIS, GOWER STREET
EDINBURGH: 100, PRINCES STREET
NEW YORK: G. P. PUTNAM'S SONS
BERLIN: A. ASHER AND CO.
LEIPSIC: BROCKHAUS
BOMBAY AND CALCUTTA: MACMILLAN AND CO., LIMITED

Cambridge:
PRINTED BY JOHN CLAY, M.A.
AT THE UNIVERSITY PRESS.

CONTENTS OF VOL. VIII.

Memoirs.

Miscellanea.

Plates.

PARTS I. AND II.

(Also Thirty-two copies of Cranial Type-Contours in pocket at the end of volume.)

PARTS III AND IV.

FRANCIS GALTON

February 16, 1822—January 17, 1911

A Founder and Consulting Editor of Biometrika.

Ἀλλὰ μὴν τὸ μέτρῳ γε καὶ λογισμῷ πιστεῦον βέλτιστον ἂν εἴη τῆς ψυχῆς. Τί μήν ; Τὸ ἄρα τούτῳ ἐναντιούμενον τῶν φαύλων ἄν τι εἴη ἐν ἡμῖν. Ἀνάγκη.

Republic, 603 A.

Some account of the lifework of Francis Galton will appear in the next issue of this Journal.

VOLUME VIII JULY, 1911 NOS. 1 AND 2

BIOMETRIKA.

A THIRD COOPERATIVE STUDY OF *VESPA VULGARIS*. COMPARISON OF QUEENS OF A SINGLE NEST WITH QUEENS OF THE GENERAL AUTUMN POPULATION.

BY E. Y. THOMSON, JULIA BELL, M.A., AND KARL PEARSON, F.R.S.

(1) *Scope and Material.*

IN our first study* the differences between queens, drones and workers from a single nest of *Vespa vulgaris* were investigated. In our second study† we compared the queens of this single nest with the queens of a general population gathered in the *spring*, i.e. after the winter selection had taken place. The main result of that comparison was that the spring population queens were about twice as variable as the nest queens. The same reduction in variability between nest and general populations had practically been reached by Dr Warren‡ in dealing with Natal termites, who attributed it largely to environmental influence. No theory of heredity seemed sufficient to account for such a large reduction, even with indefinite inbreeding. An American writer of distinction§ suggested that our general population might consist of queens from one or two nests; he did not seem to recognise that this would render more difficult of explanation than ever the marked difference in variability, and further was incompatible with the fact that the wasps were collected after winter hibernation in all parts of a large parish, many square miles in extent. On a small farm in an adjacent district within a few weeks in July, 1909, 40 nests alone were destroyed, and this was not a fruit farm, nor was the destruction considered anything unusual by the bailiff. We may safely take it that the autumn nests in the area from which we drew our spring supplies were to be reckoned rather by thousands than hundreds, and that the chances that our spring wasps came from one or two nests are absolutely negligible.

---

* *Biometrika*, Vol. v. p. 407. † *Ibid.* Vol. VII. p. 48. ‡ *Ibid.* Vol. VI. p. 329.

§ *The American Naturalist*, Vol. XLIV. p. 310, with reply by Pearson, *Ibid.* Vol. XLIV. p. 503.

A point, however, of some interest left unsettled by our earlier papers was the influence of winter hibernation on the wing characters of the queens. Is there a marked difference between the means, variations and correlations of general populations of autumn and spring queens? We endeavoured to investigate this point by collecting queens from nests taken in an adjacent district of the same county, Buckinghamshire, during the months of August and September in the autumn of 1909. The process was not an easy one as more than 100 nests had to be individually taken and a queen extracted. However, we were able to procure ultimately 100 queens, and although a few old queens may have by accident been included in this number, we have we believe practically a general population of queens of the autumn of 1909*. Slides were made of the four wings as in the previous cases, and the measurements taken on the right and left front wings. We were able to obtain 100 cases of each character measured. This must be looked upon as a small sample, but the difficulty of taking the nests without injuring the population, and the labour required in making the requisite 1400 microscopic measurements† really preclude the use of much larger samples. We propose, however, in our next wasp study to consider one or two characters only in very large samples of queens of the general spring population and of one nest, now in our possession. The remarkably small variability of *Vespa vulgaris* does, however, to some extent justify the use of small samples, for it leads to very small probable errors in absolute measurements.

(2) *Characters measured.*

These are precisely the same as those of our first and second studies. We reproduce the cuts indicating the cells measured.

We should have wished to some extent to change this series of measurements. The two rather unsatisfactory measurements, as greater experience has shown, are the maximum breadth D and the length of the cell b, i.e. B. While X (see Figs. I and III above) is often a well-marked point, there are cases in which this is not so, and the determination of the point Y most distant from X requires some care. On the whole, however, the end of the cell b (see Fig. II) is the most troublesome point to determine accurately. Some cases are like Fig. II, but there is not infrequently need for very cautious determination of the tangent line with the ocular micrometer ‡. For this reason in the present series a determination was, as we have stated above, first made with the ocular micrometer and then with a Hilger reading microscope, and any sensible cases of discrepancy between

---

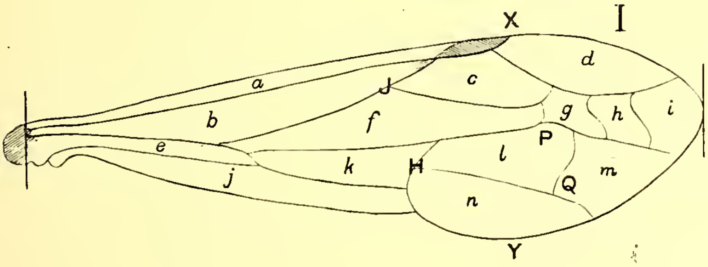
* The number of queens in the autumn nests may run from 30 to 100 or 200, and even in special cases, as those of two nests of *V. germanica* sent to us last year, to 500 and over 600 respectively!

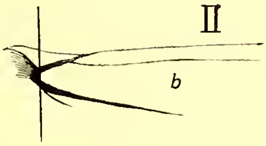
† The measurements were made in the manner indicated on p. 49 of our second study, but certain difficulties having arisen with regard to the maximum breadth D and the length of the cell b, i.e. B, these measurements were repeated with a Hilger reading microscope, and in case of sensible divergence between the two methods of measurement the measurement was again carefully examined.

‡ A low power is necessary in order to adjust the 'horizontal' line of the micrometer to the length of the cell, but this power may leave the end of the cell b not clearly defined.

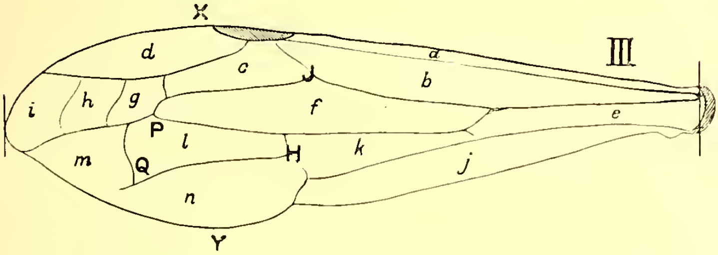
the two were then reinvestigated. Now it will be noticed (see Tables II and III) that the character *B* is the sole one in which the relative or absolute variability of the autumn queens falls below that of the spring queens. We at first believed this to be due to the possibility that the present series had been determined with greater exactitude than was adopted in the earlier series, but a remeasurement of the cell *b* of the spring queens has not justified this hypothesis.

RIGHT WING. *Vespa Vulgaris.*





LEFT WING. *Vespa Vulgaris.*



We reproduce the list of actual measurements made for reference.

(A) Total length of wing reckoned from the tip of wing outside of cell *i* to the inside of the vein at the end of the cell *b*. Cf. Figs. I and II.

(B) Greatest length of cell *b* taken inside thickened end of veins.

(C) Length of cell *f* from point to further inside point on vein *fg*.

(D) Greatest breadth of wing from *X* to wherever found, i.e. *XY*.

(E) Distance apart of *J* and *H* measured inside veins of cell *f*.

(F) Greatest length of cell *d*, measured inside cell.

(G) Distance apart of points *P* and *Q* measured inside cell *l*.

The indices determined were:

$$H = D/A, \quad K = B/A,$$
$$I = E/D, \quad L = C/A,$$
$$J = G/D, \quad M = F/A.$$

The correlations selected for comparison were those of the previous memoirs, i.e.:

A and D,	G and F,	I and K,
A and C,	G and D,	M and J,
A and E,	B and E,	L and J,
	B and F,	I and L.

Further the correlation of all the absolute lengths measured on the two wings was made as before.

(3) *On the Means of the three Series.*

Taking the absolute sizes we see a remarkable accordance between the series of spring and of autumn queens of the same species from districts some 20 miles apart. On the average the autumn queens appear to be slightly larger (1·0074 right wing and 1·0038 left wing), but no stress whatever can be laid on such a slight difference, nor on the fact that in the case of two characters the autumn series appear to be smaller. It would require far larger numbers than we have dealt with to give such differences real significance. In the case of the indices also, it is impossible to assert that there is any sensible differentiation between the queens of the general population in autumn and in spring. But the determination of the characters for the autumn population has been of great value, for we conclude that the differentiation in absolute size is not due to (*a*) local race differences—the queens of Gerard's Cross are identical in size with the queens of Checkendon and Woodcote, nor (*c*) the selective action of the winter in differentiating autumn from spring queens. Even one portion of our alternative* (*b*), individuality of the nest arising from heredity or nurture, seems excluded, because the general change of environment from Gerard's Cross to Checkendon would we suppose be greater than between two nests at any one place. We shall be able to test this point on further material in our possession, but it seems reasonable to accept as a working hypothesis that the difference between the nest queens and the general population queens in absolute size was the result of heredity in the individual nest. In the case of the indices there are no average differences on which, for our limited data, we should feel inclined to lay the least stress. Lastly, we do not find either in absolute measurement or in the indices any persistent differences of a sensible character between right and left wing for the three series. If the right side is differentiated from the left—as it certainly appears to be in the case of bone measurements in man—there are no differences of such a marked character that they are sensible on samples of the present size.

---

* See *Biometrika*, Vol. VII. p. 52.

TABLE I.

Mean Values of Characters in Nest and Population Queens.

		SPRING POPULATION QUEEN		AUTUMN POPULATION QUEEN		NEST QUEEN		RATIO Aut. *P. Q.*/*N. Q.*		RATIO Sp. *P. Q.*/*N. Q.*		RATIO Aut. *P. Q.*/Sp. *P. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes.	*A*	13·223 ± ·015	13·219 ± ·016	13·339 ± ·023	13·342 ± ·023	13·623 ± ·008	13·617 ± ·009	·9791	·9798	·9706	·9708	1·0088	1·0093
	B	7·770 ± ·010	7·818 ± ·010	7·872 ± ·012	7·884 ± ·011	8·042 ± ·006	8·058 ± ·007	·9789	·9784	·9662	·9702	1·0131	1·0084
	C	6·185 ± ·008	6·177 ± ·008	6·171 ± ·012	6·174 ± ·012	6·388 ± ·004	6·390 ± ·004	·9660	·9662	·9682	·9667	·9977	·9995
	D	3·933 ± ·006	3·950 ± ·007	4·036 ± ·011	4·019 ± ·012	3·989 ± ·003	3·983 ± ·003	1·0118	1·0090	·9860	·9917	1·0262	1·0175
	E	1·446 ± ·003	1·452 ± ·003	1·439 ± ·005	1·439 ± ·006	1·457 ± ·002	1·467 ± ·001	·9876	·9809	·9925	·9898	·9952	·9910
	F	3·031 ± ·005	3·045 ± ·005	3·043 ± ·008	3·049 ± ·007	3·175 ± ·003	3·191 ± ·003	·9584	·9555	·9546	·9542	1·0040	1·0013
	G	1·414 ± ·002	1·405 ± ·003	1·424 ± ·003	1·404 ± ·004	1·420 ± ·002	1·415 ± ·002	1·0028	·9922	·9958	·9929	1·0071	·9993
							Mean	·9835	·9803	·9763	·9766	1·0074	1·0038
Indices.	*H*	·2975 ± ·0004	·2992 ± ·0005	·3028 ± ·0007	·3013 ± ·0007	·2928 ± ·0002	·2925 ± ·0002	1·0342	1·0301	1·0161	1·0229	1·0178	1·0070
	I	·3672 ± ·0005	·3675 ± ·0005	·3560 ± ·0010	·3578 ± ·0009	·3651 ± ·0004	·3687 ± ·0003	·9751	·9704	1·0057	·9967	·9695	·9736
	J	·3605 ± ·0005	·3565 ± ·0004	·3531 ± ·0007	·3496 ± ·0008	·3560 ± ·0004	·3552 ± ·0004	·9919	·9842	1·0126	1·0037	·9795	·9806
	K	·5878 ± ·0003	·5917 ± ·0003	·5903 ± ·0006	·5910 ± ·0007	·5904 ± ·0003	·5918 ± ·0003	·9998	·9986	·9956	·9998	1·0043	·9988
	L	·4680 ± ·0003	·4680 ± ·0003	·4625 ± ·0004	·4627 ± ·0004	·4690 ± ·0002	·4694 ± ·0002	·9861	·9857	·9979	·9970	·9882	·9987
	M	·2293 ± ·0002	·2303 ± ·0002	·2281 ± ·0003	·2285 ± ·0003	·2331 ± ·0002	·2344 ± ·0002	·9785	·9748	·9837	·9825	·9948	·9922
							Mean	·9943	·9906	1·0019	1·0004	·9923	·9918

(4) *On the Variability of the Autumn and Spring Queens.*

Table II gives the standard deviations of the absolute measurements and the indices, and we see here at once most interesting and important conclusions can be drawn. We note in the first place that for absolute measurements the autumn queens are on the average 10 to 12 per cent., and for indices 18 to 22 per cent. more variable than the spring queens. Now it would seem that there is only one interpretation of this result: *The reduced variability of the spring queens as compared with the autumn queens is the result of selection after the dispersal of the nest population.*

There is only one apparent exception to this rule and that is the case of cell *b*, or measurement B*. Even including this case the reduction is still 10 to 12 per cent.

Now two points must be noted here :

(*a*) The standard deviations have been markedly reduced without sensibly altering the means.

Hence, if we assume the distributions to be practically Gaussian and suppose N_1 autumn queens to provide N_2 spring queens, the non-differential deathrate being neglected, we shall have: $N_1/(\sqrt{2\pi}\sigma_1) = N_2/(\sqrt{2\pi}\sigma_2)$, but $\sigma_1 = 1{\cdot}11\sigma_2$ say, for absolute measurements.

Hence $N_2 = {\cdot}90 N_1$, or 10 °/o of the autumn queens are on the average destroyed by a differential deathrate in the case of the absolute measurements.

In the case of the indices we have: $N_2/N_1 = \sigma_2/\sigma_1 = 100/120 = {\cdot}83$ say, or 17 °/o of the autumn queens are on the average removed by a differential deathrate depending on the proportions of the wing. In the case of the ratio of the cell *b* to the total length of the wing, i.e. $K = B/A$, the loss of variability is about 50 °/o or $N_1/N_2 = 100/152 = 66$ °/o, or 34 °/o seem to be destroyed by this ratio diverging too far from its mean or type value.

(*b*) The reduction of variability is considerably greater in the matter of the proportioning of the wing than in the case of absolute size.

Thus the exact fitness for survival of the queen during the period in which she is seeking winter quarters, hibernating and starting to form a new colony, seems to depend more considerably on the ratio of the parts of her wings than on their absolute size. That 34 °/o of queens should fail to survive, owing to the failure of the proportioning of the cell *b* to the total length of the wing between autumn and spring, will perhaps appear less wonderful to many now than it would have

---

* In order to test whether this exception arose from personal equation in determining the length of the cell *b*, the spring series were most carefully remeasured for this character with the reading microscope. There resulted :

		Cell *b*, mean	S. D.	C. of V.	Correlation J. H. and *b*
Reading microscope,	R. wing	7·856	·2023	2·58	·515
,, ,,	L. wing	7·866	·2043	2·60	·557
Ocular micrometer,	R. wing	7·770	·2099	2·70	·506
,, ,,	L. wing	7·718	·2026	2·59	·617

No argument has been based on differences of this order; and it was some satisfaction to find that in this, the most difficult measurement, independent processes agreed so well.

TABLE II.

Standard Deviations of Characters for Nest and Population Queens.

		SPRING POPULATION QUEEN		AUTUMN POPULATION QUEEN		NEST QUEEN		RATIO Aut. *P.Q.*/*N.Q.*		RATIO Sp. *P.Q.*/*N.Q.*		RATIO Aut. *P.Q.*/Sp. *P.Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes.	*A*	·3124 ± ·0109	·3165 ± ·0110	·3365 ± ·0160	·3380 ± ·0161	·1428 ± ·0060	·1464 ± ·0061	2·3564	2·3087	2·1877	2·1619	1·0771	1·0679
	B	·2099 ± ·0073	·2026 ± ·0070	·1742 ± ·0083	·1659 ± ·0079	·1089 ± ·0046	·1174 ± ·0049	1·5996	1·4131	1·9275	1·7257	·8299	·8189
	C	·1644 ± ·0057	·1678 ± ·0058	·1770 ± ·0084	·1724 ± ·0082	·0666 ± ·0028	·0704 ± ·0030	2·6577	2·4489	2·4685	2·3835	1·0766	1·0274
	D	·1343 ± ·0047	·1438 ± ·0050	·1595 ± ·0076	·1717 ± ·0082	·0525 ± ·0022	·0528 ± ·0022	3·0381	3·2519	2·5581	2·7235	1·1876	1·1940
	E	·0527 ± ·0018	·0552 ± ·0019	·0815 ± ·0039	·0825 ± ·0039	·0329 ± ·0014	·0246 ± ·0010	2·4772	3·3537	1·6018	2·2439	1·5465	1·4946
	F	·0991 ± ·0034	·1044 ± ·0036	·1124 ± ·0054	·1106 ± ·0053	·0462 ± ·0019	·0468 ± ·0020	2·4329	2·3632	2·1450	2·2308	1·1342	1·0594
	G	·0495 ± ·0017	·0524 ± ·0018	·0496 ± ·0024	·0555 ± ·0026	·0376 ± ·0016	·0385 ± ·0016	1·3191	1·4416	1·3165	1·3610	1·0020	1·0592
							Mean	2·269	2·369	2·029	2·119	1·122	1·103
Indices.	*H*	·00914 ± ·00032	·00925 ± ·00032	·01027 ± ·00049	·01044 ± ·00050	·00329 ± ·00014	·00314 ± ·00013	3·1216	3·3248	2·7781	2·9459	1·1236	1·1286
	I	·01022 ± ·00036	·01058 ± ·00027	·01416 ± ·00066	·01300 ± ·00062	·00628 ± ·00026	·00526 ± ·00022	2·2548	2·4715	1·6274	2·0114	1·3855	1·2287
	J	·01022 ± ·00036	·00881 ± ·00031	·01063 ± ·00051	·01121 ± ·00053	·00717 ± ·00030	·00733 ± ·00031	1·4826	1·5293	1·4254	1·2019	1·0401	1·2724
	K	·00614 ± ·00021	·00616 ± ·00021	·00833 ± ·00040	·01042 ± ·00050	·00422 ± ·00018	·00504 ± ·00021	1·9739	2·0675	1·4550	1·2222	1·3567	1·6916
	L	·00531 ± ·00018	·00540 ± ·00019	·00604 ± ·00029	·00550 ± ·00026	·00281 ± ·00012	·00285 ± ·00012	2·1495	1·9298	1·8897	1·8947	1·1375	1·0185
	M	·00487 ± ·00017	·00483 ± ·00017	·00497 ± ·00024	·00462 ± ·00022	·00360 ± ·00015	·00345 ± ·00014	1·3806	1·3391	1·3528	1·4000	1·0205	·9565
							Mean	2·060	2·110	1·755	1·779	1·177	1·216

done a few years ago, since the interval has taught man himself on how minute details his own safety in flying depends.

It is needless to say that these matters want testing and will be tested on far larger numbers, but the last columns of Table II seem to justify a very strong conviction that the survival of the queen wasp from autumn to spring depends very largely on the deviations in the size and proportioning of her wings not differing either in excess or defect very markedly from the type.

If we turn to the relationship of the autumn queens to queens from a single nest, we find that in general they entirely confirm the conclusions we had drawn from the spring queens; the variabilities are more than doubled when we pass from the nest queens to the general autumn population. The difficulty already referred to in our second paper is therefore emphasised when we compare nest and population queens of the same period; instead of the 20 °/₀ reduction we might anticipate we have over a 50 °/₀ reduction*. The possibility that this large reduction in variability was due to a winter selection which increased the variability (see p. 58 of our second study) is thus definitely negatived by our present work. This result further demonstrates that the selective influence of environment, as measured by the autumn to spring deathrate of queens (i.e. 10 °/₀ to 20 °/₀) is only $\frac{1}{5}$th to $\frac{2}{5}$ths as effective as the reduction, probably due to heredity, which marks the queens of the individual nest.

(5) *On the Relative Variability of the Autumn and Spring Queen Populations.*

Table III gives the coefficients of variation of all the characters dealt with. We see at once that the results drawn from the absolute variations are fully confirmed when we deal with these ratios of variability to size of character varying. We find that the autumn queens are 10 °/₀ to 20 °/₀ more variable than the spring queens; in fact but for the length of the cell *b*, the anomalous nature of which has already been pointed out†, we should have the autumn queens sensibly more variable for all characters. The coefficients of variation of the absolute sizes of the autumn queens are on the average about 3·50—still very sensibly below most values hitherto obtained for coefficients of variation, and tending to confirm the view that the wing is an exceedingly delicate organ, in which closeness to type is of supreme importance.

(6) *Coefficients of Correlation of Autumn Queens compared with Spring and Nest Queen Populations.*

Table IV contains the comparative data. Again certain very definite conclusions can be drawn.

To begin with the correlations of the indices of the autumn queens are seen to be low and varying in sign; in these characteristics they resemble more closely the nest queens than the spring queens. The conclusion therefore of our first study that the correlation between the proportions of different parts of the wings

---

* See pp. 53—8 of our second study.

† It is conceivable that the conditions of hibernation may vary the distribution of pigmentation at the end of the cell *b*, and that less concentration of pigmentation may render the position selected for the tangent (see Fig. II, p. 3) more variable.

TABLE III.

Coefficients of Variation of Characters for Nest and Population Queens.

	SPRING POPULATION QUEEN		AUTUMN POPULATION QUEEN		NEST QUEEN		RATIO Aut. *P. Q.*/*N. Q.*		RATIO Sp. *P. Q.*/*N. Q.*		RATIO Aut. *P. Q.*/Sp. *P. Q.*	
	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
A	2·36 ± ·08	2·39 ± ·08	2·52 ± ·09	2·53 ± ·09	1·05 ± ·04	1·09 ± ·05	2·40	2·32	2·25	2·19	1·07	1·06
B	2·70 ± ·09	2·59 ± ·09	2·21 ± ·07	2·11 ± ·07	1·36 ± ·07	1·46 ± ·06	1·63	1·45	1·99	1·77	·82	·81
C	2·66 ± ·09	2·72 ± ·09	2·87 ± ·10	2·79 ± ·09	1·05 ± ·04	1·10 ± ·05	2·73	2·54	2·53	2·47	1·08	1·03
D	3·41 ± ·12	3·64 ± ·13	3·95 ± ·13	4·27 ± ·14	1·31 ± ·06	1·32 ± ·06	3·02	3·23	2·60	2·76	1·16	1·17
E	3·65 ± ·13	3·80 ± ·13	5·67 ± ·19	5·73 ± ·19	2·13 ± ·09	1·68 ± ·07	2·66	3·41	1·71	2·26	1·55	1·51
F	3·27 ± ·11	3·43 ± ·12	3·69 ± ·12	3·63 ± ·12	1·45 ± ·06	1·46 ± ·06	2·54	2·49	2·26	2·35	1·13	1·06
G	3·50 ± ·12	3·73 ± ·13	3·48 ± ·12	3·96 ± ·13	2·65 ± ·11	2·70 ± ·11	1·31	1·47	1·32	1·38	·99	1·06
Mean	3·08	3·19	3·48	3·57	1·57	1·54	2·33	2·42	2·09	2·17	1·11	1·10
H	3·07 ± ·11	3·09 ± ·11	3·39 ± ·16	3·46 ± ·17	1·12 ± ·05	1·07 ± ·05	3·03	3·23	2·74	2·89	1·10	1·12
I	2·78 ± ·10	2·88 ± ·10	3·98 ± ·19	3·63 ± ·17	1·72 ± ·07	1·43 ± ·06	2·31	2·54	1·62	2·01	1·43	1·26
J	2·83 ± ·10	2·47 ± ·09	3·01 ± ·14	3·21 ± ·15	2·01 ± ·08	2·04 ± ·09	1·50	1·57	1·41	1·21	1·06	1·30
K	1·04 ± ·04	1·04 ± ·04	1·41 ± ·07	1·76 ± ·08	0·72 ± ·03	0·85 ± ·04	1·96	2·07	1·44	1·22	1·36	1·69
L	1·13 ± ·04	1·15 ± ·04	1·31 ± ·06	1·19 ± ·06	0·60 ± ·03	0·61 ± ·02	2·18	1·95	1·88	1·88	1·16	1·03
M	2·12 ± ·07	2·10 ± ·07	2·18 ± ·10	2·02 ± ·10	1·54 ± ·06	1·47 ± ·06	1·42	1·37	1·38	1·42	1·03	·93
Mean	2·17	2·12	2·55	2·55	1·28	1·25	2·07	2·12	1·75	1·77	1·19	1·22

TABLE IV.

Coefficients of Correlation for Characters of Nest and Population Queens.

	SPRING POPULATION QUEEN		AUTUMN POPULATION QUEEN		NEST QUEEN		RATIO Aut. *P. Q.*/*N. Q.*		RATIO Sp. *P. Q.*/*N. Q.*		RATIO Aut. *P. Q.*/Sp. *P. Q.*	
	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
A and *D*	·478 ± ·038	·579 ± ·033	·502 ± ·050	·580 ± ·045	·558 ± ·040	·619 ± ·037	·90	·94	·86	·94	1·05	1·00
A and *C*	·917 ± ·008	·894 ± ·010	894 ± ·014	·903 ± ·013	·838 ± ·018	·857 ± ·016	1·07	1·05	1·09	1·04	·97	1·01
A and *E*	·531 ± ·035	·647 ± ·029	·651 ± ·039	·722 ± ·032	·238 ± ·056	·259 ± ·053	2·74	2·79	2·23	2·50	1·23	1·12
G and *F*	·413 ± ·041	·396 ± ·042	·299 ± ·061	·327 ± ·060	·164 ± ·058	·051 ± ·059	1·82	6·41	2·52	7·76	·72	·83
G and *D*	·672 ± ·027	·781 ± ·019	·711 ± ·033	·694 ± ·035	·677 ± ·032	·694 ± ·031	1·05	1·00	·99	1·13	1·06	·89
B and *E*	·506 ± ·037	·617 ± ·031	·418 ± ·056	·465 ± ·053	·233 ± ·056	·280 ± ·055	1·79	1·66	2·17	2·20	·83	·75
B and *F*	·677 ± ·027	·671 ± ·027	·623 ± ·041	·578 ± ·045	·180 ± ·057	·139 ± ·058	3·46	4·16	3·76	4·83	·92	·86
Mean	·599	·655	·585	·610	·413	·414	1·83	2·57	1·95	2·92	·97	·92
I and *K*	·123 ± ·048	·054 ± ·049	− ·280 ± ·062	− ·402 ± ·057	− ·021 ± ·059	·037 ± ·059	—	—	—	—	—	—
M and *J*	·050 ± ·049	·137 ± ·048	·187 ± ·065	− ·029 ± ·067	− ·138 ± ·058	− ·185 ± ·057	—	—	—	—	—	—
L and *J*	·097 ± ·049	·132 ± ·048	·180 ± ·065	·324 ± ·060	− ·367 ± ·051	− ·186 ± ·057	—	—	—	—	—	—
I and *L*	·137 ± ·048	·203 ± ·047	·102 ± ·067	·195 ± ·065	·039 ± ·059	·194 ± ·057	—	—	—	—	—	—
Mean	·102	·131	·047	·022	− ·122	− ·035	—	—	—	—	—	—

was of small importance, seems to receive further justification*. If we turn to the correlations of absolute sizes, these appear on the average to be slightly smaller for the autumn queens than for the spring queens. Now the effect of reducing variability, i.e. direct selection of variability, is to reduce correlation. Hence if the sole difference of the autumn and spring queens were a rather stringent selection round type of the former to provide the latter, we should have anticipated a reduced correlation for the spring queens. Instead of this we find a slightly larger correlation. The only reasonable assumption to make is that there has been a *direct* selection of correlation as well as a selection round type. In other words, the nature of the differential selection of queens to survive the winter depends not only on the wings of those queens differing not widely from type, but on the existence of a closer relationship between the dimensions of the parts of the wing. The selection that has taken place tends not only to approximate the wing closer to type, but to leave its parts more closely correlated. We have thus additional evidence that natural selection is really at work in causing sensible differences in the wings of spring and autumn queens.

(7) *Comparison of Correlations between Right and Left Wings in the three Series.* (Table V.)

We notice at once that the correlations of the spring and autumn queens are sensibly higher (28 °/o to 29 °/o) than those of the nest queens; in fact in only one case—that of the anomalous cell *b* for the autumn queens—is the correlation less, and then not sensibly less, in the nest queens. This result is exactly what we should anticipate—as shewn in our second study (p. 62)—if we look upon the nest queens as a selection of decreased variability from the general population.

But if we look upon the spring queens as a selection from the autumn queens—as a group somewhat stringently selected round type as we have shewn earlier in this paper—we should solely on this ground expect a reduced correlation in the spring queens. This expectation is, however, not fulfilled, but the left and right wings of the two series of queens are on the average equally correlated. We believe this result is due to a compensating principle, i.e. not only does natural selection acting between autumn and spring reduce wing variability by concentrating on type, but there is a direct selection of proportionality in wing characters. The former selection tends to lower, the latter to raise the correlations of the spring

---

* It should be noted that there is nothing surprising about this result if we admit a high correlation of the absolute measurements. Let $i_1 = x_1/y_1$ and $i_2 = x_2/y_2$ be two indices, then their correlation is given by

$$r_{i_1 i_2} = \frac{v_{x_1} v_{x_2} r_{x_1 x_2} + v_{y_1} v_{y_2} r_{y_1 y_2} - v_{x_1} v_{y_2} r_{x_1 y_2} - v_{x_2} v_{y_1} r_{x_2 y_1}}{\sqrt{v_{x_1}^2 + v_{y_1}^2 - 2 v_{x_1} v_{y_1} r_{x_1 y_1}} \sqrt{v_{x_2}^2 + v_{y_2}^2 - 2 v_{x_2} v_{y_2} r_{x_2 y_2}}} .$$

Now in our cases the coefficients of variation v for the absolute characters do not differ very widely from each other, and the coefficients of correlation differ from ·6 by small quantities ϵ say. Thus *roughly*:

$$r_{i_1 i_2} = \frac{\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4}{2\sqrt{\cdot 4 - \epsilon_5}\sqrt{\cdot 4 - \epsilon_6}} = \frac{\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4}{\cdot 8} .$$

Hence if the ϵ's are small this is a small quantity and we might anticipate small order correlations for the indices.

TABLE V. *Correlations of Right and Left Wings in Nest and Population Queens.*

	Spring Population Queen	Autumn Population Queen	Nest Queen	Ratio Aut. *P. Q.*/*N. Q.*	Ratio Sp. *P. Q.*/*N. Q.*	Ratio Aut. *P. Q.*/Sp. *P. Q.*
A	·954 ± ·004	·989 ± ·001	·893 ± ·012	1·11	1·07	1·04
B	·911 ± ·008	·824 ± ·022	·831 ± ·018	·99	1·10	·91
C	·964 ± ·004	·977 ± ·003	·858 ± ·016	1·14	1·12	1·01
D	·862 ± ·013	·883 ± ·015	·600 ± ·038	1·47	1·44	1·02
E	·694 ± ·026	·762 ± ·028	·428 ± ·049	1·78	1·62	1·10
F	·932 ± ·006	·905 ± ·012	·605 ± ·038	1·50	1·54	·97
G	·856 ± ·013	·796 ± ·025	·787 ± ·023	1·01	1·09	·93
Mean	·882	·877	·714	1·29	1·28	1·00

queens. In the case of correlations of parts of the same wing, the raising process is apparently more emphasised than in the case of homologous parts of the two wings. But the equality of correlations in these parts before and after selection, where we should expect a reduction, strongly suggests the same process as we have indicated in the correlation of parts of the same wing, i.e. the existence of an autumn to spring selection which tends to strengthen the proportionality of the parts of the wings, a feature as important as selection about type. That such a selection really exists is evidenced by the manner in which the variability of the indices of the wing parts has been reduced by the autumn to spring selection (see Table III).

(8) *Conclusions.*

An examination of autumn queens indicates that:

(i) The chief result flowing from our second study, i.e. the remarkable reduction of variability within the nest, has no relation to the action of natural selection on the queens surviving to spring.

(ii) There is some evidence to suggest that it is a result of heredity and not of nest environment.

(iii) There is strong evidence to indicate that natural selection is actually at work reducing the deviations from type and increasing the proportionality of the parts of the wings of queen wasps between autumn and spring.

We are fully aware that the importance of the latter conclusion is so great that it needs ampler treatment, but we have further material in hand and hope to publish within a reasonable time reductions for larger numbers. The chief difficulty is the collection in autumn of a sufficient population of queens from different nests within a reasonably circumscribed area*.

---

* We are already in possession of many thousand spring queens. But 1*s*. is not an unreasonable price to pay for the taking without damage of a complete nest in the autumn, and at least 500 nests have to be taken from one district and in one season to obtain a really adequate sample of autumn queens.

PIGMENTATION OF THE HAIR AND EYES OF CHILDREN SUFFERING FROM THE ACUTE FEVERS, ITS EFFECT ON SUSCEPTIBILITY, RECUPERATIVE POWER AND RACE SELECTION.

By DAVID MACDONALD, M.B., Ch.B.

(1) *Introduction.* During the last half century considerable interest has been taken in and observations made upon the pigmentation of the hair and eyes in man, primarily with regard to the geographical distribution and race elements of the various colour classes.

Shortly after the close of the Franco-Prussian war, Virchow, during a controversy on the racial elements in Germany, induced the German government to authorize an official census of the colours of the hair and eyes of school children throughout the Empire. This seems to have been the first pigmentation survey attempted on a large scale. Later observers have, with modifications, adopted Virchow's methods. In course of time there followed pigmentation surveys of school children in most of the continental countries, and statistics of great extent and value have been accumulated and published. Prior to Virchow, Beddoe had been making personal observations and collecting data from all over the continent. It was not until 1885, however, that he published his great work* on the colour of the hair and eyes, based on personal observations on the adult populations in several districts of the United Kingdom and on the Continent. In connection with these surveys other names might well be mentioned, as Guillame in Switzerland and Ranke in Germany, but the extent of interest taken in the pigmentation of the hair and eyes has been sufficiently indicated.

Pigmentation, however, has been found to have a further importance than merely the separation of race elements. It has been associated with vitality under different conditions and also with disease. Darwin† states that complexion and liability to certain diseases are believed to run together in man and the lower animals. Thus, white terriers suffer more than terriers of any other colour from

---

* Beddoe: *The Races of Britain.*

† Darwin: *Variation of Animals under Domestication,* Vol. II. p. 227.

the fatal distemper. In Virginia white pigs perish from eating certain roots which dark pigs can eat without injury. White chickens are more subject than dark coloured chickens to certain parasitic worms. In cattle susceptibility to the attack of flies is correlated with colour, as also is the liability to be poisoned by certain plants, the white varieties suffering most severely[*]. Certain forms of blindness are said to be associated with the colour of the hair. Thus Darwin cites the case of a man with black hair and a woman with light coloured hair, both of sound constitution, who married, and had nine children, all of whom were born blind: of these children, five with dark hair and brown iris were afflicted with amaurosis: the four others, with light coloured hair and blue iris, had amaurosis and cataract combined[†].

Then again the tubercular child is loosely recognized by certain features, including the quality of the skin, the brightness of the eyes and the length and pigmentation of the eyelashes.

Beddoe states that phthisis and cancer are more prevalent among dark-haired persons[‡]. Tocher has shown[§] that on an average more persons become insane in parts of Scotland where there is an excess of light-eyed persons in the population, and in a much less degree where there is an excess of dark-haired persons. Lunacy, he states, is distinctly correlated positively to light eyes and in a much less degree to dark hair, and is distinctly correlated negatively to red hair, and in a less degree to dark eyes. There is thus a greater tendency to insanity among light-eyed and dark-haired persons, and a lesser tendency to insanity among red-haired and dark-eyed persons, compared in both cases with the general population. He goes on to say that these are only statistical facts, and offers no explanation as to how or why presence or absence of pigment comes to be associated with insanity. When he extended his inquiry[||] to discover whether the excess of any particular hair or eye colour is associated with physical or mental defects such as blindness, deafness and imbecility, he found that the distribution of cases of mental affection differs from that of the last three classes, namely, that excesses in the number of cases of imbecility, blindness and deafness, occur in the region of excesses of blue eyes and of dark and jet black hair.

It is well known that in the lower animals pigmentation has an important function. It seems, especially in the organs of sense, to be essential to their full development. Albinos in all species are apt to be defective in keenness of sense. Darwin gives numerous examples of the defective senses of such non-pigmented animals. White cats with blue eyes, he states, are almost always deaf. He cites a remarkable case, in which the iris at the end of four months began to grow dark coloured, and then for the first time the cat began to hear[¶]. Histological exami-

---

[*] Darwin: *Origin of Species*, p. 236.

[†] Darwin: *Variation of Animals under Domestication*, Vol. II. p. 328.

[‡] Beddoe: *Races of Britain*, p. 224.

[§] Tocher: "Anthropometry of Scottish Insane." *Biometrika*, Vol. v. p. 344.

[||] Tocher: "Pigmentation Survey of School Children in Scotland." *Biometrika*, Vol. VI. p. 198.

[¶] Darwin: *Variation of Animals under Domestication*, Vol. II. p. 329.

nations have shown that among other defects in the internal ear of such cats the walls of the perilymph chamber lack pigmentation. In a recent histological examination* of a case of albinism, in a child aged ten weeks, no abnormality was found save a complete absence of pigment in the brain, eyes, internal ear, suprarenal gland, skin and hair. Commenting on this case, Pearson† refers to an interesting suggestion. He remarks that the disappearance of superficial pigment is one of the marks of senility, that it affects the hair and eyes alike. Is it possible, he asks, that this loss of superficial pigment can ever be accompanied by a loss of internal pigment, possibly in the case of the brain centres? Senile imbecility, and in its milder form senility, might possibly be associated with a weakening of the intensity of pigmentation in certain of the brain centres. Is it conceivable that any forms of imbecility are associated with defective brain pigmentation? Ophthalmoscopic investigation shows a high percentage of incomplete albinism of the eye in the insane. Pearson also suggests that the lack of pigmentation in the internal ear as shown in the albinotic cat might possibly be associated with deafness.

This association of pigmentation with certain classes of disease seems not unreasonable when it is considered that the ectoderm gives rise not merely to the hair and epidermis, but also to the whole central and peripheral nervous system and other important structures.

The endogenous pigments of the body may be divided into two groups, haemoglobin and its derivatives and other metabolic pigments. The former is outside the scope of this investigation; of the latter the most important group is the melanins. These are dark, black or brown and reddish brown pigments, and exist normally in the hairs, the choroid coat of the eye, the deeper cells of the malpighian layer of the skin, in the chromatophores of the upper layers of the corium and also in the membranes of the brain, especially in the neighbourhood of the choroid plexus. The normal production of melanin in members of the human family has its extremes represented by the fair-haired Saxon and the swarthy negro. The differences in colour here are due, not to the presence or absence of the cells themselves, but to variations in the amount of pigment therein deposited. In this respect, therefore, the negro differs physiologically rather than anatomically from the European.

A physiological increase in the pigmentation is observed also in pregnant women, even among the fair, and is most marked in brunettes; a similar pigmentation is observed in many cases of exophthalmic goitre and in certain neurotic states. What is regarded as a pigmentation of the same order is met with in Addison's disease. This condition is generally held to be an affection of the abdominal sympathetic system, induced most commonly by disease of the suprarenal bodies. The medulla of the suprarenal body is produced by an ingrowth of cell groups

---

* J. E. Adler and J. MacIntosh: "Histological Examination of a Case of Albinism." *Biometrika*, Vol. VII. p. 237.

† Pearson: "Note on Internal Albinism." *Biometrika*, Vol. VII. p. 246.

derived from the sympathetic system: these cell groups are derived from the ectoderm. It would seem that in general there is some close connection between pigmentation and the ectoderm, and although pigment is found in certain mesoblastic structures, such as the choroid and stroma of the iris in the eye, the cells of the perilymphatic spaces in the internal ear and the cortex of the suprarenal body, yet even these are closely associated with the ectoderm, and it might ultimately be found that the pigmented cells are ectodermal in origin. The case of complete albinism already referred to is open to this interpretation; certainly the majority of these normally pigment-containing structures are ectodermal in origin, and it might be concluded that the ectodermal derivatives are the chief pigment-producing structures in the body. Tocher's findings regarding the relation of pigmentation to insanity suggest an etiological association in this connection, the hair and brain both being ectodermal structures.

From general observation it must be concluded that there is some association between the colour of the hair and the type of skin in the individual. The fresh complexion, easily freckled, met with in red-haired, and the olive and more sallow skin occurring in dark-haired persons, are well known. If we have this association between the colour of the hair and the skin, one of the chief excretory organs of the body, it might reasonably be expected that some definite relationship could be found between pigmentation and the ability to withstand disease. Beddoe found that, in his experience as a physician, on the whole, dark-complexioned children showed more tenacity of life than fair ones*.

The same author remarks† that pale shades in the hair, and in the eyes also, are the results of a defect of secretion, but that it does not necessarily follow that they are a mark of weakness. Several facts, however, might point in that direction, such as the physical and constitutional inferiority of albinos, the comparative lightness of the hair of children and the changes which take place in disease, generally in the direction of dullness and paleness of hue.

Brownlee‡ found that the Jewish children in Glasgow exhibited a considerable susceptibility to scarlet fever and diphtheria, associated with a high degree of recuperative power.

Ripley§, commenting on the large proportion of dark-haired and dark-eyed persons in the continental towns, states that it is not improbable that there is in the dark hair and eye some indication of vital superiority, for it requires energy and courage, physical as well as mental, not only to break the ties of home, but also to maintain one's self afterwards under the stress of city life. After discussing the defects of sense in albinos he goes on to say that these facts tend to justify the conclusion that pigmentation, if not absolutely necessary, at least conduces to acuteness of sense, and that when abundantly present is often

---

* Beddoe: *The Races of Britain*, p. 223.

† Beddoe: *Ibid.*, p. 224.

‡ Brownlee: *Report of the City of Glasgow Fever and Smallpox Hospitals, Belvidere*, 1906.

§ Ripley: *The Races of Europe*, p. 557.

an index of vitality. He sums up by stating that the tendency of present knowledge certainly points in the direction of some relation between pigmentation and general physiological and mental vigour. On the other hand, it cannot be forgotten that many conquering and ruling races have been fair, and have subdued races which have been dark. To mention only a few, the Goths, Angles, Norsemen and Saxons, were all dominant races in their time.

From the observation of a million soldiers of mixed nationalities in the American Federal Army, Baxter* formed the opinion that although nations of superior stature exhibit a majority of blondes, yet in detail among themselves the dark-complexioned exhibit a slight superiority in stature and girth of chest over the fair complexioned. He concludes that stature depends on race and not upon complexion, and that it does not appear that any recognizable relation exists between circumference of chest and stature when complexion is made the basis of classification. On the other hand†, when the various and numerous diseases and injuries for which recruits were rejected are considered in regard to complexion, he finds that almost without exception men of light complexion were more affected than those of dark. He states that, in regard to this almost invariable rule applying to complexion, the fact is submitted without comment. Ripley‡ gives Baxter as his authority for stating that the brunette type, on the whole, opposed a greater resistance to disease and offered more hope of recovery from injuries in the field. Boudin§ states that, in the French army which invaded Russia, soldiers having a dark complexion, from the southern parts of Europe, withstood the intense cold better than those with lighter complexions from the north. He remarks that this fact is contrary to the opinion generally held.

Darwin|| states that the colour of the skin and hair is sometimes correlated in a surprising manner with a complete immunity from the action of certain vegetable poisons and from the attacks of certain parasites. After discussing the immunity of the negro from the yellow fever so destructive in tropical America, and from the fatal intermittent fevers that prevail in parts of the African coast, he says that it is a mere conjecture that this immunity is in any degree correlated with the colour of the skin. The conjecture, however, seemed to him not improbable, and he obtained permission to transmit tables to the surgeons of the various regiments on foreign service asking for particulars of the colour of the hair of all the men in their regiments and also of those who suffered from the various tropical fevers. In this way he hoped to find out whether any relation existed between the colour of the hair and constitutional liability to tropical diseases. Unfortunately he received no returns, and at present there exists great divergence of opinion as to whether

---

* Baxter: *Statistics, Medical and Anthropological, of the Provost Marshal General's Bureau*, Washington, 1875, Vol. II. p. 24.

† Baxter: *Ibid.*, Vol. I. p. 72.

‡ Ripley: *Races of Europe*, p. 558. The latter statement I cannot find in the pages referred to by Ripley, nor indeed in any of Baxter's records.

§ Boudin: *Traité de Géographie Médicale*, Tom. I. p. 406.

|| Darwin: *Descent of Man*, p. 193.

light-haired persons with florid complexions or dark-haired persons with sallow complexions suffer less from the diseases of tropical countries.

With a view to ascertaining how far any of these statements or suggestions could be justified in the case of some of the acute fevers, and also to find if any one pigmentation type was more liable to one or any of these fevers than another, the present inquiry was begun. This inquiry was made possible by comparison by the publication of a pigmentation survey of school children in Scotland in 1908*.

(2) *Methods of Observation.* Four of the acute fevers, scarlet fever, diphtheria, measles and whooping cough, have been chosen on which to make these observations. The material to work upon has been collected in the Glasgow Corporation Fever Hospitals, mainly at Ruchill during the past year, 1909—10, and for a few months previously at Belvidere. The colours of the hair and eyes of children suffering from these diseases were carefully noted on admission to hospital by the physician under whose charge they came. These cases in every instance were consecutive, and every case entering hospital was recorded except for a short period during which the observations were interrupted by sickness on the medical staff. No selection was made. Tocher's analytical tables for hair and eye colours were closely followed. (See *Biometrika*, Vol. VI. pp. 133 and 134.)

These tables are probably as accurate as can be verbally devised, but nevertheless they are defective in some ways. In the various colour classes the bulk of cases can be easily placed, but there are others which are more difficult. There is no method of distinguishing mixed colours, fair hair tending to become medium or medium hair tending to dark. Eye colour is more easily distinguished, but considerable difficulty has been found in determining the true blue. Taking the pure deep blue as representing this class in accordance with Tocher's table, it is found that only 5·3 °/₀ of the hospital cases have blue eyes, compared with 11·2 °/₀ of blue eyes in Tocher's statistics for the children inhabiting the area from which the hospital cases were drawn. This suggests some difference in the personal equation of the observers. Beddoe† remarks on the difficulty of determining the limits of the blue eye. Tocher's pigmentation survey takes Glasgow as a whole, but also divides it into sub-districts. Belvidere and Ruchill Hospitals drain different districts. This might have led to difficulties of comparison, but it was found that these districts corresponded closely to Tocher's sub-divisions.

It will be seen from the table below, constructed from Tocher's statistics for the Glasgow sub-divisions, that there is very little difference in the percentages of hair and eye colours between the Belvidere and Ruchill areas. Taking this fact into consideration, these areas are for convenience sometimes considered separately and sometimes combined in this inquiry. From the same table it will be seen also that the percentages of hair and eye colours of boys and girls taken as separate units show only a slight difference.

---

* J. F. Tocher: "Pigmentation Survey of School Children in Scotland." *Biometrika*, Vol. VI. p. 129.

† Beddoe: *The Races of Britain*, p. 77.

TABLES I *a* AND I *b*. *Showing the Percentages of the various Hair and Eye Colours of the School Children in the Areas drained by Belvidere and Ruchill Hospitals.*

(*a*) Hair Colour.

	Black	Dark	Medium	Fair	Red
Ruchill Area, Boys	0·9	25·4	46·5	21·7	5·6
,, ,, Girls	1·2	28·0	44·1	21·6	5·0
,, ,, Boys and Girls combined	1·1	26·7	45·3	21·7	5·3
Belvidere Area, Boys	1·3	26·8	45·5	21·5	5·0
,, ,, Girls	1·2	28·6	44·8	20·8	4·6
,, ,, Boys and Girls combined	1·3	27·7	45·1	21·1	4·8
Belvidere and Ruchill Areas combined (Boys and Girls)	1·2	27·2	45·2	21·4	5·0

(*b*) Eye Colour.

	Dark	Medium	Light	Blue
Ruchill Area, Boys	24·2	32·5	31·6	11·7
,, ,, Girls	24·6	32·3	30·8	12·2
,, ,, Boys and Girls combined	24·4	32·4	31·2	12·0
Belvidere Area, Boys	24·9	35·5	29·6	10·1
,, ,, Girls	24·3	35·5	29·4	10·9
,, ,, Boys and Girls combined	24·6	35·5	29·5	10·5
Belvidere and Ruchill Areas combined (Boys and Girls)	24·5	34·0	30·3	11·2

Tocher* states that on an average any excess or defect in the boy population from the general mean in any locality is accompanied in about 70—90 °/$_o$ of the cases by a corresponding excess or defect in the girl population and vice-versa. Such being the case, boys and girls have been combined in this inquiry. In scarlet fever alone are the boys and girls treated as separate units.

In all, 3535 observations have been made, namely: 1864 on scarlet fever patients, 700 on diphtheria patients, 661 on measles patients and 310 on whooping cough patients.

On the Possibility of Comparing the Hospital Results with those of Tocher. In comparing the results obtained in hospital with those of Tocher, a difficulty is met with in regard to the age difference of the subjects of these investigations. In hospital the majority of the children on whom observations were made, were between the ages of 2 and 12. Tocher gives the age of the school children considered in his survey as between 6 and 18, but the bulk are between 6 and 12 years. The difference in age is considerable, and might be thought to give rise to inaccuracy when comparing the results of these different observations, but authority can be given for considering the age difference as of little importance and liable

---

* Tocher: "Pigmentation Survey of School Children in Scotland." *Biometrika*, Vol. VI. p. 199.

at the most to give rise to only a very small error, where the comparison is based on the pigmentation of the hair and eyes.

Ripley* states that a great defect in all such investigations on children lies in the tendency to a darkening of the hair and eyes with growth; that from 10°/₀—20°/₀ of blonde children at maturity develop darker hair or eyes. Beddoe† remarks, however, that the broad results are not affected by this flaw. He gives statistics of school children under and over 11 years of age, showing a very slight augmentation of the darker colours in the older children. Pearson‡, collating statistics, concludes that there is little change in hair or eye colour in children under 14, but after 14 there is even before 19 a more marked change, the correlations still, however, remaining low. This change is much more considerable in the case of hair than of eye colour, though sensible in both. Commenting on Dr Pfitzner's results on the pigmentation of the hair and eyes of children in Lower Elsass, he states that they are quite comprehensible if there be a positive correlation between fairness and disease in childhood, and that this is exactly what British school children show—*there is a correlation between health and darkness of hair colour.* "Hence if we do not follow up individuals, noting their pigmentation at different ages, but simply correlate age of different individuals with hair colour, we are liable to exaggerate the correlation between age and pigmentation, and this will be especially the case if we use hospital returns. Hence it is probable that our neglect of a selective death rate, based upon the known correlation between general health and pigmentation, really emphasises the values found for correlation between pigmentation and age. Further, while it is probable that if we take adult life into account we should find this correlation increased, the value deduced from Pfitzner's observations may be safely considered to mark in the first place a selective death rate, i.e. a correlation between fitness *in childhood* and dark pigmentation."

In conclusion Pearson, commenting on a table exhibiting the results for age and hair and eye colour of British school girls from 7—19 years of age, states that "it would seem doubtful, having regard to the paucity of individuals dealt with, whether we can assert significant changes in the percentages of medium-eyed girls having fair or brown hair at different ages. Nor would it be wise to insist that the changes of percentages in red-haired girls with light or dark eyes are significant. Red-haired girls with medium eyes seem to become continuously fewer with age: light-eyed girls with fair hair become significantly fewer, and brown-haired girls with light eyes more numerous. Dark-eyed girls with fair or brown hair become significantly fewer and dark-eyed girls with dark hair become more numerous, and probably light-eyed girls with dark hair also. The medium-eyed girls with dark hair remain much the same in percentage. Thus, except in the case of red-haired girls, those with medium eye colour change least; the fair-haired girls with

---

* Ripley: *The Races of Europe*, p. 222.

† Beddoe: *The Races of Britain*, p. 77.

‡ Pearson: "Miscellanea." *Biometrika*, Vol. III. pp. 464, 465.

light eyes tend to become brown or even dark, and the dark-eyed girls with fair or brown hair to become dark-haired. How far these changes are influenced by a selective death rate still remains to be determined."

It would therefore be expected that, if there is any discrepancy from the age difference in comparing the results of this inquiry with those of Tocher's survey of school children, a proportion of fair and possibly medium dark takes the place of medium and dark at the more advanced age. An appreciable but not large number of the children observed in hospital are considerably younger than the children on whom Beddoe and Pearson based their results as given above. All the evidence, however, tends to show that the colour of the hair and of the eyes to a lesser degree darkens with age, in which case a slight excess of fair-haired children should be found entering hospital, when compared with Tocher's statistics for school children, unless such a result be upset by some correlation between the colour of the hair and susceptibility to the diseases considered. This excess is certainly present, except in scarlet fever, but it is not great, the percentage of fair-haired children varying from 19·3 °/$_0$ in scarlet fever to 27·8 °/$_0$ in measles as compared with Tocher's figure for school children for the same area of 21·4 °/$_0$. How much of this discrepancy is accounted for by the age difference, and how much by some correlation between the colour of the hair and susceptibility to the diseases considered cannot be definitely stated. As will be shown later, however, there is a distinct negative correlation between dark hair and susceptibility, and there is no reason why the excess of fair-haired children entering hospital should not be due as much to some such correlation between fair hair and susceptibility as to any error from age difference.

(3) *The Incidence of the Diseases considered in the different pigmentation Types.* It was thought that, by comparing the percentages of the various hair and eye colours of children suffering from scarlet fever, diphtheria, measles and whooping cough with the percentages of the various hair and eye colours given by Tocher for the areas corresponding to the areas from which these children were drawn, some definite result might be obtained, indicating the pigmentation type most liable to suffer from any of these diseases.

As will be seen from Table II there is, with regard to the colour of both hair and eyes, a striking similarity between the percentages of each colour attacked by each of the fevers considered, and these percentages differ considerably from Tocher's percentages of the general population for the same area.

With regard to the pigmentation of the hair, in every case the medium is considerably in excess, the dark deficient and the fair and red about equal when compared to their proportional representation in the general population. The colour of the eyes shows a similar result, the medium considerably in excess, the dark and blue deficient, the light being the only variable colour, being in excess in diphtheria and whooping cough and deficient in scarlet fever and measles as compared with the general population.

TABLES II *a* AND II *b*. *Showing the Percentages of the various Hair and Eye Colours in the Diseases considered as compared with the General Population.*

(*a*) Hair Colour.

	Black	Dark	Medium	Fair	Red
General population ...	1·2	27·2	45·2	21·4	5·0
Scarlet Fever	0·6	15·5	59·4	19·3	5·0
Diphtheria	0·1	13·9	57·7	24·1	4·2
Measles	0·0	12·9	55·5	27·8	3·8
Whooping Cough	0·6	11·3	61·3	23·6	3·2

(*b*) Eye Colour.

	Dark	Medium	Light	Blue
General population ...	24·5	34·0	30·3	11·2
Scarlet Fever	15·0	62·0	17·8	5·2
Diphtheria	15·2	44·4	35·3	5·1
Measles	14·5	60·2	22·1	3·2
Whooping Cough	15·2	38·1	35·8	11·0

Scarlet Fever. In considering the scarlet fever cases, as elsewhere, boys and girls and Belvidere and Ruchill patients have been grouped together, and compared with Tocher's statistics for the combined areas. In addition the boys and girls in the Ruchill cases have been separated and compared with Tocher's statistics for the boys and girls in the Ruchill area. This accounts for the slight difference in the theoretical* percentages.

From Table III it appears that whether the boys and girls be treated as separate units or combined, the results are similar. Both combined and individually medium hair is considerably in excess, dark and jet black considerably and fair hair slightly deficient and red hair about equal when compared to their proportional representation in the general population. The greater percentage of medium and the smaller percentage of dark-haired cases in hospital, in boys as compared with girls, has a corresponding difference in the general population. It would seem then that the medium-haired child is susceptible to scarlet fever to a greater extent than, the dark and jet black-haired to a less extent than, and the red and fair-haired child almost equally with its proportional representation in the general population.

---

* The term "theoretical percentage" refers to the percentage given in Tocher's statistics for the same area. The term "theoretical numbers" refers to the numbers which would occur if the cases were admitted to hospital in the same percentage of each colour as in Tocher's statistics for the same area.

TABLE III. *Hair Colour. Showing the Percentages and Numbers of the various Hair Colours of Children suffering from Scarlet Fever, as compared with the General Population.*

	Black	Dark	Medium	Fair	Red	Totals
1. Belvidere and Ruchill (combined):						
Actual numbers	12	289	1109	360	94	1864
Theoretical numbers	22	507	842	399	94	1864
Actual percentage	0·6	15·5	59·4	19·3	5·0	100
Theoretical percentage	1·2	27·2	45·2	21·4	5·0	100
2. Ruchill, *Girls:*						
Actual numbers	3	137	389	138	38	705
Theoretical numbers	9	198	311	152	35	705
Actual percentage	0·4	19·5	55·2	19·6	5·4	100
Theoretical percentage	1·2	28·0	44·1	21·6	5·0	100
3. Ruchill, *Boys:*						
Actual numbers	5	77	360	113	35	590
Theoretical numbers	5	150	274	128	33	590
Actual percentage	0·8	13·1	61·0	19·2	5·9	100
Theoretical percentage	0·9	25·4	46·5	21·7	5·6	100

TABLE IV. *Eye Colour. Showing the Percentages and Numbers of the various Eye Colours of Children suffering from Scarlet Fever, as compared with the General Population.*

	Dark	Medium	Light	Blue	Totals
1. Belvidere and Ruchill (combined):					
Actual numbers	280	1156	331	97	1864
Theoretical numbers	457	634	564	209	1864
Actual percentage	15·0	62·0	17·8	5·2	100
Theoretical percentage	24·5	34·0	30·3	11·2	100
2. Ruchill, *Girls:*					
Actual numbers	90	400	70	30	590
Theoretical numbers	143	192	186	69	590
Actual percentage	15·3	67·8	11·9	5·0	100
Theoretical percentage	24·2	32·5	31·6	11·7	100
3. Ruchill, *Boys:*					
Actual numbers	123	433	117	32	705
Theoretical numbers	174	228	217	86	705
Actual percentage	17·5	61·4	16·6	4·5	100
Theoretical percentage	24·6	32·3	30·8	12·2	100

In scarlet fever whether boys and girls be considered individually or combined, the medium eye is very much in excess, the blue and light eye and to a less extent the dark eye considerably deficient, compared to their proportional representation in the general population.

This would indicate that the medium-eyed child is more susceptible to scarlet fever than the dark-eyed child and, even more so, than the blue and light-eyed child. The medium-eyed girl would seem to be more susceptible than the medium-eyed boy, the light-eyed girl being less susceptible to a corresponding degree.

Diphtheria.

TABLE V. *Hair Colour. Showing the Percentages and Numbers of the various Hair Colours of Children suffering from Diphtheria, as compared with the General Population.*

	Black	Dark	Medium	Fair	Red	Totals
Belvidere and Ruchill:						
Actual numbers	1	97	404	169	29	700
Theoretical numbers ...	8	191	316	150	35	700
Actual percentage ...	0·1	13·9	57·7	24·1	4·2	100
Theoretical percentage ...	1·2	27·2	45·2	21·4	5·0	100

In diphtheria medium hair is considerably and fair hair slightly in excess, red hair is slightly and dark and jet black hair considerably deficient, compared to their proportional representation in the general population. This would indicate that the medium-haired child is susceptible to diphtheria to a greater extent than, the dark and jet black-haired child to a less extent than, and the red and fair-haired child about equally with, its proportional representation in the general population.

TABLE VI. *Eye Colour. Showing the Percentages and Numbers of the various Eye Colours of Children suffering from Diphtheria as compared with the General Population.*

	Dark	Medium	Light	Blue	Totals
Belvidere and Ruchill:					
Actual numbers	106	311	247	36	700
Theoretical numbers ...	171	238	212	79	700
Actual percentage	15·2	44·4	35·3	5·1	100
Theoretical percentage ...	24·5	34·0	30·3	11·2	100

In diphtheria the medium eye and, to a less extent, the light eye are considerably in excess, the blue eye and, to a less extent, the dark eye considerably deficient, compared to their proportional representation in the general population. This would indicate that the medium-eyed child and, to a less extent, the light-eyed child is more susceptible to diphtheria than the dark-eyed child and, even more so, than the blue-eyed child.

Measles.

TABLE VII. *Hair Colour. Showing the Percentages and Numbers of the various Hair Colours of Children suffering from Measles, as compared with the General Population.*

	Black	Dark	Medium	Fair	Red	Totals
Ruchill only:						
Actual numbers ...	0	85	367	184	25	661
Theoretical numbers ...	7	176	299	144	35	661
Actual percentage ...	0·0	12·9	55·5	27·8	3·8	100
Theoretical percentage...	1·1	26·7	45·3	21·7	5·3	100

In measles medium hair and, to a less extent, fair hair, are considerably in excess, red hair and, even more so, dark and jet black hair considerably deficient, compared to their proportional representation in the general population.

This would indicate that the medium-haired child, and, to a less extent, the fair-haired child, is more susceptible to measles than the red-haired child and, even more so, than the dark and jet black-haired child.

TABLE VIII. *Eye Colour. Showing the Percentages and Numbers of the various Eye Colours of Children suffering from Measles, as compared with the General Population.*

	Dark	Medium	Light	Blue	Totals
Ruchill only :					
Actual numbers	96	398	146	21	661
Theoretical numbers ...	161	215	206	79	661
Actual percentage	14·5	60·2	22·1	3·2	100
Theoretical percentage ...	24·4	32·4	31·2	12·0	100

In measles the medium eye is considerably in excess, the blue eye and, to a less extent, the dark and light eye, considerably deficient, compared to their proportional representation in the general population. This would indicate that the medium-eyed child is much more susceptible to measles than the dark and light-eyed child and, even more so, than the blue-eyed child.

Whooping Cough.

TABLE IX. *Hair Colour. Showing the Percentages and Numbers of the various Hair Colours of Children suffering from Whooping Cough, as compared with the General Population.*

	Black	Dark	Medium	Fair	Red	Totals
Belvidere and Ruchill :						
Actual numbers	2	35	190	73	10	310
Theoretical numbers ...	4	84	140	66	16	310
Actual percentage	0·6	11·3	61·3	23·6	3·2	100
Theoretical percentage ...	1·2	27·2	45·2	21·4	5·0	100

In whooping cough medium hair is considerably and fair hair slightly in excess, red hair is considerably, and dark and jet black hair markedly, deficient, as compared to their proportional representation in the general population. This would indicate that the medium-haired child and, to a much less extent, the fair-haired child, is more susceptible to whooping cough than the red-haired child and much more so, than the dark and jet black-haired child.

TABLE X. *Eye Colour. Showing the Percentages and Numbers of the various Eye Colours of Children suffering from Whooping Cough, as compared with the General Population.*

	Dark	Medium	Light	Blue	Totals
Belvidere and Ruchill:					
Actual numbers	47	118	111	34	310
Theoretical numbers ...	76	105	94	35	310
Actual percentage	15·2	38·1	35·8	11·0	100
Theoretical percentage ...	24·5	34·0	30·3	11·2	100

In whooping cough the light eye and the medium eye are in excess but not to any marked extent, the blue eye is equal and the dark eye deficient when compared to their proportional representation in the general population. This would indicate that the light-eyed child and the medium-eyed child are more susceptible to whooping cough than the blue-eyed child and considerably more so than the dark-eyed child.

Thus a striking similarity is maintained throughout in these four fevers as regards the susceptibility of the various hair and eye colours.

Hair Colour. In every case there is a marked excess of medium hair and a marked deficiency of dark and jet black hair. In measles there is a considerable, and in diphtheria and whooping cough a slight excess of fair hair. In scarlet fever there is a slight deficiency of fair hair. Red hair is deficient except in scarlet fever, where it equals its proportional representation in the general population.

The conclusion would seem to be that in scarlet fever, diphtheria, measles and whooping cough, the medium-haired child is more liable to become infected than the red-haired and much more so than the dark and jet black-haired child, the fair-haired child occupying an intermediate position as regards infection.

Eye Colour. In every case there is an excess of medium eyes, not so marked in whooping cough, and a considerable deficiency in blue and dark eyes, except in whooping cough, where blue eyes are equal to their proportional representation in the general population. The light eye occupies an intermediate position, being in excess in whooping cough and diphtheria and deficient in scarlet fever and measles. The conclusion would seem to be that in scarlet fever, diphtheria, measles and whooping cough the medium-eyed child is more liable to become infected than the dark-eyed child and, even more so, than the blue-eyed child, the light-eyed child occupying an intermediate position as regards infection; the light-eyed child seems to be more susceptible to diphtheria and whooping cough than to scarlet fever and measles.

(4) *The Recuperative Power of the various pigmentation Types in the Diseases considered.* Throughout this section no attempt has been made to separate boys and girls or Belvidere and Ruchill cases.

To determine the recuperative power of the various pigmentation types in the diseases considered, the number of patients, the percentage of severe cases and the percentage of deaths in each type have been tabulated. The number of patients includes the severe cases and deaths. The percentage of severe cases includes the percentage of deaths. What have been termed the severe cases will require further definition. In scarlet fever, measles and whooping cough, severity is indicated by the incidence of complications. The complications are chiefly represented in scarlet fever by nephritis, arthritis, adenitis and otitis media, in measles by laryngitis, broncho-pneumonia, subsequent tuberculosis and otitis media, and in whooping cough by convulsions, broncho-pneumonia and subsequent tuberculosis. These complications indicate the severity of the attack. In diphtheria, however, there are so many cases with a toxaemia considerably above the average, which yet develop no actual complications, that it has been thought more accurate to consider the degree of toxaemia without reference to complications. In diphtheria, therefore, any case with marked toxaemia, that is, with a toxaemia more intense than the average, has been tabulated as severe.

1. *Recuperative Power and Pigmentation of the Hair.* During a two years' experience of children in fever hospitals and before any attempt had been made to arrive at any statistical proof on the subject, I had formed, more or less unconsciously, the opinion that the fair-haired child tended to be more severely attacked by and to succumb more readily to the acute fevers, and that the dark-haired child tended to be less severely attacked and offered more resistance to the disease. How far this impression is confirmed by fact will be seen from the tables below.

It appears that in each of the diseases considered the fair-haired children show the greatest percentage of severe cases and of deaths, and not only is this so, but the greater severity and higher mortality in fair-haired children is marked and constant. The only exception is the slightly greater percentage of severe cases of diphtheria in red and medium-haired children. Next, but not so pronounced, in order of severity and mortality comes the red-haired class. Only in the percentage of deaths in diphtheria, which is remarkably small, and of severe cases in whooping cough is this order changed. But the low death rate in diphtheria is fully made up by the large number of severe cases, and the small number of red-haired children with whooping cough renders the observation of little value. The severity and mortality in medium-haired children occupies a mean between the high severity and death rates among the fair-haired and, in a less degree, the red-haired children and the comparatively low severity and death rates among the

TABLE XI. *Hair Colour. Showing the Number of Patients, the Percentage of Severe Cases and the Percentage of Deaths in the various Hair Colours in the diseases considered.*

	Black	Dark	Medium	Fair	Red
Scarlet Fever:					
Number of cases	12	289	1109	360	94
Percentage of severe cases ...	8·3	14·2	17·4	20·3	19·2
Percentage of deaths ...	0·0	2·1	3·2	5·3	3·2
Diphtheria:					
Number of cases	1	97	404	169	29
Percentage of severe cases ...	0·0	32·0	40·1	39·6	41·4
Percentage of deaths ...	0·0	9·3	10·6	12·4	3·4
Measles:					
Number of cases	0	85	367	184	25
Percentage of severe cases ...	0·0	16·4	16·4	23·9	20·0
Percentage of deaths ...	0·0	4·7	7·7	13·6	12·0
Whooping Cough:					
Number of cases	2	35	190	73	10
Percentage of severe cases ...	0	11·4	20	21·9	10·0
Percentage of deaths ...	0	0·0	10	13·7	10·0

dark and jet black-haired children. This position is changed in diphtheria, where the fair-haired children have a slightly smaller percentage of severe cases and red-haired children a smaller percentage of deaths, and in whooping cough where the red-haired have a smaller percentage of severe cases. As with the fair-haired the position of the dark and jet black-haired children is marked and constant, but in their case in respect of smaller severity and death rates. The only exception is the lower death rate in diphtheria and the lower severity rate in whooping cough among red-haired children.

To sum up, the dark and jet black-haired child occupies one pole, the pole of less severity and mortality, the fair-haired and, to a less degree, the red-haired child occupies the other pole, the pole of greater severity and mortality, while the mean is represented by the medium-haired child. This will be seen conveniently in Table XII, giving the hair colours in order of ascending severity and death rates in the different diseases considered.

2. *Recuperative Power and Pigmentation of the Eyes.* Taking into consideration the conclusions drawn between the severity and mortality of the diseases and the pigmentation of the hair, it should follow by analogy that the dark-eyed and the light and blue-eyed children should occupy the extreme poles, the dark-eyed that of less severity and mortality and the light and blue-eyed that of greater severity and mortality, with the medium-eyed children representing the mean. As will be seen from Table XIII this would hardly be accurate in detail. On the other hand, if we group together the

TABLE XII. *Giving from left to right the Hair Colours in order of ascending Severity and Death Rates in the diseases considered.*

(i) Severity Rates.

Disease	Hair Colour						
Scarlet Fever ...	Black	—	Dark	—	Medium	Red	Fair
Diphtheria ...	Black	—	Dark	Fair	Medium	Red	—
Measles	Black	—	Dark	—	Medium	Red	Fair
Whooping Cough ...	Black	Red	Dark	—	Medium	—	Fair

(ii) Death Rates.

Disease	Hair Colour					
Scarlet Fever ...	Black	—	Dark	Medium	Red	Fair
Diphtheria	Black	Red	Dark	Medium	—	Fair
Measles	Black	—	Dark	Medium	Red	Fair
Whooping Cough ...	Black	—	Dark	Medium	Red	Fair

TABLE XIII. *Eye Colour. Showing the Number of Patients, the Percentage of Severe Cases, and the Percentage of Deaths in the various Eye Colours in the diseases considered.*

	Dark	Medium	Light	Blue
Scarlet Fever:				
Number of cases	280	1156	331	97
Percentage of severe cases ...	17·1	16·8	19·0	20·6
Percentage of deaths ...	1·4	3·6	4·2	4·1
Diphtheria:				
Number of cases	106	311	247	36
Percentage of severe cases ...	35·8	37·6	42·9	30·6
Percentage of deaths ...	13·2	10·3	10·1	8·3
Measles:				
Number of cases	96	398	146	21
Percentage of severe cases ...	17·7	17·3	22·0	23·8
Percentage of deaths ...	7·3	8·8	11·0	9·5
Whooping Cough:				
Number of cases	47	118	111	34
Percentage of severe cases ...	21·2	16·9	22·5	11·8
Percentage of deaths ...	10·6	7·6	12·6	5·9

TABLE XIV. *Giving from left to right the Eye Colours in order of ascending Severity and Death Rates in the diseases considered.*

(i) Severity Rates.

Disease	Eye Colour					
Scarlet Fever ...	—	Medium	Dark	—	Light	Blue
Diphtheria ...	Blue	—	Dark	Medium	Light	—
Measles	—	Medium	Dark	—	Light	Blue
Whooping Cough ...	Blue	Medium	Dark	—	Light	—

(ii) Death Rates.

Disease	Eye Colour						
Scarlet Fever ...	—	Dark	—	Medium	—	Blue	Light
Diphtheria ...	Blue	—	Light	Medium	Dark	—	—
Measles	—	Dark	—	Medium	—	Blue	Light
Whooping Cough ...	Blue	—	—	Medium	Dark	—	Light

dark and medium-eyed children and the light and blue-eyed children it is found (Table XV) that the dark and medium-eyed group represent the less severity and mortality, while the light and blue-eyed group represent the greater severity and mortality. The only exception to this is the death rate in diphtheria, which is slightly lower in the light and blue-eyed group. When the eye colours are considered separately, however, the conclusion is not so definite. The dark-eyed child has not the same advantage over the medium-eyed child as the dark-haired child has over the medium-haired child. Only in the death rate in scarlet fever is there a decided advantage, and there is also a slight advantage in the death rate in measles and the severity rate in diphtheria. In the other diseases both the severity and death rates are lower in medium-eyed children, but the difference is not great. The light-eyed child shows the highest severity and mortality, the only exceptions being the severity rate in scarlet fever and measles, which is rather lower than in blue-eyed children, and the death rate in diphtheria, which is rather lower than in both medium and dark-eyed children. The blue-eyed child is not at all constant; in diphtheria and whooping cough it has the smallest severity and death rates, whereas in scarlet fever and measles it has the greatest percentage of severe cases and, with the exception of light-eyed children, of deaths. It would seem therefore, that the advantage is still in favour of the medium and dark-eyed children, but that the blue-eyed children are not so liable to severe attacks of diphtheria and whooping cough.

3. *Recuperative Power and Pigmentation of the Hair and Eyes combined.* The various combinations of hair and eye colours are so numerous and the number of cases in some combinations so small that only the three main ones have been

considered here, namely, the dark-haired children with dark eyes, the medium-haired children with medium eyes, and the fair-haired children with light eyes.

TABLE XV. *Eye Colour. Showing the Number of Patients, the Percentage of Severe Cases, and the Percentage of Deaths in Dark and Medium-eyed Children grouped together and Light and Blue-eyed Children grouped together.*

Disease	Dark and Medium	Light and Blue
Scarlet Fever:		
Number of cases	1436	428
Percentage of severe cases ...	16·9	19·4
Percentage of deaths ...	3·2	4·2
Diphtheria:		
Number of cases	417	283
Percentage of severe cases ...	37·2	41·3
Percentage of deaths ...	11·0	9·9
Measles:		
Number of cases	494	167
Percentage of severe cases ...	17·4	25·3
Percentage of deaths ...	8·5	12·3
Whooping Cough:		
Number of cases	165	145
Percentage of severe cases ...	18·2	20·0
Percentage of deaths ...	8·5	11·0

TABLE XVI. *Hair and Eye Colour. Showing the Number of Patients, the Percentage of Severe Cases and the Percentage of Deaths in the three main Hair and Eye combinations.*

	Dark hair and dark eyes	Medium hair and medium eyes	Fair hair and light eyes
Scarlet Fever:			
Number of cases	145	826	165
Percentage of severe cases ...	12·4	16·6	19·4
Percentage of deaths ...	1·4	3·6	4·9
Diphtheria:			
Number of cases	40	223	97
Percentage of severe cases ...	37·5	39·9	44·3
Percentage of deaths ...	12·5	11·7	16·5
Measles:			
Number of cases	39	260	70
Percentage of severe cases ...	20·6	16·1	25·7
Percentage of deaths ...	5·1	8·0	14·3
Whooping Cough:			
Number of cases	16	89	39
Percentage of severe cases ...	12·5	19·1	28·3
Percentage of deaths ...	0·0	7·9	17·9

TABLE XVII. *Giving from left to right the combined Hair and Eye Colours in order of ascending Severity and Death Rates in the diseases considered.*

(i) Severity Rates.

Disease	Hair and Eye Colour			
Scarlet Fever ...	—	Dark dark	Medium medium	Fair light
Diphtheria ...	—	Dark dark	Medium medium	Fair light
Measles	Medium medium	Dark dark	—	Fair light
Whooping Cough	—	Dark dark	Medium medium	Fair light

(ii) Death Rates.

Disease	Hair and Eye Colour			
Scarlet Fever ...	—	Dark dark	Medium medium	Fair light
Diphtheria ...	Medium medium	Dark dark	—	Fair light
Measles	—	Dark dark	Medium medium	Fair light
Whooping Cough	—	Dark dark	Medium medium	Fair light

From Tables XVI and XVII it is seen that the result is striking. The dark-haired dark-eyed children occupy one pole, that of less severity and lower mortality; the fair-haired light-eyed children occupy the opposite pole, that of greater severity and greater mortality, and not only is this so but the greater severity and greater mortality in the fair-haired light-eyed child is marked and constant in each of the diseases considered. The medium-haired medium-eyed child occupies the mean between these poles. The only exceptions are found in measles where the medium-haired medium-eyed children show a smaller percentage of severe cases, and in diphtheria where they show a slightly smaller percentage of deaths than the dark-haired dark-eyed children.

It has been found then that the dark and jet black-haired children oppose greater resistance to the diseases than the red-haired children and, even more so, than the fair-haired children, while the medium-haired children occupy an intermediate position. It has also been seen that the medium and dark-eyed children are less severely attacked than the light and blue-eyed children and, further, that combining the hair and eye colours the dark-haired dark-eyed children show considerably more resistance to the diseases than the fair-haired light-eyed children, with the medium-haired medium-eyed children occupying an intermediate position.

From this it must be concluded that the dark-haired dark-eyed type have higher recuperative powers and offer greater resistance to the diseases than the fair-haired light-eyed type, and that, in the various gradations between the extreme dark and extreme fair types, the closer the type approximates to fair, the less recuperative power it has and the less resistance is offered in the diseases here considered.

(5) *Special Cases.*

1. The incidence of nephritis in the various pigmentation types in scarlet fever.

The possible connection between the pigmentation of the hair and eyes and the skin has already been considered. The type of skin, as one of the chief excretory systems in the body, might be expected to have an effect on the incidence of nephritis. In 1295 cases of scarlet fever in Ruchill Hospital 32 or 2·5 °/₀ suffered from true scarlatinal nephritis. In Table XVIII the percentages of those occurring in the various pigmentation types are given. The evidence is not very conclusive, but here again the darker type seems to offer more resistance.

TABLE XVIII. *Showing the Number of Cases and the Percentage developing Nephritis in the various Hair and Eye Colours in Scarlet Fever.*

(i) Hair Colour.

	Black	Dark	Medium	Fair	Red
Number of cases	8	214	749	251	73
Percentage with nephritis ...	0·0	1·9	2·7	2·0	4·1

(ii) Eye Colour.

	Dark	Medium	Light	Blue
Number of cases	213	833	187	62
Percentage with nephritis ...	2·3	2·3	3·2	3·2

2. The incidence of paralysis and the necessity for tracheotomy in diphtheria.

The possible association between the ectoderm and pigmentation has already been mentioned. The peripheral nerves being ectodermal in origin, it was thought that some relationship might be found between the incidence of paralysis in diphtheria and the pigmentation of the hair and eyes. The evidence again is not very conclusive. In Table XIX the percentage of cases with subsequent paralysis, chiefly palatal, and the percentage of cases requiring tracheotomy in 535 children suffering from diphtheria have been tabulated with reference to the colour of their hair and eyes. The figures bear out what has already been said with regard to the severity of the disease, the dark and medium types being less severely attacked; exceptions to this are found in the high percentage of cases of paralysis in dark-haired and in dark-haired dark-eyed children, and in the low percentage of cases of paralysis in blue-eyed children.

TABLE XIX. *Showing the Number of Cases and the Percentage developing subsequent Paralysis, also the Percentage requiring Tracheotomy performed, in the various Hair and Eye Colours in Diphtheria.*

(i) Hair Colour.

	Black	Dark	Medium	Fair	Red
Number of cases	1	78	303	134	19
Percentage with paralysis ...	0·0	6·4	3·9	4·5	10·5
Percentage requiring tracheotomy	0·0	5·1	4·9	6·0	10·5

(ii) Eye Colour.

	Dark	Medium	Light	Blue
Number of cases	86	242	178	29
Percentage with paralysis ...	4·7	4·1	5·6	3·5
Percentage requiring tracheotomy	4·7	3·7	7·9	6·9

(iii) Hair and Eye Colour Combined.

	Dark hair and dark eyes	Medium hair and medium eyes	Fair hair and light eyes
Number of cases	31	169	73
Percentage with paralysis ...	9·7	3·6	4·1
Percentage requiring tracheotomy	3·2	3·0	4·1

(6) *Pigmentation as a Factor in Race Selection.* By comparing the percentages in each hair or eye class of the total number of severe cases with recovery with the percentage of that class in the general population, an indication is given as to how far that class is handicapped by subsequent disability after severe illness, and by comparing the percentages in each class of the total number of deaths with the percentage of that class in the general population, an indication is given as to how far that class is selected for extermination. The effect of selection is so much alike in each of the diseases considered that it is unnecessary to consider them separately.

(i) *Hair Colour.*

Table XX shows that the medium-haired class is undoubtedly selected both for handicap by its greater number of severe cases with subsequent disability, and for extermination by its greater number of deaths.

This is the case in every one of the diseases considered, but to a less extent in measles where the fair-haired class suffers more severely. At the expense of the medium-haired and, to a much less extent, of the fair-haired, the black and dark-haired class is least selected for such handicap and extermination. The fair-haired and the red-haired class suffer about equally, the fair-haired rather more and the red-haired rather less than their proportional representation in the general population. In measles, as has been mentioned, the fair-haired class suffers most severely.

This result is somewhat at variance with the idea commonly held that the fair-haired class is being exterminated in the towns. That there is some degree of adverse selection is obvious from Table XX, but there is yet more adverse selection among the medium-haired class. It would seem therefore that the current belief of the inability of the fair-haired person to stand the town life is not supported to any great extent by fever statistics.

TABLE XX. *Hair Colour. Showing the Percentages of the various Hair Colours in the General Population and in Hospital Patients, also the Percentages in each Colour of the total number of severe cases with recovery and of deaths.*

	Black	Dark	Medium	Fair	Red
Percentage in general population:					
Belvidere and Ruchill areas	1·2	27·2	45·2	21·4	5·0
Scarlet Fever:					
Percentage in hospital patients	0·6	15·5	59·4	19·3	5·0
Percentage of total severe cases with recovery	0·4	13·4	59·9	20·6	5·7
Percentage of total deaths	0·0	9·4	56·3	29·7	4·7
Diphtheria:					
Percentage in hospital patients	0·1	13·9	57·7	24·1	4·2
Percentage of total severe cases with recovery	0·0	11·1	60·2	23·2	5·6
Percentage of total deaths	0·0	12·2	58·1	28·4	1·4
Whooping Cough:					
Percentage in hospital patients	0·6	11·3	61·3	23·6	3·2
Percentage of total severe cases with recovery	0·0	13·8	65·6	20·7	0·0
Percentage of total deaths	0·0	0·0	63·3	33·3	3·3
Percentage in general population:					
Ruchill area only	1·1	26·7	45·3	21·7	5·3
Measles, Ruchill only :					
Percentage in hospital patients	0·0	12·9	55·5	27·8	3·8
Percentage of total severe cases with recovery	0·0	15·9	50·8	30·2	3·1
Percentage of total deaths	0·0	6·6	46·7	41·7	5·0

Tocher* found that there was a positive correlation between the death rate and medium hair, and another between the death rate and dark eyes in Scotland. This result, he states, was to be expected, since density is similarly associated with colour and the denser the population, the greater the death rate; the denser the

---

* Tocher: "Pigmentation Survey of School Children in Scotland." *Biometrika*, Vol. VI. 1908—9, p. 188.

population, the greater the excess of medium hair, and therefore the greater the excess of medium hair, the greater the death rate. This, however, is obviously not the sole factor, as it is found that the rate at which medium-haired children develop infectious disease is considerably greater than the average.

(ii) *Eye Colour.*

Table XXI shows that the effect of selection in the various eye colours follows closely that of the hair colours. The medium-eyed class by a very large margin is selected both for handicap by its large number of severe cases with subsequent disability and for extermination by its large number of deaths. The only exception to this is in whooping cough where the light-eyed class suffer more severely. With the exception of the light-eyed class in diphtheria and whooping cough the other colours all suffer less by handicap or extermination than their proportion in the general population, and this happens at the expense of the medium-eyed class.

TABLE XXI. *Eye Colour. Showing the Percentages of the various Eye Colours in the General Population and in Hospital Patients, also the Percentages in each Colour of the total number of severe cases with recovery and of deaths.*

	Dark	Medium	Light	Blue
Percentage in general population:				
Belvidere and Ruchill Areas	24·5	34·0	30·3	11·2
Scarlet Fever:				
Percentage in hospital patients	15·0	62·0	17·8	5·2
Percentage of total severe cases with recovery	16·8	58·4	18·7	6·1
Percentage of total deaths	6·2	65·7	21·9	6·2
Diphtheria:				
Percentage in hospital patients	15·2	44·4	35·3	5·1
Percentage of total severe cases with recovery	12·1	42·9	40·9	4·1
Percentage of total deaths	18·9	43·2	33·8	4·1
Whooping Cough:				
Percentage in hospital patients	15·2	38·1	35·8	11·0
Percentage of total severe cases with recovery	17·2	38·0	38·0	6·9
Percentage of total deaths	16·7	30·0	46·7	6·7
Percentage in general population:				
Ruchill area only	24·4	32·4	31·2	12·0
Measles, Ruchill only:				
Percentage in hospital patients	14·5	60·2	22·1	3·2
Percentage of total severe cases with recovery	15·9	54·0	25·4	4·8
Percentage of total deaths	11·7	58·4	26·7	3·3

Next to the medium, the light-eyed class is selected for handicap and extermination, the dark-eyed less so, and the blue-eyed least of all. This order is maintained throughout with well marked degrees of diminution of adverse selection from the medium-eyed to the blue-eyed class.

(iii) *Hair and Eye Colour combined.* The determination of the effect of selection on the race with regard to hair and eye colour combined is not at present possible as no statistics are available of the different combinations of hair and eye colour in the school children of Glasgow*.

(7) *Pigmentation and Geographical Distribution of Disease.* In connection with pigmentation and the severity of these diseases, the following facts given by Clemow† are of interest. "Scarlet fever," he states, "is essentially a disease of temperate climates. In the tropics it is almost unknown. The influence of race is uncertain. People so ethnologically distinct as the Chinese, the natives of South Africa and the inhabitants of the principal European countries all suffer considerably from the disease. But it is certain that some races are more susceptible than others. The statistics of recent censuses in the United States of America tend to show that the disease is less prevalent and less fatal among the negroes and Red Indians than among the whites. In the few cases of scarlet fever observed in India, almost all have occurred among Europeans, and a very small number in natives of the country. In Egypt the disease is rare, but the infection is not infrequently imported, and when it does attack Egyptian children is of a mild character.

Diphtheria bears a close resemblance to scarlet fever in its distribution. The influence of race has never been fully determined. All the great divisions in the human family, including pure Mongols and full-blooded negroes, seem to be susceptible to the disease, though probably both their susceptibility to attack and their power of recovery vary greatly. In China, for example, the disease is said to be much more intense and fatal in natives than in European residents, while in the United States the white races suffer much more than the black.

Measles is one of the most widely prevalent of all diseases. In its relation to race, it appears to be as indifferent as in its relation to most other external conditions. All races are susceptible, and it seems to be as capable of attacking the Chinaman, the Hindu and the Negro as the European. It appears, on the whole, to be decidedly less common in the African Negro than in most other races.

Whooping cough has an extremely wide distribution. Racial susceptibility appears to be a factor of little importance, for all races are affected, though some more severely than others. In the United States the Negro inhabitants fall victims to it much more readily than those of other races."

It would appear from Clemow's facts that scarlet fever and diphtheria are less prevalent and less fatal among the darkly pigmented races, but on the other hand measles and whooping cough seem to be equally severe, irrespective of colour.

It seems that the statistics on pigmentation on which the results of this inquiry are based as regards susceptibility and recuperative power, although probably

---

* [They exist in Tocher's original schedules and no doubt could be easily extracted. ED.]

† Clemow: *The Geography of Disease.*

holding approximately for the European races, do not necessarily hold good among other races. The dark-haired Chinaman, for instance, suffers more severely from diphtheria than the European. It would be unwise, therefore, to draw any conclusions by analogy as to susceptibility and recuperative power in these fevers among the various races according to their pigmentation.

(8) *Conclusions.* From the statistics already given it has been shown in the four acute fevers here considered, namely, scarlet fever, diphtheria, measles and whooping cough, that among Glasgow school children :

1. The medium-haired child is more liable to become infected than the red-haired and, much more so, than the dark-haired and jet black-haired child. The fair-haired child occupies an intermediate position as regards infection.

2. The medium-eyed child is more liable to become infected than the dark-eyed and, much more so, than the blue-eyed child. The light-eyed child occupies an intermediate position as regards infection. The light-eyed child appears to be more susceptible to diphtheria and whooping cough than to scarlet fever and measles.

3. The dark-haired and jet black-haired child has higher recuperative power than the red-haired and, much more so, than the fair-haired child. The medium-haired child occupies an intermediate position as regards recuperative power.

4. The medium-eyed and dark-eyed child has higher recuperative power than the light-eyed and blue-eyed child.

5. Combining the hair and eye colours we find that the dark-haired dark-eyed child has considerably more recuperative power than the fair-haired light-eyed child. The medium-haired medium-eyed child occupies an intermediate position as regards recuperative power.

6. In the various gradations between the extreme dark and extreme fair types, the closer the type approximates to fair, the less recuperative power it has and the less resistance it offers to the diseases.

7. The medium-haired class is undoubtedly selected both for handicap by its greater number of severe cases with subsequent disability, and for extermination by its greater number of deaths. The jet black and dark-haired class is least selected for such handicap and extermination, while the fair-haired class is selected rather more and the red-haired class rather less than their proportion in the general population.

8. The medium-eyed class is undoubtedly selected both for handicap by its greater number of severe cases with subsequent disability and for extermination by its greater number of deaths. The light-eyed class is next in order adversely selected, the dark-eyed class less so and the blue-eyed class least of all.

[*Editorial Note.* Owing to the absence of Dr Macdonald from England it has not been possible to consult him as to several points in his most interesting paper. We think it might have been desirable to lay further stress on the possibility of considerable personal equation in the estimation of the "medium" class in both hair and eye colour. That 34 °/$_0$ of the general population only should be of medium eye colour but 62 °/$_0$ of the scarlet fever population seems a very startling difference to attribute wholly to pigmentation. It does not accord with data reduced in the Biometric Laboratory and shortly to be published bearing on this point. In that material there was only *one* determination of the pigmentation of the general and the fever populations. It seems possible therefore that some of the above difference is due to personal equation which has led to the emphasising of the medium hair and eye colours in the fever population. Should this view be confirmed later, it would affect Dr Macdonald's conclusions (1), (2), (7) and (8), but not the important results (3), (4), (5) and (6), which depend upon estimates made solely in the fever hospital. There is some differentiation in age between Tocher's and Macdonald's populations. This is shown in the following table:

Mean Ages.

Hair	Fair	Red	Medium	Dark	Jet Black	Totals	Tocher
Fever Cases, Boys	8·5	9·9	9·1	10·2	12·2	9·5	10·1
" " Girls	8·7	8·8	9·5	10·3	10·5	9·4	10·2

Eyes	Blue	Light	Medium	Dark	Totals	Tocher
Fever Cases, Boys	7·6	9·0	9·7	10·0	9·6	10·1
" " Girls	8·7	8·6	9·5	9·8	9·4	10·2

It will thus be seen that the lighter haired children are younger than the darker haired, and that this is also substantially the case for the eye colour, the lighter being younger than the darker eyed. The fever population is 5 to 10 months younger than the general school population of Glasgow and we should accordingly expect it to be somewhat less dark than the latter. We should expect, as the fair pass with age into the medium class, that there would be a less percentage of medium in the fever than in the general population. Thus if there has been no personal equation, the conclusions (1), (2), (7) and (8) must be still further emphasised. K. P.]

FIRST RESULTS FROM THE OXFORD ANTHROPOMETRIC LABORATORY.

[*A paper read before the British Association* (*Section D*) *at Sheffield,* 1910.]

By E. SCHUSTER, D.Sc.

This Laboratory was instituted in the Department of Comparative Anatomy in January 1908 by Professor G. C. Bourne. Its objects are defined in its circulars as follows:

(1) To obtain a statistical survey of the physical development of undergraduates, which would in itself have a permanent value as a record which might be compared with similar records obtained in other places and possibly in future ages.

(2) To ascertain whether any degree of interconnection or correlation exists between mental and physical characters.

(3) To ascertain what bodily changes or development take place during a man's residence in Oxford as an undergraduate, and whether such changes depend at all on what games he plays, what school he reads for, and so on.

(4) To obtain data by which exact measures can be made of the resemblance between brothers and between first cousins.

As will be shown some progress has already been made with regard to Nos. 1 and 3, but in the present paper no attempt will be made to deal with Nos. 2 and 4.

In order to attain the objects here detailed it was proposed that each undergraduate should be measured twice, once near the beginning and once near the end of his career. The number of men who have actually been measured twice is at present only 89 so that the changes in individuals cannot be satisfactorily dealt with, but the material has been used in this way:—it has been divided into groups according to the age of the subject at the time of measurement, and all those measured twice are included twice, once in the group corresponding to their age at first measurement and once in the group corresponding to their age at second measurement. Thus reckoned there are 959 men included in my tables. 29 of 18 years of age, 330 of 19, 209 of 20, 137 of 21, 95 of 22, and 59 of 23 and over The average values for each measurement employed have been determined for each of these six groups and their standard deviations as a measure of variability. In addition a large number of correlation coefficients between pairs of measurements have been calculated for the five largest groups.

Measurements and Test Applied.

(1) *Spot Pattern Test.* This test was devised by Dr McDougall and included in our schedule at his suggestion; he considers that it forms a measure of the power of concentration. Its object is to find how soon a simple pattern may be correctly reproduced. The pattern, of which a sample is shown below, is made by pricking holes in a square of cardboard. The holes lie in equidistant rows and columns, the distances between them being obtained by first plotting them at the points of intersection of the lines on sectional paper. The number of rows and columns is limited to five so that the number of possible places for the spots is 25. The first pattern attempted has nine spots placed irregularly in these positions. The cardboard square is placed in a wooden frame which brings it to the level of the eyes of the subject who sits before it, behind is placed a large instantaneous self-setting photographic shutter and behind that again a carefully shaded electric light. When the shutter is worked, the subject sees the pattern for a small fraction of a second as a number of bright spots arranged like a constellation. The procedure is for the operator to show the pattern once and then to read the simple directions. After this the pattern is shown again five times at intervals of about two seconds; the subject is then required to attempt to map the spots on sectional paper with the same sized squares as that used in making the pattern. Areas are ruled off on the paper of the right size for the purpose. The first attempt is generally a failure, though one or two successes are recorded; the pattern is then shown again five times and a fresh attempt is made, if this is also unsuccessful the procedure is again repeated, and so on till the map is correctly drawn, except that in order to save time no one was allowed more than ten attempts.

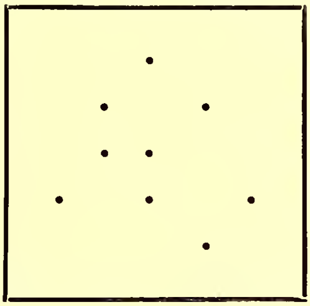


Figure showing one of the patterns employed in the text.
It can be used any way up.

The actual record of how many people succeeded at the first attempt, how many at the second attempt and so on is not of much interest, but what is of interest are the correlations found to exist between success at this test and intellectual eminence as measured by examination. The fourfold table method was employed for measuring these correlations, but it is only claimed that the coefficients so found give a rough idea of the degree of correlation.

As a specimen table the following may be taken:

Spot Pattern Test and Class in final Schools.

	Classes I and II	Classes III and IV and Pass	Totals
5 tries or less ...	27	26	53
6 tries or more ...	18	29	47
Totals ...	45	55	100

$r = \cdot 2$.

This table includes all those who had taken their final Schools by the end of 1909, the following year's results having appeared too late to be included. It will be seen that the first and second class men are distinctly better than those included in Classes III or IV or who obtained pass degrees, since 60 per cent. of the former and only 46 per cent. of the latter mapped the pattern correctly in five attempts or less. The correlation coefficient calculated from this table is ·2 which indicates a low degree of correlation. It should perhaps be explained that when the correlation is perfect, i.e. when a variation of any particular degree of the one variable is always associated with a variation of the same degree of the other variable, then the coefficient is equal to unity; when no correlation exists it is equal to 0.

Since the numbers are small and the correlation is low, the conclusion drawn from this one table could not be regarded as safe if considered independently, but as it receives corroboration from the study of the results of scholarship examinations it may be accepted with fair confidence.

The correlation between the possession of a scholarship or exhibition and success at the test under consideration is positive for each age group, though as the numbers in each such group are small it has a considerable variation. The average value for all age groups taken together is ·22. Here again the correlation coefficients are not high, nor, since they are based on small numbers, are they particularly constant; but some degree of positive correlation is exhibited by every age group, so that the probability of the result being partly a chance one is so small as to be negligible.

Further, an investigation was made of the relation between an undergraduate subject of study and his success at this test, with the result that those reading science or mathematics were found to be superior to those reading other subjects.

The correlations were in this case:

Age 18 $r = \cdot 19$ Age 21 $r = \cdot 24$
Age 19 $r = \cdot 08$ Age 22 $r = \cdot 02$
Age 20 $r = \cdot 19$ Average $r = \cdot 14$

Here the correlation is lower than in the case previously considered, but again is positive in every group.

As it had been suggested to me that the test might be effected by eyesight, the correlations with acuity of vision were determined and are here shown:

Age 18 $r = - \cdot 06$ Age 21 $r = + \cdot 11$
Age 19 $r = - \cdot 04$ Age 22 $r = - \cdot 01$
Age 20 $r = - \cdot 12$ Average $r = - \cdot 02$

The minus signs indicate a negative correlation, i.e. that bad eyesight shows to slight advantage, but since the numerical values are very small and the sign is not constant throughout, it may be concluded that eyesight does not appreciably affect the results of this test.

(2) *Acuity of Vision* is tested by means of a card on which are printed ten lines of type of heights ranging from rather over 4½ inches in the top line to ⅛th inch in the bottom line. The height of the type is so adjusted that it subtends an angle of five minutes in the eye at the following distances:

Top line	80 metres	6th line	9 metres
2nd „	36 „	7th „	6 „
3rd „	24 „	8th „	4 „
4th „	18 „	9th „	3 „
5th „	12 „	10th „	2 „

A person with normal sight should be able to read each line at the distances named with each eye separately. Since the card is shown to the subject at a distance of six metres, he should be able to read down to the seventh line. As a matter of fact, a considerable number of men can read accurately with either eye as far as the ninth line; but we have not as yet found anyone who can get as far as the tenth.

In recording the results of this test, the smallest type which can be read is noted and the distance at which it should be read by a person of normal sight is entered in the schedule. For example, if the entry stands "Right Eye 80, Left Eye 36," it means that the subject, standing at a distance of six metres from the card, can read the top line only with his right eye and the second line also with his left. A person of normal sight could have read these at distances of 80 and 36 metres respectively. (½) entered against the number signifies that only a certain proportion of the letters in the line referred to were read correctly.

This test is that usually employed as a preliminary by oculists, in making an examination of a patient's eyesight. Short sight and astigmatism are detected by it but not distinguished from one another as different defects of vision; long sight may pass unnoticed.

I give a table in which the men tested are classed in four grades according to the sight of the right eye; a similar table for the left eye would be practically the same, since the correlation between the two eyes is very high.

		4 or better	4½ and 6	6½—24	24½ or worse	Totals
Age 18	Number	52	28	30	19	129
	Percentage	40 °/o	22 °/o	23 °/o	15 °/o	
Age 19	Number	113	79	70	64	326
	Percentage	35 °/o	24 °/o	21 °/o	20 °/o	
Age 20	Number	87	50	39	32	208
	Percentage	42 °/o	24 °/o	19 °/o	15 °/o	
Age 21	Number	50	37	34	12	133
	Percentage	38 °/o	28 °/o	26 °/o	9 °/o	
Age 22	Number	37	19	20	19	95
	Percentage	39 °/o	20 °/o	21 °/o	20 °/o	
Average	percentages	39 °/o	24 °/o	22 °/o	18 °/o	

This table shows that about a quarter of the whole number have eyesight of what oculists consider to be of the normal keenness, while the remainder are almost equally distributed above and below. Of those below the normal 18 °/o have distinctly bad eyesight. The fact that this test is made is of advantage to some, at any rate, of the men who come to the laboratory, for many of those with faulty vision are quite unaware of it till it is thus pointed out, although possibly suffering considerable inconvenience therefrom. The visit to an oculist which is recommended may in consequence conduce to their greater comfort and to the preservation of their sight.

(3) *Measurements of Physical Development.*

Lung Capacity is measured by means of a spirometer. This instrument was copied with slight modifications from that used by Sir Francis Galton in his anthropometric laboratory. It consists of an outer cylinder filled with water, in which there rests an inverted inner cylinder. The subject is asked to take a deep breath and then to blow down a flexible tube which is connected in such a way with the inner cylinder that all the air passes into the latter. The inner cylinder is counterpoised so that the air passing into it causes it to rise, the amount of the rise being proportional to the volume of the air. A pointer is attached to its top, which as the cylinder rises passes up a vertical scale divided into millimetres. The diameter of the cylinder was so arranged that a rise of one mm. corresponds with entrance of 20 c.c. of air. The measurement of volume in this way is not quite accurate, as it is not possible without somewhat elaborate arrangements to get the inner cylinder counterpoised correctly for all positions, since the more it rises the less it is supported by the water in the outer cylinder. Thus when it rises above a certain level the air contained in it becomes compressed to an extent slightly greater than the atmospheric pressure, and the more it rises the greater this additional pressure becomes. Thus in the case of men with good chest development the lung capacities are slightly understated.

In applying this test three blows are allowed and the mean of them is taken. Sometimes the first blow is a failure, owing to the subject not quite understanding what he has to do, and in this case it is not recorded but a fourth attempt is made instead.

Stature is measured by means of a special instrument in which there are two platforms, a lower fixed one, and an upper hinged seat which may be brought down on to fixed brackets at such a height that its top surface is exactly 500 mm. above the lower platform or put out of the way when it is not wanted. The subject first removes his boots and is asked to stand on the lower platform, a sliding block, massive in its proportions but counterpoised, is brought down on to the top of his head and his stature read off a millimetre scale in the usual way. The upper platform is then brought into position, he is asked to sit on it and the height of the top of his head in this position is measured. By deducting the height of the seat (500 mm.) the length of his body without the legs is obtained. He then kneels on the lower platform and another record is taken; in this case his whole length except that part below the knees is measured; by a little subtraction the length of the leg and the length of the thigh are arrived at. Two ratios are then calculated, namely the ratio of length of leg to the whole stature, expressed in the tables in $\frac{1}{100}$ths of the stature, and the ratio of the thigh length to the leg length expressed in $\frac{1}{100}$ths of the leg length.

Weight is measured on an Avery weighing machine in stones, pounds and ounces. The subject is fully dressed but without his boots.

Strength of Pull is measured by means of a spring balance, working up to 500 lbs., suspended by a staple driven into the wall. To the free end of the balance a wire rope is attached, which passes under two pulleys fastened to the floor and ends in a cross-piece resting close to the ground. The subject is required to stand over this handle, stooping so as to reach it, and to pull it steadily upwards as far as he can; then to hold it in this position for five seconds.

It is not proposed to give a detailed account of all the means, standard deviations and correlation coefficients which have been calculated. These are recorded more clearly and satisfactorily in a tabular form. But an attempt will be made to point out the more interesting facts which the tables show.

Strength of Pull. The average pull for the 18 year old group is 185·2 lbs., this rises fairly steadily with age; at 21 years 216·4 lbs. is reached, then it drops slightly to 211·4 at 22 and 212·9 at 23. As the probable errors of the means for the last three ages range from about 3 to 4½ lbs. no significance need be attached to this slight falling off.

The lung capacity starting at its lowest in the 18 year old group with an average of 4184 c.c. also rises to its highest at 21 years where 4438 c.c. is reached, then follows a slight drop succeeded again by another rise.

The weight is also at its lowest at 18 years of age when the average is 10 st. 7 lbs. 9 ozs., this rises to 10 st. 12 lbs. at 19, and to 11 st. 2 lbs. at 21, then as in the case of the pull there is a drop of the average. Professor Bourne tells me that his experience with rowing men is something of the same kind, after 18 years there is a sudden rise of ½ stone followed subsequently by a slight drop.

These three measurements are fairly closely correlated one with another, the average correlation coefficients being: strength of pull and weight ·46, strength of pull and lung capacity ·37, lung capacity and weight ·59.

Knowing the correlation between each pair of three variables it is possible to find by means of a formula the correlation between any pair for a constant value of the third variable. In this case for instance it might perhaps be said that there is no independent correlation between strength of pull and lung capacity and that the apparent correlation is due to the fact that people with large lung capacities are also heavy and that their weight gives them an advantage at the pull test. That they do derive some advantage from their weight is perfectly true, but the application of the formula teaches us that for people of equal weight, the correlation between lung capacity and strength of pull is ·14.

To take for purposes of comparison the relations of weight, pull and stature.

Here we have the correlation between stature and pull ·21, between pull and weight ·46 and between stature and weight ·66. With a positive correlation between stature and pull it looks as if tall men had an advantage, but again applying the formula we find that for men of equal weight the correlation between stature and pull is actually negative, namely − ·13, and that the tall men are at a distinct disadvantage. If we took from them besides the advantage of extra weight which their extra stature provides them with, the increased lung capacity which also goes with it, we would find that this disadvantage was greater still. The disadvantage itself may be either due to the fact that our apparatus is more difficult for a tall man properly to apply his strength at, or it may be that the tall men are more apt to be weedy and weak in the back than the short, at any rate during the early part of their lives. A certain amount of evidence in favour of the latter supposition is given by a comparison of the correlation coefficients of the different age groups, in the 18 year old group we find the correlation between stature and weight, lung capacity and strength of pull severally smaller that in the succeeding ages. The correlation between stature and weight is ·50 for the 18 year olds, ·63 for the 19 year olds and goes up to ·76 for those aged 21. As the P.E. in the latter case is ·02 and in the former ·04, this difference is probably significant.

The average stature is 1751·6 mm. for the 18 year old group and goes up to 1767·2 mm. for the 19 year old men, while 1778·6 is recorded for those of 21 years of age. The latter figure is probably an accidentally high one since the succeeding years show lower averages. The numbers are really too small to give a satisfactory curve of growth.

The mean ratio of length of leg to stature is fairly constant through all the ages considered, the lowest average recorded being 47·01 and the highest 47·46. The mean ratio of thigh to leg is even more constant, ranging between 46·44 and 46·65.

The correlation between leg ratio in stature is well marked, its average value being ·44, which means that for any particular age group increase in stature is due to a great extent to increased length of leg. On the other hand it must be pointed out that although the mean stature appears to increase with the age the mean length of leg does not, thus it may be argued that growth after the age of 18 concerns the body rather than the legs.

As thigh ratio is sensibly correlated with leg ratio it would appear that people with disproportionately long legs also are apt to have disproportionately long thighs.

(4) *Head Measurements.*

(1) *Greatest Length.* Taken from the most prominent point of the glabella or prominence in the mid line between the two eyebrows to the most distant point in the middle line on the back of the head, known as the occipital point.

(2) *Greatest Breadth.* Measured wherever it can be found above the plane of the ear-holes. The calipers must be held in such a way that the two points lie in the same vertical and horizontal planes.

(3) *Auricular Height.* Two points are placed firmly in the ear-holes, and a third point lying in the same vertical plane is brought down on to the top of the head. The distance between this point and the line joining the other two is read off by means of a scale.

(4) *Maximum Circumference.* Measured by passing a steel tape over the glabella in front and the occipital point behind.

(5) *Sagittal Arc.* Measured by passing the tape from the glabella in front, over the top of the head in the middle line to the inion behind. The inion is a bony projection, developed to very varying extent in different people, which lies in the middle line, just above the area of the skull, to which the fleshy muscles at the back of the neck are attached.

(6) *Transverse Arc.* Measured by passing the tape over the top of the head in a vertical plane from one preauricular point to the other. The preauricular point is the point immediately in front of the tragus, or little projection of ear which lies in front of the ear-hole.

Two cranial indices are deduced from these measurements, namely, the ratio of the greatest breadth to the greatest length, and of the auricular height to the greatest length.

A Flower's Cranimeter was used for measuring the greatest length and the greatest breadth, while the auricular height was taken with a special instrument not hitherto described.

This instrument consists essentially of a long brass box with open ends which forms a bearing for two flat bars. The bars can slide in or out of the box in the direction of its longitudinal axis, but they are constrained to move inwards or outwards to an equal extent as they each carry, where they face one another in the box, a rack which is in mesh with one and the same pinion. Running downwards from the outer ends of the bars and at right angles to them are long arms, to the lower ends of which are attached the ear points. Sliding through bearings in the top and bottom of the box is a round rod, which is brought down on to the top of the subject's head, on it is engraved a scale by which the distance from the top of the head to the line joining the ear points may be arrived at. The object of the two racks and the pinion is to ensure that the end of the rod comes down on to the middle line of the head. This instrument is too heavy to be held in the hand of the operator in the usual way and therefore is suspended from a sort of double gallows and counterpoised.

The numbers available are not sufficient to enable me to make any very definite statement about the changes in head measurements with age, but there is some indication that the length increases during the period dealt with. The mean head length at 18 is 195·2 mm., at 21 196·05, and at 27 197·4 mm. The breadth for these ages 152·9, 152·7, 153·1, respectively, while the highest mean head breadth recorded is, at 22, namely 153·66, so that this measurement shows little if any increase. Consequently the cranial index (breadth to length) drops slightly, being 78·40 for the youngest group, at 77·59 for the oldest. The auricular height shows a very slight tendency to rise, while its index falls slightly. The correlation between stature and head length is ·31, which is more than twice as great as that between stature and head breadth (·14). Consequently there is a slight negative correlation between this and the cranial index B/L. (– ·13.)

The correlation between stature and auricular height is about the same as that between stature and head length, and therefore the auricular height index shows itself as practically independent of stature.

TABLE OF MEANS AND OF STANDARD DEVIATIONS WITH GENERAL AVERAGES OF AGES 18 TO 22; 23 YEARS AND OVER.

TABLE I. *Table of Means**.

Character	Age 18, No. 129	Age 19, No. 330	Age 20, No. 209
Strength of Pull ...	185·2 ± 2·7 lbs.	200·9 ± 1·7 lbs.	204·8 ± 2·2 lbs.
Lung Capacity ...	4184 ± 34 c.c.	4278 ± 22 c.c.	4268 ± 29 c.c.
Weight	10 st. 7 lbs. 9 oz. ± 14 oz.	10 st. 12 lbs. 1 oz. ± 10 oz.	10 st. 11 lbs. 9 oz. ± 12 oz.
Stature	1751·6 ± 3·4 mm.	1767·2 ± 2·5 mm.	1766·2 ± 3·2 mm.
Length of Leg ...	840·2 ± 2·6 mm.	835·0 ± 1·6 mm.	835·2 ± 2·2 mm.
Length of Thigh ...	392·9 ± 1·6 mm.	388·6 ± ·9 mm.	389·2 ± 1·2 mm.
Ratio of Leg to Stature	47·46 ± ·08	47·23 ± ·05	47·27 ± ·06
Ratio of Thigh to Leg	46·65 ± ·06	46·50 ± ·04	46·56 ± ·05
Head Length *L* ...	195·23 ± ·37 mm.	195·66 ± ·22 mm.	195·15 ± ·31 mm.
Head Breadth *B* ...	152·90 ± ·27 mm.	152·63 ± ·18 mm.	152·04 ± ·25 mm.
Auricular Height *H*	136·48 ± ·36 mm.	136·52 ± ·21 mm.	135·97 ± ·28 mm.
Cranial Index *B/L*...	78·40 ± ·18	78·06 ± ·11	77·97 ± ·14
Cranial Index *H/L*...	69·95 ± ·20	69·80 ± ·12	69·77 ± ·14
Horizontal Circumference of Head	560·76 ± ·79 mm.	562·29 ± ·51 mm.	560·90 ± ·73 mm.
Sagittal Arc ...	345·93 ± ·81 mm.	347·35 ± ·48 mm.	346·67 ± ·66 mm.
Transverse Arc ...	361·77 ± ·69 mm.	361·23 ± ·41 mm.	360·35 ± ·62 mm.

Character	Age 21, No. 137	Age 22, No. 95	Age 23 and over, No. 59	General Average
Strength of Pull ...	216·4 ± 2·9 lbs.	211·4 ± 3·6 lbs.	212·9 ± 4·4 lbs.	205·3 lbs.
Lung Capacity ...	4438 ± 39 c.c.	4301 ± 37 c.c.	4418 ± 59 c.c.	4315 c.c.
Weight	11 st. 2 lbs. ± 1 lb.	10 st. 12 lbs. 10 oz. ± 1 lb. 3 oz.	10 st. 11 lbs. 12 oz. ± 1 lb. 6 oz.	10 st. 11 lbs. 15 oz.
Stature	1778·6 ± 4·1 mm.	1757·5 ± 4·6 mm.	1769·2 ± 6·2 mm.	1765·0 mm.
Length of Leg ...	840·3 ± 2·9 mm.	826·7 ± 3·0 mm.	838·1 ± 4·4 mm.	836 mm.
Length of Thigh ...	391·1 ± 1·5 mm.	385·6 ± 1·6 mm.	391·1 ± 2·3 mm.	389·8 mm.
Ratio of Leg to Stature	47·23 ± ·08	47·01 ± ·08	47·38 ± ·11	47·26
Ratio of Thigh to Leg	46·44 ± ·06	46·58 ± ·07	46·63 ± ·10	46·56
Head Length *L* ...	196·05 ± ·38 mm.	196·82 ± ·43 mm.	197·41 ± ·49 mm.	196·05 mm.
Head Breadth *B* ...	152·69 ± ·28 mm.	153·66 ± ·38 mm.	153·11 ± ·38 mm.	152·84 mm.
Auricular Height *H*	136·90 ± ·30 mm.	136·86 ± ·41 mm.	137·01 ± ·51 mm.	136·62 mm.
Cranial Index *B/L*...	78·00 ± ·17	78·08 ± ·20	77·59 ± ·22	78·02
Cranial Index *H/L*...	69·94 ± ·17	69·58 ± ·21	69·35 ± ·21	69·73
Horizontal Circumference of Head	563·64 ± ·89 mm.	565·53 ± 1·06 mm.	565·47 ± 1·02 mm.	563·10 mm.
Sagittal Arc ...	349·67 ± ·74 mm.	348·57 ± ·95 mm.	350·17 ± 1·10 mm.	348·06 mm.
Transverse Arc ...	361·45 ± ·67 mm.	364·90 ± ·87 mm.	364·12 ± 1·01 mm.	362·30 mm.

---

* [The following comparative data for 1000 Cambridge and 493 Scottish Undergraduates may be of interest:

	Cambridge		Aberdeen	
	Mean	Standard Deviation	Mean	Standard Deviation
Strength of Pull	84·0 ± ·27 lbs.	12·7 ± ·2 lbs.	—	—
Weight ...	10 st. 12·8 lbs. ± ·35 lbs.	16·55 ± ·25 lbs.	—	—
Stature ...	1749 ± 1·4 mm.	64·6 ± 1·0 mm.	1717 ± 1·8 mm.	59·4 ± 1·3 mm.
Head Length ...	193·5 ± ·13 mm.	6·16 ± ·09 mm.	194·8 ± ·17 mm.	5·73 ± ·12 mm.
Head Breadth...	154·0 ± ·11 mm.	5·06 ± ·08 mm.	153·4 ± ·14 mm.	4·69 ± ·10 mm.
Head Index ...	79·6 ± ·06	2·999 ± 0·044	78·8 ± ·09	2·79 ± ·06

See *R. Soc. Proc.* Vol. 66, p. 26, *Biometrika*, Vol. I. pp. 188 et seq., and *Proc. Anat. and Anthrop. Soc.* University of Aberdeen (Macdonell), 1906–8.

It will be seen at once that the measurement of pull at Oxford and Cambridge are made by different methods and are wholly incomparable. Cambridge appears in weight and breadth of head to exceed

TABLE II. *Table of Standard Deviations.*

Character	Age 18, No. 129	Age 19, No. 330	Age 20, No. 209
Strength of Pull ...	44·66 ± 1·88 lbs.	45·99 ± 1·20 lbs.	46·17 ± 1·52 lbs.
Lung Capacity ...	566·2 ± 23·8 c.c.	602·5 ± 15·8 c.c.	611·6 ± 20·2 c.c.
Weight	1 st. 1 lb. 1 oz. ± 10 oz.	1 st. 2 lbs. 12 oz. ± 7 oz.	1 st. 2 lbs. 12 oz. ± 9 oz.
Stature	57·92 ± 2·43 mm.	62·22 ± 1·63 mm.	68·09 ± 2·25 mm.
Length of Leg ...	43·40 ± 1·82 mm.	43·84 ± 1·15 mm.	46·24 ± 1·52 mm.
Length of Thigh ...	26·83 ± 1·13 mm.	24·45 ± ·64 mm.	24·98 ± ·82 mm.
Ratio of Leg to Stature	1·300 ± ·055	1·335 ± ·035	1·287 ± ·043
Ratio of Thigh to Leg	1·076 ± ·045	1·123 ± ·029	1·017 ± ·034
Head Length L ...	6·30 ± ·26 mm.	5·98 ± ·16 mm.	6·73 ± ·22 mm.
Head Breadth B ...	4·62 ± ·19 mm.	4·88 ± ·13 mm.	5·25 ± ·17 mm.
Auricular Height H	6·08 ± ·25 mm.	5·73 ± ·15 mm.	5·93 ± ·20 mm.
Cranial Index B/L ...	3·08 ± ·13	2·99 ± ·08	3·00 ± ·10
Cranial Index H/L ...	3·38 ± ·14	3·16 ± ·08	3·02 ± ·10
Horizontal Circumference of Head	13·38 ± ·56 mm.	13·73 ± ·36 mm.	15·65 ± ·52 mm.
Sagittal Arc ...	13·65 ± ·57 mm.	12·85 ± ·34 mm.	14·12 ± ·46 mm.
Transverse Arc ...	11·71 ± ·49 mm.	11·16 ± ·29 mm.	13·19 ± ·44 mm.

Character	Age 21, No. 137	Age 22, No. 95	Age 23 and over, No. 59	General Average
Strength of Pull ...	50·25 ± 2·04 lbs.	51·73 ± 2·53 lbs.	50·31 ± 3·12 lbs.	48·18 lbs.
Lung Capacity ...	683·1 ± 27·8 c.c.	540·9 ± 26·5 c. c.	674·8 ± 41·9 c.c.	613·2 c.c.
Weight	1 st. 3 lbs. 3 oz. ± 11 oz.	1 st. 2 lbs. 13 oz. ± 13 oz.	1 st. 1 lb. 15 oz. ± 1 lb.	1 st. 2 lbs. 6 oz.
Stature	71·44 ± 2·91 mm.	66·20 ± 3·24 mm.	70·59 ± 4·38 mm.	66·08 mm.
Length of Leg ...	49·53 ± 2·02 mm.	42·78 ± 2·09 mm.	49·63 ± 3·08 mm.	45·90 mm.
Length of Thigh ...	25·73 ± 1·05 mm.	23·15 ± 1·13 mm.	26·66 ± 1·66 mm.	25·30 mm.
Ratio of Leg to Stature	1·311 ± ·053	1·141 ± ·056	1·261 ± ·078	1·273
Ratio of Thigh to Leg	1·021 ± ·042	1·030 ± ·050	1·085 ± ·067	1·059
Head Length L ...	6·59 ± ·27 mm.	6·20 ± ·30 mm.	5·55 ± ·34 mm.	6·23 mm.
Head Breadth B ...	4·84 ± ·20 mm.	5·51 ± ·27 mm.	4·39 ± ·27 mm.	4·92 mm.
Auricular Height H	5·27 ± ·21 mm.	5·97 ± ·29 mm.	5·79 ± ·36 mm.	5·80 mm.
Cranial Index B/L ...	2·98 ± ·12	2·94 ± ·14	2·55 ± ·16	2·92
Cranial Index H/L ...	3·00 ± ·12	3·08 ± ·15	2·42 ± ·15	3·01
Horizontal Circumference of Head	15·51 ± ·63 mm.	15·23 ± ·75 mm.	11·65 ± ·72 mm.	14·19 mm.
Sagittal Arc ...	12·87 ± ·52 mm.	13·84 ± ·68 mm.	12·48 ± ·77 mm.	13·30 mm.
Transverse Arc ...	11·74 ± ·47 mm.	12·49 ± ·61 mm.	11·46 ± ·71 mm.	11·96 mm.

Oxford, but Oxford men are taller and longer headed. Thus they are more dolico-cephalic than the Cambridge men, 78·0 as against 79·6. Macdonell's 3000 criminals gave a cephalic index of 78·5. The average head length of 196 mm. agrees fairly well with Pearson's measurements at University College, as the table below will indicate (*Phil. Trans.* Vol. 196 A, p. 251), U = Horizontal Circumference.

	L	B	H	U	B/L
British Association Measurements	198·8	155·0	130·9	—	78·0
University College, London Staff	196·4	153·5	134·8	—	78·2
Oxford Undergraduates ...	196·0	152·8	136·6	563·6	78·0
Scottish Undergraduates ...	194·8	153·4	132·3	562·6	78·8
Cambridge Undergraduates ...	193·5	154·0	?	—	79·6
English Criminals	191·4	150·3	?	—	78·5

EDITOR.]

TABLE III.

*Table of Correlation Coefficients**.

Correlation Coefficient	Age 18, No. 129	Age 19, No. 330	Age 20, No. 209	Age 21, No. 137	Age 22, No. 95	General Average
Stature and Length of Leg ...	+·87±·01	+·86±·01	+·89±·01	+·91±·01	+·91±·01	+·89
„ and ratio of Leg to Stature	+·44±·05	+·37±·03	+·42±·04	+·50±·04	+·45±·06	+·44
„ and Length of Thigh ...	+·80±·02	+·74±·02	+·82±·02	+·82±·02	+·83±·02	+·80
„ and ratio of Thigh to Leg	+·31±·05	+·08±·04	+·16±·05	−·06±·06	+·06±·07	+·11
„ and Weight	+·50±·04	+·63±·02	+·68±·03	+·76±·02	+·72±·03	+·66
„ and Lung Capacity ...	+·44±·05	+·65±·02	+·55±·03	+·64±·03	+·59±·04	+·57
„ and Strength of Pull ...	+·01±·06	+·25±·03	+·22±·04	+·27±·05	+·28±·06	+·21
„ and Head Length *L* ...	+·22±·06	+·31±·03	+·35±·04	+·36±·05	+·33±·06	+·31
„ and Head Breadth *B* ...	+·05±·06	+·17±·04	+·19±·05	+·10±·06	+·17±·07	+·14
„ and Auricular Height *H*	+·09±·06	+·28±·03	+·31±·04	+·32±·05	+·42±·06	+·28
„ and Cranial Index *B/L* ...	−·06±·06	−·10±·04	−·15±·05	−·23±·05	−·10±·07	−·13
„ and Cranial Index *H/L* ...	+·01±·06	+·05±·04	+·03±·05	+·02±·06	+·21±·07	+·06
Head Length and Sagittal Arc of Head	+·65±·03	+·64±·02	+·67±·03	+·64±·03	+·65±·04	+·65
Head Length and Transverse Arc of Head	+·32±·05	+·33±·03	+·54±·03	+·42±·04	+·53±·05	+·43
Head Length and Horizontal Circumference of Head	+·83±·02	+·80±·01	+·86±·01	+·86±·01	+·80±·03	+·83
Strength of Pull and Weight ...	+·37±·05	+·52±·03	+·45±·04	+·47±·04	+·48±·05	+·46
„ „ and Lung Capacity	+·44±·05	+·40±·03	+·37±·04	+·32±·05	+·31±·06	+·37
„ „ and ratio of Leg to Stature	−·19±·06	−·05±·04	−·07±·05	+·00±·06	+·27±·06	−·01
Ratio of Thigh to Leg and ratio of Leg to Stature	+·27±·06	+·43±·03	+·29±·04	+·30±·05	+·18±·07	+·29
Lung Capacity and Weight ...	+·55±·04	+·62±·02	+·52±·03	+·66±·03	+·62±·04	+·59

---

* [The following results for 1000 Cambridge Undergraduates may be of interest for purposes of comparison. See Pearson, *R. Soc. Proc.* Vol. 66, p. 26, Dec. 1899, Macdonell, *Biometrika*, Vol. I. pp. 188, 202.

Stature and Weight	+·49±·02	Stature and Head Length ...	+·28±·02
Stature and Pull	+·30±·02	Stature and Head Breadth ...	+·15±·02
Weight and Pull	+·56±·02		
Stature and Head Index ...	−·08±·02	3000 Criminals	
Weight and Head Index ...	+·01±·02	Stature and Head Length ...	+·34±·02
Pull and Head Index	+·04±·02	Stature and Head Breadth ...	+·18±·02

EDITOR.]

ON THE CORRELATION BETWEEN SOMATIC CHARACTERS AND FERTILITY: ILLUSTRATIONS FROM THE INVOLUCRAL WHORL OF *HIBISCUS*.

By J. ARTHUR HARRIS, Ph.D., Carnegie Institution of Washington, U.S.A.

INTRODUCTORY.

In several places* Pearson has pointed out the importance of knowing whether fertility be correlated with somatic characters. Recently† he sums up his experience of this matter as follows:

> "I should be surprised to find a large inheritance of fertility now, just as I should be surprised not to find a large inheritance of any somatic character. And on the evidence we have at present, I should not expect to find a correlation between fertility and a somatic character; if it does exist in any case, I should anticipate very rapid changes going on in that species."

One of the phases of the investigations of fertility and fecundity in plants which have occupied me for the past several years is the relationship between the somatic characters and fertility and fecundity. The time now seems ripe to present the results of certain limited phases of this work. I select for the present paper results of studies on the involucral whorl of Malvaceous genus *Hibiscus*.

At the outset I must confess that I took up the work with a prejudice in favor of a sensible relationship between the number of bracts and the fertility of the fruit. The intimate spatial association of the involucral whorl and the carpellary whorl seemed justification enough for this hypothesis. Furthermore I believe botanists generally would have concurred in this opinion. Certain it is that Baur‡ writes with confidence of the factors producing a chance variation curve in the size of the bean seed, and includes number of pods and number of assimilating leaves on the branch among them, although no weighings are given in substantiation. One of the most prominent European botanists intimated to me that a correlation

---

* For instance *Proc. Roy. Soc.* Vol. LXIX. p. 301; *Phil. Trans. Roy. Soc.* Vol. CXCII. p. 258.

† *Biometrika*, Vol. VII. pp. 369—370, 1910.

‡ Baur, E.: *Ber. Deutsch. Bot. Ges.* Vol. XXV. pp. 444—446, 1910.

between the number of pods on a plant and the fertility of the pods was so *a priori* evident that its demonstration on the bases of actual material was superfluous.

The determination of the degree of interdependence of somatic characters and fertility in plants presents some special difficulties as compared with the working out of the same problem on zoological material. The chief obstacle is the fact that in the higher plants the reproductive organs are not so specialized nor so clearly differentiated from the other organs as in the higher animals. It is, therefore, difficult to obtain "somatic" characters which are not directly connected up with the reproductive organs either (1) as a means of support, (2) as a conducting system for plastic materials, or (3) as a direct source of food supply.

This difficulty has been incurred in the attempts which have been made to measure the relationship between the vegetative and reproductive organs. In the case of the correlation between diameter of the stem and fruit length in *Oenothera* studied by de Vries* it must be remembered that the stem not only served to support the fruits but transported the material of which they were built up and bore the leaves which are largely concerned in its manufacture. In the study of the relationship between length of flowering stalk and number of flowers in *Nothoscordium* and *Allium*† where a correlation of about ·500 was found, it is certainly quite impossible to regard the length of the stem as a strict "somatic" character and the number of flowers as an independent "fertility" character‡, so intimate is the structural relationship of the two. The same is true for the interdependence between the number of leaves and flowers per inflorescence in *Spiraea Vanhouttei* to be published later. In *Sanguinaria*§, where a correlation between length of peduncle and number of ovules produced and seeds developing per fruit has been demonstrated, there is perhaps less reason to think of a necessary structural interdependence of the characters.

But when one takes the relationship between the number of fruits or flowers per inflorescence and the number of ovules or seeds maturing per fruit there is no patent morphological reason why there should be a correlation between number of fruits per inflorescence and number of ovules per fruit, although the biologist might say that the competition of a number of fruits above the average for plastic material would result in a negative correlation for a number of fruits and number of seeds per fruit.

Very low correlations have been found for both number of flowers and number of fruits per inflorescence and number of seeds per fruit in *Celastrus*‖ and for number of ovaries per inflorescence and number of ovules per ovary in *Cercis* and in a large series of unpublished observations on *Staphylea*—now nearly ready for the press.

---

* *Die Mutationstheorie*, Vol. I. p. 113, 1901.

† Harris, J. Arthur: *Ann. Rep. Mo. Bot. Gard.* Vol. XX. pp. 105—115, 1909.

‡ Although for convenience I have occasionally referred to them as such.

§ Harris, J. Arthur: *Biometrika*, Vol. VII. pp. 314—317, 1910.

‖ Harris, J. Arthur: *Ann. Rep. Mo. Bot. Gard.* Vol. XX. p. 122, 1909.

MATERIAL.

The involucral bracts of *Hibiscus* seem particularly adapted for an investigation of the relationship between a somatic character and fertility.

Without having as far as we know any important physiological relationship to the essential organs of the flower, the involucral whorl is nevertheless in the closest ontogenetic association with them*.

Diagrams of involucral whorls of *Hibiscus*. Figures 1 and 2 show involucre *in situ* with the calyx and matured fruit; others show involucre only. Fig. 1, *H. moscheutos;* fig. 2, *H. syriacus;* fig. 3, *H. militaris;* fig. 4, *H. trionum;* fig. 5, *H. manihot.* All are natural size.

---

* This statement is meant to apply only to the species discussed in this paper and is not intended to exclude the ecological function of protection to the immature flower bud. In some species of plants the involucral whorl is so large and chlorophyllaceous that it may take some part in photosynthesis, possibly contributing sensibly to the development of the seeds. Again, in other forms the involucre may undergo extensive modifications during—and possibly in consequence of—the development of the fruit. In the species discussed neither of these possibilities need to be taken into consideration. The figures will make the structures perfectly clear.

The technique of counting was quite simple. Variation in the number of bractlets is not discontinuous but continuous, if by continuity one understands the presence of transitional conditions. These intermediate stages—appearing sometimes as one or more bracts conspicuously smaller than the others, sometimes as large apparently compound or longitudinally fused bractlets—are relatively rare in five of the six species considered. In the preparation of the tables for our present use, it has seemed sufficient to throw them into the nearest integral class. An exception is made in the case of *Hibiscus manihot.* The bracts in this form are not subulate or filiform but broad and very irregular in size and completeness of division. Considerable numbers of fruits have been halved between two adjoining bract classes*.

In the species in which the number of aborted ovules could be counted, both the number of ovules formed and the number of seeds developing per locule were recorded. Only fruits with the same number of locules were included in any correlation table†.

Analysis of Data.

Hibiscus militaris, Cav.

A series of *Hibiscus militaris* was collected on the margins of a lake in the low Illinois flood plain of the Mississippi river across from the Carondelet ferry at St Louis. The collection was made at about the middle of September and cannot safely be regarded as quite typical of the species in that habitat because the greater number of the pods had already dehisced and the collection is there-

TABLE I.

Hibiscus militaris.

Seeds Developing per Locule.

Number of Bracts.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	Totals
8	—	—	—	—	—	—	—	—	—	1	2	1	1	—	—	—	—	5
9	—	—	—	—	—	—	—	1	2	1	2	3	3	3	—	—	—	15
10	1	1	—	2	10	11	5	12	9	26	33	58	74	56	26	1	—	325
11	1	1	—	2	6	14	12	29	31	69	124	141	220	174	93	19	4	940
12	—	—	4	5	3	16	21	34	35	58	140	186	235	157	68	19	4	985
13	—	2	3	5	1	7	6	12	34	39	69	114	121	112	60	9	1	595
14	—	—	—	1	—	1	1	3	1	7	17	22	22	17	13	—	—	105
15	—	—	—	—	—	—	—	—	—	—	2	1	2	—	—	—	—	5
Totals	2	4	7	15	20	49	45	91	112	201	389	526	678	519	260	48	9	2975

---

* In all the forms except *Hibiscus manihot*, the number of bracts furnishes a fairly satisfactory index to the amount of bract tissue, but in this species the irregularity in size is so great that number cannot safely be taken as representing mass.

† Since we are dealing with integral variates, Sheppard's modification was not applied.

In calculating the probable error of the constants for the bracts and for correlations involving the number of bracts, the actual number of fruits was used—not the number weighted with the locules as tabled in many cases. For the probable errors of the means and standard deviations of number of ovules or seeds per locule N was taken as the actual number of locules.

fore composed chiefly of the late fruits from ordinary individuals or of the fruits of late-flowering plants. Considering the results obtained from other species of Hibiscus, I think these facts throw no doubt on the conclusions stated in this paper, but they must be borne in mind by anyone considering the data as establishing a "place mode" for the species.

Altogether 595 fruits were available for countings of the number of bracts, number of sepals and number of seeds maturing per locule. Of the 595 involucres only 29 were noted as at all abnormal: in all of these two adjoining bractlets seemed slightly grown together at the base.

Table I shows the correlation between the number of bracts and the number of seeds developing in the five individual locules. I find

$$M_b = 11{\cdot}719 \pm {\cdot}029,\quad \sigma_b = 1{\cdot}050 \pm {\cdot}021,\quad V_b = 8{\cdot}961,$$
$$M_s = 11{\cdot}114 \pm {\cdot}028,\quad \sigma_s = 2{\cdot}300 \pm {\cdot}020,\quad V_s = 20{\cdot}699,$$
$$r_{bs} = {\cdot}052 \pm {\cdot}028,$$

a value not significant with regard to its probable error. The means of arrays show no evidence of non-linearity, hence r can be regarded as a satisfactory description of the relationship.

Hibiscus lasiocarpus, Cav.

On the shores of the same lakes *H. lasiocarpus* was associated in great abundance with the preceding species. The fruits seem to dehisce a little earlier, and I was only able to secure 89 perfect ones. Table II shows the correlation between the number of bracts and the number of seeds per locule. The constants are:

$$M_b = 11{\cdot}910 \pm {\cdot}078,\quad \sigma_b = 1{\cdot}098 \pm {\cdot}056,\quad V_b = 9{\cdot}218,$$
$$M_s = 20{\cdot}542 \pm {\cdot}210,\quad \sigma_s = 6{\cdot}753 \pm {\cdot}153,\quad V_s = 32{\cdot}876,$$
$$r_{bs} = {\cdot}005 \pm {\cdot}072.$$

Hibiscus moscheutos, L.

The Swamp Rose Mallow is the third of the three native species of the North-eastern United States. A collection of these fruits from 257 inflorescences was made in a small marshy meadow near Milburn, New Jersey, September 5, 1908. The relationship between the number of bractlets and the number of seeds developing per locule appears in Table III. Analysis gives:

$$M_b = 12{\cdot}630 \pm {\cdot}026,\quad \sigma_b = 1{\cdot}134 \pm {\cdot}019,\quad V_b = 8{\cdot}980,$$
$$M_s = 23{\cdot}449 \pm {\cdot}046,\quad \sigma_s = 4{\cdot}440 \pm {\cdot}032,\quad V_s = 18{\cdot}934,$$
$$r_{bs} = {\cdot}043 \pm {\cdot}023.$$

TABLE II.

Hibiscus lasiocarpus.

Seeds Developing per Locule.

Number of Bracts.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	Totals
9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	5
10	—	—	—	—	—	—	—	—	—	—	—	3	2	3	3	1	1	—	1	2	3	5	—	—	2	2	—	1	1	—	—	—	—	—	—	—	—	30
11	—	—	2	—	1	1	1	8	7	3	3	10	5	5	6	9	2	5	11	8	4	6	6	4	4	1	1	1	1	—	1	—	—	—	3	—	1	120
12	1	4	1	5	4	2	2	3	4	2	2	2	6	15	4	17	13	10	10	8	12	9	6	6	10	2	3	—	6	1	—	—	—	—	—	—	—	170
13	1	1	—	1	—	2	3	2	3	2	—	5	5	3	4	4	6	12	7	5	5	4	4	3	1	1	1	3	3	3	—	—	1	4	1	—	—	100
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
15	—	—	—	—	—	—	—	—	3	2	—	—	—	—	1	1	—	1	—	2	—	—	1	1	1	—	1	—	—	1	—	—	—	—	—	—	—	15
Totals	2	5	3	6	5	5	6	13	17	9	5	20	18	26	18	33	23	29	30	26	26	26	17	15	18	6	6	5	11	5	1	—	1	4	4	—	1	445

TABLE III.

Hibiscus moscheutos.

Seeds Developing per Locule.

Number of Bracts.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	Totals
9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	2	—	—	—	—	—	—	—	—	—	—	—	5
10	—	—	—	—	—	—	—	1	2	—	1	—	3	1	2	1	3	4	5	3	6	3	5	6	7	8	7	4	6	1	1	—	—	—	—	—	80
11	—	—	—	1	—	—	—	1	1	—	1	2	3	6	8	1	9	9	21	21	21	40	40	47	57	56	53	52	39	29	10	6	2	4	—	—	540
12	2	1	1	—	—	—	2	—	3	2	6	2	6	6	8	12	32	34	46	52	64	100	132	151	162	159	129	110	78	47	35	14	4	—	—	—	1400
13	2	1	—	—	—	1	2	4	1	5	2	5	9	11	11	22	16	28	39	60	82	99	94	113	129	133	139	106	92	68	49	22	10	8	1	1	1365
14	2	—	—	—	1	—	1	2	—	1	1	6	6	3	5	4	14	12	17	24	25	36	52	61	65	70	73	61	38	31	19	13	4	3	—	—	650
15	2	—	—	—	1	—	—	—	—	—	—	—	5	2	1	3	4	2	4	1	7	9	17	23	13	32	26	28	11	6	8	7	3	—	—	—	215
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	5	6	1	2	2	2	1	1	2	—	—	—	—	—	—	25
Totals	8	2	1	1	2	1	5	8	7	8	11	15	32	29	35	43	78	90	134	162	205	293	347	402	437	460	429	362	265	184	122	62	23	15	1	1	4280

Hibiscus trionum, L.

The three preceding species are tall-growing native perennials. *H. trionum* is an adventitious herb, widely disseminated in the Eastern United States—sometimes a rather serious weed.

TABLE IV.

Hibiscus trionum.

Ovules per Locule.

Number of Bracts.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
8	—	1	4	—	—	—	—	—	—	5
9	—	—	5	10	6	—	2	2	—	25
10	2	9	31	57	54	20	21	1	—	195
11	—	8	40	40	80	45	25	2	—	240
12	1	7	26	60	115	60	35	5	1	310
13	—	1	8	13	22	13	1	11	1	70
Totals	3	26	114	180	277	138	84	21	2	845

TABLE V.

Hibiscus trionum.

Seeds Developing per Locule.

Number of Bracts.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	Totals
8	—	—	—	—	—	1	4	—	—	—	—	—	5
9	—	—	—	—	—	3	10	7	1	3	1	—	25
10	—	—	—	1	5	26	40	55	45	15	8	—	195
11	—	—	3	3	7	24	65	37	59	24	16	2	240
12	2	—	1	6	10	24	52	83	80	36	16	—	310
13	—	—	—	—	—	6	11	16	21	7	4	5	70
Totals	2	—	4	10	22	84	182	198	206	85	45	7	845

As material I have 169 fruits taken from a large number of plants growing in a very poor weed-infested pasture near Mt. Hermon, Athens Co., Ohio, in the fall of 1907. The data are given in Tables IV—V, and the constants for number of bracts and ovules and seeds per locule are:

$$M_b = 11{\cdot}225 \pm {\cdot}053, \quad \sigma_b = 1{\cdot}024 \pm {\cdot}038, \quad V_b = 9{\cdot}129,$$

$$M_o = 7{\cdot}857 \pm {\cdot}032, \quad \sigma_o = 1{\cdot}374 \pm {\cdot}023, \quad V_o = 17{\cdot}491,$$

$$M_s = 7{\cdot}058 \pm {\cdot}037, \quad \sigma_s = 1{\cdot}589 \pm {\cdot}026, \quad V_s = 22{\cdot}576,$$

for bracts and ovules $r = {\cdot}186 \pm {\cdot}050, \quad r/Er = 3{\cdot}51,$

for bracts and seeds $r = {\cdot}106 \pm {\cdot}051, \quad r/Er = 2{\cdot}08.$

These values are both low, but possibly the first is significant.

Hibiscus syriacus, L.

This tall exotic shrub grows admirably in many of our gardens where it is sometimes used as a hedge plant. For about five years I have been engaged in a study of some of the phases of fertility and fecundity in these fruits and the results bearing on our present problem may be considered here.

The material consists of three collections from a fine row of young shrubs in the Missouri Botanical Garden at St Louis, and one from much older plants near the shore at Cold Spring Harbor. The St Louis collections were made in the autumns of 1905, 1907 and 1908. In 1905, 1000 fruits were taken as a random sample from about 35 shrubs in a hedge in the main garden. In 1907, the shrubs were studied during the flowering season as well, with the result that one was found to bear purple instead of white flowers; this was omitted and about 100 fruits each from 34 white-flowered individuals taken—altogether 3393 fruits. In 1908, it was impossible because of the pressure of other work to take up *Hibiscus* until many of the fruits had dehisced. Furthermore, the shrubs had grown so tall that it had been necessary to cut them back during the season of growth, and since whole inflorescences were wanted the collection was chiefly confined to those lower on the plant which had escaped the shears. Again the same 34 plants were visited; 100 fruits could be obtained from some but not from all, so only 2364 were secured.

TABLE VI. *Hibiscus syriacus.*

Total Ovules per Pod.

	20	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*
M. B. G., 1905 ...	—	—	—	—	3	2	4	18	45	107	188	125	81	80	80
M. B. G., 1907 ...	3	—	—	3	4	8	12	37	70	261	1025	379	272	247	205
M. B. G., 1908 ...	—	—	1	—	—	3	5	4	22	88	875	335	263	160	142
C. S. H., 1908, A	—	—	—	—	—	—	—	7	7	22	57	48	60	71	68
C. S. H., 1908, B	—	—	—	—	—	—	—	—	—	—	—	1	5	2	6

TABLE VI. (*continued*).

	35	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	Totals
M. B. G., 1905 ...	85	54	54	34	27	13	—	—	—	—	—	—	—	—	—	—	—	1000
M. B. G., 1907 ...	191	173	140	141	121	85	14	1	1	—	—	—	—	—	—	—	—	3393
M. B. G., 1908 ...	118	101	90	57	54	43	3	—	—	—	—	—	—	—	—	—	—	2364
C. S. H., 1908, A	70	117	151	201	294	322	67	17	5	7	1	1	—	—	1	—	—	1594
C. S. H., 1908, B	12	18	20	17	29	15	23	33	50	53	64	84	67	58	5	2	1	565

The Cold Spring Harbor fruits were a random sample from large plants perhaps ten feet in height, forming a close untrimmed hedge about six rods long. An estimate of the number of individuals was not made because of the large amount of branching from the ground. These plants—so far as flowers still remained at the time I visited them—belonged to a dark purple variety.

TABLE VII.

Hibiscus syriacus.

Frequency of Total Number of Seeds per Pod.	M. B. G., 1905	M. B. G., 1907	M. B. G., 1908	C. S. H., 1908, A	C. S. H., 1908, B
0	—	5	1	1	—
1	—	20	4	—	—
2	4	57	14	2	—
3	5	74	24	1	—
4	3	80	37	2	—
5	6	98	39	1	3
6	14	81	49	6	1
7	14	95	46	2	2
8	23	103	44	6	1
9	20	88	30	9	1
10	21	84	58	13	1
11	35	88	60	19	5
12	23	84	57	21	4
13	42	82	58	29	2
14	51	120	61	28	6
15	37	129	61	31	11
16	51	128	78	34	5
17	44	121	68	47	18
18	42	118	88	43	11
19	58	96	78	52	11
20	47	151	108	54	12
21	55	142	99	67	14
22	39	141	101	76	16
23	50	152	126	78	27
24	53	177	124	86	24
25	48	151	134	109	28
26	36	162	128	90	23
27	40	123	127	78	28
28	38	113	102	92	26
29	25	101	118	69	19
30	20	71	81	102	21
31	21	53	52	79	17
32	10	39	39	68	32
33	9	26	28	64	24
34	5	19	17	46	34
35	7	8	15	31	22
36	2	5	3	27	25
37	2	5	4	13	20
38	—	2	3	11	19
39	—	1	—	3	23
40	—	—	—	3	9
41	—	—	—	1	10
42	—	—	—	—	5
43	—	—	—	—	1
44	—	—	—	—	3
45	—	—	—	—	—
46	—	—	—	—	—
47	—	—	—	—	1
Totals	1000	3393	2364	1594	565

TABLE VIII.

Hibiscus syriacus.

Number of Ovules per Locule.

Series	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	Totals
M. B. G., 1905 ...	3	5	63	365	2476	1334	754	—	—	—	5000
M. B. G., 1907 ...	—	3	67	687	9799	4005	2359	45	—	—	16965
M. B. G., 1908 ...	1	—	11	234	7655	2643	1258	18	—	—	11820
C. S. H., 1908, A ...	—	4	20	136	1171	2050	4321	243	25	—	7970
C. S. H., 1908, B ...	—	5	18	91	600	1098	1509	62	5	2	3390

TABLE IX.

Hibiscus syriacus.

Number of Seeds Developing per Locule.

Series	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
M. B. G., 1905	105	365	669	921	1009	918	729	210	74	—	—	5000
M. B. G., 1907	1129	1839	2319	2535	2986	2975	2324	697	156	5	—	16965
M. B. G., 1908	438	840	1307	1671	2178	2451	2122	662	150	1	—	11820
C. S. H., 1908, A	59	168	495	888	1397	1743	1632	1146	429	12	1	7970
C. S. H., 1908, B	26	97	240	468	620	749	658	402	129	1	—	3390

TABLE X.

Hibiscus syriacus.

Series	M. B. G., 1905			M. B. G., 1907			M. B. G., 1908			C. S. H., 1908, A			C. S. H., 1908, B		
	Pods	Total Ovules	Total Seeds	Pods	Total Ovules	Total Seeds	Pods	Total Ovules	Total Seeds	Pods	Total Ovules	Total Seeds	Pods	Total Ovules	Total Seeds
4	—	—	—	—	—	—	1	30	27	—	—	—	—	—	—
5	—	—	—	35	1125	534	42	1336	690	14	509	337	4	154	111
6	236	7726	4614	732	23651	12442	620	20256	12686	335	12355	8365	74	3178	2138
7	400	12978	7806	1478	47958	26735	991	31740	20370	689	25417	16856	214	9318	6036
8	281	9009	5531	900	29016	16659	518	16390	10449	425	15882	10454	215	9338	6077
9	67	2101	1254	216	7011	4008	159	5093	3161	83	3149	2194	48	2095	1351
10	15	475	297	29	958	585	32	1000	612	14	542	401	10	447	296
11	—	—	—	3	99	54	1	28	14	—	—	—	—	—	—
12	1	35	26	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1000	32324	19528	3393	109818	61017	2364	75873	48009	1560	57854	38607	565	24530	16009

The Cold Spring Harbor plants presented in a marked degree the peculiarity of producing a high percentage of six-loculed fruits. In the St Louis material only a very few of these were found; it seems worth while to consider both types: in the Tables, *A* designates the five-loculed and *B* the six-loculed fruits. The data appear in Tables VI—X*. The means and variabilities are shown for the bracts in Table XI, and for number of ovules and seeds in Table XII. The means for the total number per fruit are unnecessary since they may be at once obtained from those for the individual locules by multiplication. The correlation coefficients are laid side by side in Table XIII.

TABLE XI.

Constants for Number of Bracts.

Series of Material	Mean and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation
Missouri Botanical Garden, 1905	7·2290 ± ·0201	·9448 ± ·0142	13·069
Missouri Botanical Garden, 1907	7·1854 ± ·0106	·9130 ± ·0075	12·707
Missouri Botanical Garden, 1908	7·0969 ± ·0135	·9716 ± ·0095	13·691
Cold Spring Harbor, 1908, A ...	7·1731 ± ·0152	·8870 ± ·0107	12·365
Cold Spring Harbor, 1908, B ...	7·4584 ± ·0258	·9104 ± ·0183	12·207

TABLE XII.

Constants for Ovules and Seeds.

Series of Material	Number per Locule			Number per Fruit	
	Mean and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation	Standard Deviation and Probable Error	Coefficient of Variation
Missouri Botanical Garden, 1905					
Ovules	6·4648 ± ·0085	·8922 ± ·0060	13·800	3·1776 ± ·0479	9·830
Seeds	3·9056 ± ·0167	1·7564 ± ·0118	44·9723	7·0506 ± ·1063	36·105
Missouri Botanical Garden, 1907					
Ovules	6·4732 ± ·0041	·8063 ± ·0030	12·456	3·3035 ± ·0270	10·207
Seeds	3·5966 ± ·0101	1·9549 ± ·0072	54·352	8·4124 ± ·0689	46·780
Missouri Botanical Garden, 1908					
Ovules	6·4190 ± ·0044	·7158 ± ·0031	11·151	2·8223 ± ·0277	8·793
Seeds	4·0617 ± ·0115	1·8566 ± ·0081	45·711	7·9675 ± ·0782	39·233
Cold Spring Harbor, 1908, A ...					
Ovules	7·4220 ± ·0066	·8718 ± ·0047	11·747	3·3045 ± ·0395	8·905
Seeds	4·9547 ± ·0131	1·7297 ± ·0092	34·910	6·8514 ± ·0819	27·656
Cold Spring Harbor, 1908, B ...					
Ovules	7·2360 ± ·0106	·9170 ± ·0075	12·673	3·9436 ± ·0791	9·083
Seeds	4·7224 ± ·0105	·9108 ± ·0075	19·287	7·8900 ± ·2239	27·846

---

* Knowing the distribution of the three characters we can calculate the coefficient of correlation from tables showing the total ovules and total seeds associated with each class of bracts. The method needs no explanation to professional statisticians. A description of the details for the use of biologists will be published in the *American Naturalist*.

TABLE XIII. *Correlation for Bracts and Fertility.*

Series of Material	Correlation for Bracts and Number per Locule	Correlation for Bracts and Number per Fruit
Missouri Botanical Garden, 1905		
Ovules	$- \cdot 0753 \pm \cdot 0212$	$- \cdot 1057 \pm \cdot 0211$
Seeds	$\cdot 0287 \pm \cdot 0213$	$- \cdot 0039 \pm \cdot 0213$
Missouri Botanical Garden, 1907		
Ovules	$\cdot 0039 \pm \cdot 0116$	$\cdot 0048 \pm \cdot 0116$
Seeds	$\cdot 0603 \pm \cdot 0115$	$\cdot 0700 \pm \cdot 0115$
Missouri Botanical Garden, 1908		
Ovules	$- \cdot 0830 \pm \cdot 0138$	$- \cdot 1053 \pm \cdot 0137$
Seeds	$- \cdot 0638 \pm \cdot 0138$	$- \cdot 0075 \pm \cdot 0139$
Cold Spring Harbor, 1908, A		
Ovules	$\cdot 0252 \pm \cdot 0171$	$- \cdot 0336 \pm \cdot 0171$
Seeds	$\cdot 0032 \pm \cdot 0171$	$\cdot 0041 \pm \cdot 0171$
Cold Spring Harbor, 1908, B		
Ovules	$\cdot 0488 \pm \cdot 0283$	$\cdot 0683 \pm \cdot 0282$
Seeds	$- \cdot 0110 \pm \cdot 0284$	$- \cdot 0077 \pm \cdot 0284$

The constants in this Table show considerable irregularity; perhaps some differ from 0 by an amount greater than can be legitimately attributed to the probable errors of random sampling. But from the table as a whole, with most of its values insignificant in comparison with their probable errors and both negative and positive in sign, only one conclusion is possible—that there is no correlation between the number of bracts and fertility.

Hibiscus manihot, L.

This was one of the species represented in considerable numbers in my cultures at the Station for Experimental Evolution in the summer of 1908. The data are given for the relationships for bracts and number of ovules per locule in Tables XIV and XV. These give:

$$M_b = 5 \cdot 173 \pm \cdot 011, \quad \sigma_b = \cdot 802 \pm \cdot 008, \quad V_b = 15 \cdot 503,$$

$$M_o = 17 \cdot 419 \pm \cdot 014, \quad \sigma_o = 2 \cdot 312 \pm \cdot 010, \quad V_o = 13.272,$$

$$M_s = 12 \cdot 029 \pm \cdot 027, \quad \sigma_s = 4 \cdot 276 \pm \cdot 019, \quad V_s = 35 \cdot 549,$$

$$r_{bo} = - \cdot 1067 \pm \cdot 0138, \quad r/E_r = 6 \cdot 73,$$

$$r_{bs} = - \cdot 0733 \pm \cdot 0139, \quad r/E_r = 5 \cdot 31.$$

The statistical significance of the constants cannot close our eyes to the fact that they are negative in sign and of a very low numerical order.

TABLE XIV. *Hibiscus manihot.* Number of Bracts.

Number of Ovules per Locule.	*3*	*4*	*5*	*6*	*7*	*8*	*9*	Totals
0	—	1	—	—	—	—	—	1
1	—	—	—	—	—	—	—	—
2	—	1	—	—	—	—	—	1
3	—	—	1	1	—	—	—	2
4	—	1	—	—	—	—	—	1
5	—	—	1	—	—	—	—	1
6	—	—	1	—	—	—	—	1
7	—	—	4	2	—	—	—	6
8	—	4	2	2	1	—	—	9
9	—	2	4·5	4·5	1	—	—	12
10	—	4	8	6	4	—	—	22
11	—	6	24·5	10·5	2·5	·5	—	44
12	—	24·5	39	40·5	8	1	—	113
13	—	46	108·5	98	26·5	5·5	·5	285
14	—	84	256·5	219	35·5	10	1	606
15	1	176	508	356·5	71	9	·5	1122
16	3	353·5	788·5	520	91	11	—	1767
17	2	370	909	539·5	87	6·5	—	1914
18	1	417	1009·5	541	54	·5	—	2023
19	4	353·5	779	379·5	38·5	4·5	—	1559
20	5·5	263	676	288	20	2·5	—	1255
21	2·5	103	360	156	5·5	1·5	·5	629
22	—	34	136	70	1	—	—	241
23	·5	8·5	35·5	13·5	1	—	—	59
24	·5	5·5	6·5	3·5	—	—	—	16
25	—	—	2	3	—	—	—	5
26	—	—	2·5	2·5	—	—	—	5
27	—	—	—	—	—	—	—	—
28	—	—	—	1	—	—	—	1
Totals	20	2257·5	5662·5	3257·5	447·5	52·5	2·5	11700

TABLE XV. *Hibiscus manihot.* Number of Bracts.

Number of Seeds Developing per Locule.	*3*	*4*	*5*	*6*	*7*	*8*	*9*	Totals
0	—	14·5	3·5	18·5	6	1	—	75
1	—	14	60·5	33·5	8	2	—	118
2	—	25	81	49·5	13·5	4	—	173
3	—	36	119·5	58·5	18	3·5	·5	236
4	—	46	119	98	24	4	—	291
5	—	53	135	87·5	17·5	3·5	·5	297
6	—	63	171·5	118·5	18·5	4·5	—	376
7	·5	57·5	198	110	11	4	—	381
8	—	61·5	206	100·5	12	1	—	381
9	—	77·5	249	155·5	13	4·5	·5	500
10	1	131	283·5	169·5	11·5	4	·5	601
11	—	151	391	238·5	20·5	1	—	802
12	2	226·5	487	328	52·5	1	—	1097
13	2	265	637	352	47	5·5	·5	1309
14	2·5	305·5	674	367·5	48	4·5	—	1402
15	4	250·5	601·5	326·5	53	3·5	—	1239
16	2·5	180	543	283	41·5	—	—	1050
17	2·5	146	342	183	20·5	1	—	695
18	·5	93·5	202·5	90·5	9	—	—	396
19	2	47	81·5	52·5	2	—	—	185
20	·5	10·5	32	24·5	·5	—	—	68
21	—	2·5	11·5	7	—	—	—	21
22	—	·5	1	2·5	—	—	—	4
23	—	—	·5	1·5	—	—	—	2
24	—	—	—	1	—	—	—	1
Totals	20	2257·5	5662·5	3257·5	447·5	52·5	2·5	11700

SUMMARY.

From the standpoint of genetic or reproductive selection as well as from that of general physiology a knowledge of the correlation between somatic characters and fertility is of great interest.

This correlation is not so easily determined on botanical as on zoological material, because of the obvious structural interdependence between many vegetative characters and the reproductive organs.

The involucral whorl of *Hibiscus* seems a good subject for such an investigation; although intimately associated with the fruit there seems to be no reason for supposing an important physiological nexus.

An examination of the fruits of six species of *Hibiscus* represented by ten series of material and involving over sixty-five thousand countings of ovules or seeds or both, for individual locules yields statistics which justify the following conclusion The correlations between number of bracts and number of ovules or seeds are sometimes statistically significant with regard to their probable errors but are uniformly of such low magnitude that no practical biological importance is to be attached to them.

COLD SPRING HARBOR,
LONG ISLAND,
August 23, 1910.

ANTHROPOMETRY OF MODERN EGYPTIANS.

By J. I. CRAIG, M.A., F.R.S.E. Director of the Computation Office,
Egyptian Survey Department.

(1) When in 1905 the Egyptian Government decided to raise the level of the Aswan Dam, they determined to make a thorough archaeological survey of the stratum of Nubia that would be affected by the elevation of the water-level when the reservoir was filled to its highest capacity. Although a large amount of work was done by non-official bodies, the chief share was carried out under Government auspices, and the general control of this portion was entrusted to the Director-General of the Survey Department. The services of Professor Elliot Smith, F.R.S., were retained for the investigation of the anatomical finds and he has described the first season's work in a report already published*. The measurements of the skulls and other bones found were made by Professor Elliot Smith or his assistants, Dr Wood Jones and Dr Douglas Derry, and the measurements were handed over to the Computation Office of the Survey Department to be dealt with statistically. At the same time some measurements of skulls from other parts of Egypt were also handed over for reduction.

(2) Professor Elliot Smith fully realized that there would be a considerable gain if the measurements of the various series of skulls could be compared with a modern series, and accordingly by the good offices of Dr Harold Nolan, medico-legal expert to the Egyptian Government, and of Harvey Pasha, Commandant of Police in Cairo, a series of 10,000 measurements of modern Egyptian criminals was obtained from the Anthropometric Bureau, and handed to the writer. These records were taken from the whole collection absolutely at random in the first instance. The original intention was to select from them only the cephalic records pertaining to subjects originating in Nubia and Giza, for comparison with the figures belonging to the ancient skulls, but on the writer representing the benefit that would accrue if all the measurements could be systematically reduced, permission was accorded to apportion a small sum from the budget of the Computation Office to this work. To all these gentlemen the writer desires to present his acknowledgments for their help.

(3) The series of records originally obtained included only thirty from subjects of undoubted Nubian origin, and Monsieur Aupest, Director of the

---

* *The Archaeological Survey of Nubia*, Vol. ii. Cairo, 1910.

Anthropometrical Bureau, kindly searched his collection and sent a further sixty-five.

The series included a few records of women and boys or youths under twenty. Both of these classes were rejected, but this was the only conscious selection operative on the statistics, which therefore deal with adult male Egyptian criminals, or rather adult male Egyptians accused of crime.

It may be objected that criminality is in itself a determinating factor of selection, but the objection does not hold in Egypt. Here it cannot be said that there exists a definite criminal class, and criminals are rather amateurs than professionals. This state of things is in all probability due to the easy conditions under which the lower classes live. There is practically no abject poverty, and but little drunkenness among them, and two of the most frequent incentives to crime are thus eliminated. It is, however, possible that wealth may have acted indirectly as a selecting factor, for it is without doubt still true that the number of witnesses for the defence is sometimes regulated by the depth of the defendant's purse; but in some respects this selection, if it does exist, will result in a distinct gain. The wealthier classes are generally, though not always, of foreign—Turkish, Albanian, Circassian, Tunisian, etc.—descent, while the poorer classes, on the other hand, are mainly autochthonous, but subject to a possible slight admixture with immigrant negro blood in the south and the foreign blood in the north*. It may be concluded, then, that the statistics are representative of the Egyptian and Nubian races with their local variations.

(4) The measurements available are the length and breadth of the head, the stature, the lengths of the left cubit, of the left middle finger and of the left foot. The photograph is not attached, nor are the stigmata noted. Mutilations, including in this term tattoo marks, are common. The cephalic measurements are made in the usual manner. In noting the stature, a sudden, firm pressure is exerted on the abdomen, with the result that the subject draws himself slightly more erect, and it should be mentioned that all the measurements are taken between the hours of 9 a.m. and 12 noon, so that the stature has not undergone the diminution to which it is subject later in the day. When the lengths of the cubit and foot are being determined, the body is thrown forward, so that the weight is supported almost entirely by the left arm and flexed left leg.

(5) One of the largest collections of anthropometrical statistics relating to Egypt is that made by Professor C. S. Myers† from the measurements of Egyptian soldiers, but this collection can hardly be described as a random sample of the population. The recruits are selected from a large number for physique, including

---

* Incidentally it may be mentioned that there is an opinion prevalent in Egypt that foreigners cannot settle in the country, but die out gradually. I am not aware that any statistics have been published bearing on this question, but the crude material for a discussion of the fertility of the Northern races in Egypt would be available if access could be obtained to the various consular registers. The results of such an investigation could not fail to be of considerable anthropogeographical interest.

† *Journ. Anthrop. Inst.* XXXVI. (1906), p. 237.

stature and chest measurement as the chief factors. The statistics here discussed show that, for natives of Alexandria at least, there is a positive correlation of 0·112 ± 0·022 between stature and cephalic index (which might be expected to be independent of stature*) on 643 subjects†. If we may extend this result to the rest of Egypt proper, it follows of course that Professor Myers' series, although not primarily selected by cephalic index, are still so selected to the extent implied in the above coefficient of correlation. The evaluation of the correlations for other parts of the country will form the subject of a later investigation. A further selection was made by rejection of the Copts from his statistics‡, and also by eliminating those subjects whose parents were natives of different mudirias (provinces), but this last procedure is probably not altogether objectionable, since it tends to assure homogeneity of the material.

(6) The arrangement of the subjects of the present paper into classes according to place of origin has been made by birthplace without reference to the origin of the parents, on which no information is available, and it becomes necessary to consider to what extent migration may have affected the figures.

The statistics resulting from the census of 1907 show that, while there is a large influx from the provinces into the towns, there is but little inter-migration between the provinces themselves. Thus out of a total native male population of 306,000 in Cairo, 105,000 (341 per thousand) were born outside the city, and of these 8,700 (28·6 per thousand) came from Girga, one of the most active centres of emigration; but for Qaliubia, a province which lies just to the north of Cairo, the figures give 15,000 (75 per thousand) male immigrants out of a total native male population of 201,000, and of these only 760 (3·8 per thousand) came from Girga. Railway statistics for Egypt show that the number of third-class passengers has increased very considerably in recent years, from which we may reasonably conclude that migration also has increased, so that the figures given above may be considered as an upper limit. In the provinces therefore, migration is so small that its effects (except possibly when integrated through long intervals) may be neglected. Where the influence is appreciable, as in the towns, it will tend to produce a recession towards the general mean of the population.

(7) *Reduction of the Cephalic Index§ to the Cranial Index.* The primary object of this paper was to give data for the comparison of the modern Egyptian

---

* [Stature and cephalic index correlation = − ·08 ± ·02 for Cambridge Undergraduates, = − ·13 for Oxford Undergraduates (see *Biometrika*, Vol. VIII. p. 51), and the interracial correlation between stature and cephalic index was found by Tschepourkowsky from two different series to be − ·18 and − ·22 respectively (*Biometrika*, Vol. IV. p. 288). It has usually been supposed that there is a small *negative* correlation between stature and cephalic index, due to the fact that the taller races are more dolichocephalic. EDITOR.]

† Since this was written the coefficient of correlation for stature and cephalic index in Nubia has been found to be − 0·237 ± 0·065.

‡ Since this was written the differences between Copts and Moslems in the present statistics have been found to be very small.

§ Throughout this paper "cephalic index" will be employed to refer to the measures on the head over the flesh, and "cranial index" to refer to measures on the skull.

people with the ancient Egyptians, and it is accordingly necessary to consider how far this is possible.

Deniker* says: "La mesure principale, l'indice céphalique, ne paraît pas toujours correspondre sur le crâne et sur le vivant. A priori la tête à l'état vivant devrait avoir un indice un peu plus fort que la crâne, les muscles de la région temporale étant plus épais que ceux de la région sus-occipitale et frontale; cependant, les expériences faites à ce sujet sont contradictoires. D'après Broca il faut soustraire deux unités à l'indice pris sur le vivant pour obtenir l'indice sur le crâne; c'est encore l'opinion de MM. Stieda et Houzé, et d'un grand nombre d'anthropologistes, tandis que MM. Mantegazza et Weisbach préconisent la réduction de trois unités; Virchow et Topinard n'en admettent aucune...... Cependant d'une façon générale on peut admettre la différence de deux unités entre les indices du crâne et du vivant."

(8) It is reasonable to expect that there may exist a correlation between the shape of the head and that of the skull, and on certain assumptions the correlation may be demonstrated.

Let l, b, be the length and breadth respectively of the head;

λ, β, the amounts to be subtracted from the length and breadth to obtain these measurements for the skull;

y, x, the cranial and cephalic indices respectively.

Then by definition

$$x = 100b/l, \quad \text{and} \quad y = 100(b-\beta)/(l-\lambda) = (100b/l) \times (1-\beta/b)/(1-\lambda/l).$$

Since the magnitudes of β/b and of λ/l are of the order of 8·5/144 and 7/190 respectively, we may write this equation:—

$$y = x(1-\beta/b+\lambda/l) - \text{other terms}.$$

The other terms will be small, and may be allowed for by assigning a mean value, so that the equation becomes

$$y = mx - c,$$

where

$$m = 1 - \beta/b + \lambda/l.$$

The ratio of β/b is in general greater than that of λ/l, so that m is a fraction slightly less than unity. In Egyptian bodies, Dr Douglas Derry has found that $\beta = 8·5$, $\lambda = 7$, $b =$ about 144 and $l = 191$†. Hence in this case $m = 0·976$ approximately, and the reduction is about $0·024x$ or about two units.

(9) Since this theoretical reasoning suggests that the formula $y = mx - c$ is capable of giving results not inconsistent with practice, I have assumed its truth, and have used it to find average values of m and c. Deniker‡ has given 43 cases

---

* *Races et peuples de la terre* (Paris, 1900), p. 86.

† Elliot Smith, *loc. cit.* p. 25. [Dr Derry's results seem rather smaller than those for Europeans: see Lee and Pearson, *Phil. Trans.* Vol. 196, A, 1901, p. 250 *et seq.* Cf. Gladstone on post-mortem cases, *Biometrika*, Vol. IV. p. 110 *et seq.*, however. EDITOR.]

‡ *Loc. cit.* pp. 667 *et seq.*

where both cranial and cephalic indices have been found for the same race. The means are in general of different weights for the two indices, but no attention was paid to the difference, and equal weights were assumed.

The coefficient of correlation between the indices is $+0{\cdot}818 \pm 0{\cdot}034$. The mean cephalic index is 80·90 and the mean cranial index 78·32; the respective standard deviations are 3·663 and 4·282 (so that skulls are more variable than living heads), and the equation to the line of regression is:

$$\Delta\alpha_s = 0{\cdot}9562\Delta\alpha_h,$$

where $\Delta\alpha_s$ and $\Delta\alpha_h$ are the deviations of the cranial and cephalic indices respectively from their means.

The equation may be written also in the form:

$$\alpha_s = 0{\cdot}9562\alpha_h + 0{\cdot}96,$$

which, it will be noticed, does not altogether agree with the form of that obtained from the above theory, since the absolute term differs in sign.

The amount to be subtracted from the cephalic index is $(\alpha_h - \alpha_s)$, or $0{\cdot}0438\alpha_h - 0{\cdot}96$, an expression which gives the following corrections:

$\alpha_h = 60$	$-(\alpha_h - \alpha_s) = -1{\cdot}67$
70	$-2{\cdot}11$
80	$-2{\cdot}54$
90	$-2{\cdot}88.$

For average skulls, therefore, the resulting correction has a value intermediate between those mentioned by Deniker.

(10) The accompanying figure exhibits the general dependence of the one index on the other, and had a few outlying discrepant cases—mostly of small weight—been rejected, the general agreement would have been more marked.

The conclusion is that we may reduce the mean cephalic index by subtracting 2·5 units on the average*.

(11) The material has been classified by mudirias (provinces), and governorships (large towns), and with the original purpose for which the work was undertaken in mind, I have subdivided the subjects from Aswan mudiria into those from Aswan Town, those born north of the town, and those born to the south of it, who may be considered as Nubians proper. The population of Aswan Town is very heterogeneous, and besides a large number of Nubians there are many Egyptians, and a greater proportion of Sudanese than in any other town. The effect of this heterogeneity is manifested in the relative greatness of the coefficient of variation which is the greatest in all the tables except the first (i.e. that of head length).

---

* [This is not really applicable to the case of the *individual* head; it is an interracial and not an intraracial result. EDITOR.] An investigation of the intraracial correspondence is in progress at the School of Medicine, Cairo.

The people of Daqahlia, Qaliubia, and especially Sharqia, are generally considered to have a considerable strain of Arab (Bedawi) blood. The Fayum, which is the most isolated portion of the country, was colonized by Macedonian and other Greeks in Ptolemaic times: it is for anthropologists to say whether the dolichocephaly of that province is the result of this foreign colonization, or is due to the isolation of the province and the freedom of the inhabitants from admixture.

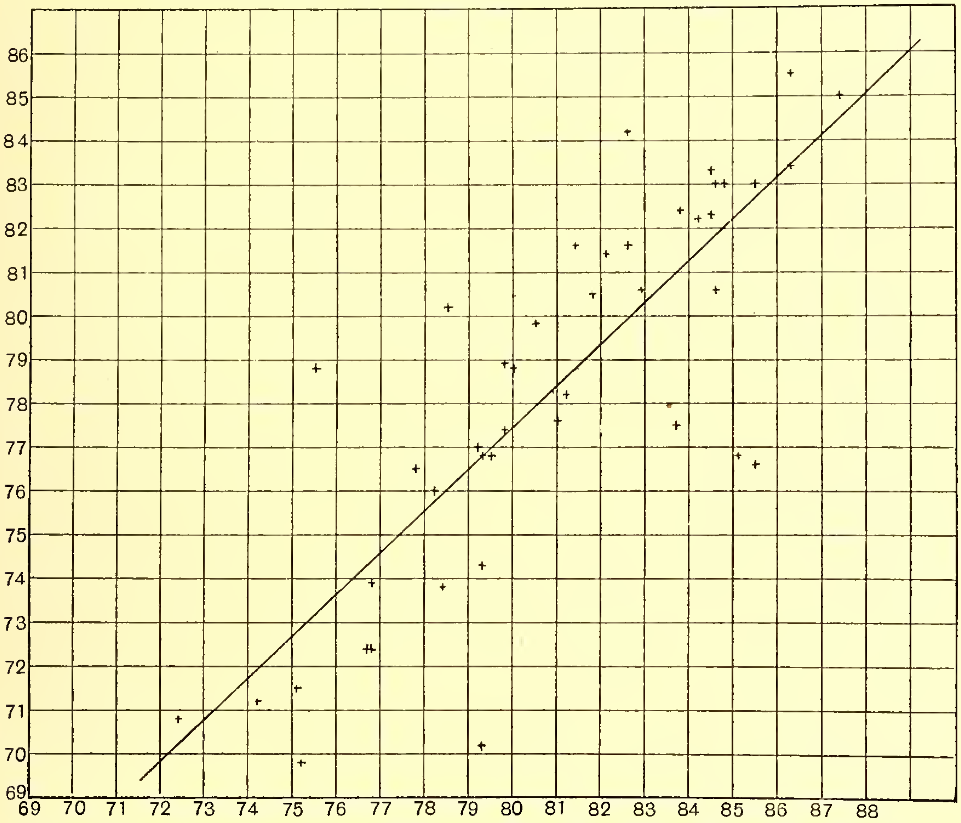


FIG. 1. Diagram showing the relation between cranial index (a_s) and cephalic index (a_h).
$a_s = 0{\cdot}9562a_h + 0{\cdot}96$.

(12) For purposes of computation, the material was classified into arrays with intervals selected so that the whole of the population of each mudiria would fall into from 12 to 20 arrays. In the case of the two smaller groups of 52 and 95 from Aswan, the means, etc., were derived from the expanded and not the condensed statistics. To ensure the accuracy of the arithmetic, all means, standard deviations and correlations have been computed twice with independent bases, which generally differed by the unit of one array. The extra labour has, of course, been considerable, but the gain in certainty warrants it.

(13) The tables are self-explanatory. n is the number of subjects in each district; M is the mean; σ the standard deviation; V the coefficient of variation; E_M, E_σ and E_V the respective probable errors of these quantities.

In what follows, a difference between two similar quantities will be considered significant if it exceeds thrice the probable error of the difference. (The odds against the chance occurrence of such an event are 22 to 1.)

(14) The following inferences may be of interest:

1. It has been stated* that an urban population is, in general, more brachycephalic than the adjoining rural population. This general law holds for Egypt also, at least as far as the large towns dealt with here go. The cephalic indices for Alexandria and Beheira are 76·29 ± 0·08 and 75·20 ± 0·09; a difference of 1·09 ± 0·12. For Cairo and Qaliubia the difference is 0·51 ± 0·13 and for Cairo

Length of Head.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	189·74±0·16	5·99±0·11	3·16±0·06
Cairo	802	190·46±0·14	5·96±0·10	3·13±0·05
Canal	127	190·61±0·34	5·74±0·24	3·01±0·13
Beheira	526	191·18±0·17	5·89±0·12	3·08±0·06
Gharbia	1104	190·97±0·12	6·01±0·09	3·15±0·05
Menufia	717	191·06±0·15	6·04±0·11	3·16±0·06
Daqahlia	504	190·35±0·20	6·65±0·14	3·49±0·07
Sharqia	515	190·79±0·18	6·16±0·13	3·23±0·07
Qaliubia	295	190·82±0·23	5·90±0·16	3·09±0·09
Giza[a]	326	191·66±0·22	5·75±0·15	3·00±0·08
Fayum	413	191·20±0·20	5·92±0·14	3·09±0·07
Beni Suef	384	191·70±0·19	5·65±0·14	2·95±0·07
Minia	491	191·73±0·17	5·71±0·12	2·98±0·06
Assiut	887	190·91±0·13	5·84±0·09	3·06±0·05
Girga[b]	610	191·51±0·16	6·03±0·12	3·15±0·06
Qena	824	191·19±0·14	6·01±0·10	3·15±0·05
(1) Aswan North	115	191·78±0·40	6·32±0·28	3·30±0·15
(2) Aswan Town	52	188·90±0·48	5·09±0·34	2·69±0·19
(3) Aswan South[c, d, e] ...	95	189·66±0·40	5·71±0·28	3·01±0·15
Aswan: (1 + 2 + 3) ...	262	190·44±0·25	6·00±0·18	3·15±0·09
Skull Measurements.				
[a] Necropolis, close to Great Pyramid, 8 miles from Cairo, about 3500 B.C.	128	184·02±0·32	5·41±0·23	2·94±0·12
[b] Predynastic, from Naga ed-Deir, 100 miles north of Thebes, about 5000 B.C.	50	183·26±0·50	5·20±0·35	2·84±0·19
[c] From Pits at Shellal, east bank of river, 1st cataract, about 1500 B.C.	111	184·69±0·92	6·52±0·65	3·53±0·35
[d] Nubians, both sides river, as far as 60 miles south of Aswan, about 2000 B.C.	41	182·32±0·56	5·30±0·40	2·91±0·22
[e] Island of Biga, south of 1st cataract, Christian cemetery, 4th–7th century A.D.	81	182·90±0·46	6·08±0·32	3·32±0·18

---

* Pearson, *Chances of Death*, Vol. I, p. 288.

Breadth of Head.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	144·66 ± 0·13	5·00 ± 0·09	3·46 ± 0·07
Cairo	799	144·43 ± 0·11	4·72 ± 0·08	3·27 ± 0·06
Canal	127	145·13 ± 0·34	5·59 ± 0·24	3·85 ± 0·16
Beheira	526	143·61 ± 0·14	4·64 ± 0·10	3·23 ± 0·07
Gharbia	1104	143·53 ± 0·10	4·85 ± 0·07	3·38 ± 0·05
Menufia	717	143·56 ± 0·12	4·63 ± 0·08	3·22 ± 0·06
Daqahlia	504	143·99 ± 0·15	4·87 ± 0·10	3·38 ± 0·07
Sharqia	516	143·63 ± 0·14	4·85 ± 0·10	3·37 ± 0·07
Qaliubia	295	143·71 ± 0·19	4·78 ± 0·13	3·33 ± 0·09
Giza[a]	326	143·16 ± 0·17	4·63 ± 0·12	3·24 ± 0·09
Fayum	413	141·85 ± 0·16	4·75 ± 0·11	3·35 ± 0·08
Beni Suef	384	142·69 ± 0·17	4·88 ± 0·12	3·42 ± 0·08
Minia	491	142·61 ± 0·14	4·55 ± 0·10	3·19 ± 0·07
Assiut	887	142·52 ± 0·10	4·43 ± 0·07	3·04 ± 0 05
Girga[b]	610	142·28 ± 0·12	4·40 ± 0·08	3·09 ± 0·06
Qena	824	142·32 ± 0·11	4·75 ± 0·08	3·34 ± 0·06
(1) Aswan North	115	143·98 ± 0·30	4·77 ± 0·21	3·32 ± 0·15
(2) Aswan Town	52	143·48 ± 0·55	5·91 ± 0·39	4·12 ± 0·27
(3) Aswan South[c, d, e] ...	95	145·16 ± 0·35	5·10 ± 0·25	3·51 ± 0·17
Aswan : (1 + 2 + 3) ...	262	144·31 ± 0·22	5·18 ± 0·15	3·59 ± 0·11
♂ Skull Measurements.				
[a] (as above)	128	139·02 ± 0·30	5·09 ± 0·21	3·66 ± 0·15
[b] („)	50	128·76 ± 0·38	3·97 ± 0·27	3·08 ± 0·21
[c] („)	110	135·41 ± 0·30	4·70 ± 0·21	3·47 ± 0·16
[d] („)	41	132·93 ± 0·46	4·39 ± 0·33	3·30 ± 0·25
[e] („)	81	138·04 ± 0·46	6·08 ± 0·32	4·40 ± 0·23

Left Middle Finger.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	115·56 ± 0·15	5·61 ± 0·11	4·86 ± 0·09
Cairo	802	114·07 ± 0·15	6·13 ± 0·10	5·37 ± 0·09
Canal	127	113·59 ± 0·34	5·72 ± 0·24	5·03 ± 0·21
Beheira	525	115·84 ± 0·17	5·85 ± 0·12	5·05 ± 0·11
Gharbia	1104	115·23 ± 0·12	6·04 ± 0·09	5·24 ± 0·08
Menufia	716	115·06 ± 0·15	5·79 ± 0·10	5·03 ± 0·09
Daqahlia	504	115·66 ± 0·17	5·79 ± 0·12	5·01 ± 0·11
Sharqia	516	115·16 ± 0·18	6·19 ± 0·13	5·37 ± 0·11
Qaliubia	295	115·86 ± 0·25	6·32 ± 0·18	5·45 ± 0·15
Giza...	326	115·89 ± 0·21	5·55 ± 0·15	4·79 ± 0·13
Fayum	413	115·19 ± 0·18	5·57 ± 0·13	4·82 ± 0·11
Beni Suef	384	115·57 ± 0·20	5·68 ± 0·14	4·92 ± 0·12
Minia	491	116·13 ± 0·18	5·90 ± 0·13	5·08 ± 0·11
Assiut	887	116·23 ± 0·13	5·78 ± 0·09	4·97 ± 0·08
Girga	610	116·63 ± 0·16	5·70 ± 0·11	4·88 ± 0·09
Qena	823	115·85 ± 0·14	5·76 ± 0·10	4·97 ± 0·08
(1) Aswan North	115	117·57 ± 0·40	6·36 ± 0·28	5·41 ± 0·24
(2) Aswan Town	52	114·44 ± 0·65	6·99 ± 0·46	6·11 ± 0·40
(3) Aswan South	95	115·74 ± 0·46	6·58 ± 0·32	5·68 ± 0·28
Aswan : (1 + 2 + 3) ...	262	116·29 ± 0·28	6·68 ± 0·20	5·75 ± 0·17

Left Foot.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	261·49±0·31	11·75±0·22	4·50±0·08
Cairo	802	257·67±0·30	12·40±0·21	4·81±0·08
Canal	127	258·38±0·78	13·10±0·55	5·07±0·21
Beheira	525	261·21±0·36	12·14±0·25	4·65±0·10
Gharbia	1104	259·16±0·25	12·43±0·18	4·80±0·07
Menufia	717	258·33±0·32	12·67±0·23	4·91±0·09
Daqahlia	504	260·20±0·37	12·22±0·26	4·69±0·10
Sharqia	516	258·69±0·37	12·62±0·26	4·88±0·10
Qaliubia	295	260·38±0·51	12·96±0·36	4·98±0·14
Giza	326	259·96±0·44	11·91±0·31	4·58±0·12
Fayum	413	259·13±0·39	11·74±0·28	4·53±0·11
Beni Suef	384	258·71±0·42	12·29±0·30	4·75±0·12
Minia	491	259·55±0·37	12·10±0·26	4·66±0·10
Assiut	887	259·87±0·27	11·83±0·19	4·55±0·07
Girga	610	261·53±0·32	11·84±0·23	4·53±0·09
Qena	824	259·65±0·28	11·99±0·20	4·62±0·08
(1) Aswan North	115	262·36±0·81	12·92±0·58	4·92±0·22
(2) Aswan Town	52	255·17±1·38	14·78±0·98	5·79±0·38
(3) Aswan South	95	258·55±0·93	13·44±0·66	5·20±0·25
Aswan: (1+2+3)	262	259·55±0·57	13·78±0·41	5·31±0·16

Left Cubit.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	462·05±0·54	20·26±0·38	4·39±0·08
Cairo	802	457·52±0·51	21·52±0·36	4·70±0·08
Canal	127	458·97±1·22	20·41±0·86	4·44±0·19
Beheira	525	468·57±0·62	21·08±0·44	4·50±0·09
Gharbia	1104	465·38±0·44	21·49±0·31	4·62±0·07
Menufia	717	466·45±0·55	21·73±0·39	4·66±0·08
Daqahlia	504	467·21±0·63	21·14±0·45	4·52±0·10
Sharqia	516	468·10±0·64	21·64±0·45	4·62±0·10
Qaliubia	295	467·72±0·84	21·32±0·59	4·56±0·13
Giza	326	470·42±0·79	21·23±0·56	4·51±0·12
Fayum	413	469·25±0·70	21·18±0·50	4·51±0·11
Beni Suef	384	467·54±0·74	21·39±0·52	4·58±0·11
Minia	491	470·00±0·67	22·12±0·48	4·70±0·10
Assiut	887	472·58±0·50	21·89±0·35	4·63±0·07
Girga	610	474·29±0·58	21·17±0·41	4·46±0·09
Qena	824	474·96±0·51	21·77±0·36	4·58±0·08
(1) Aswan North	115	479·50±1·45	23·04±1·02	4·80±0·21
(2) Aswan Town	52	464·48±2·59	27·73±1·83	5·96±0·39
(3) Aswan South	95	468·86±1·86	26·94±1·32	5·75±0·28
Aswan: (1+2+3)	262	472·66±1·09	26·23±0·77	5·55±0·16

Stature.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		cm.	cm.	
Alexandria	643	166·62 ± 0·16	5·97 ± 0·11	3·59 ± 0·07
Cairo	802	165·79 ± 0·14	6·03 ± 0·10	3·64 ± 0·06
Canal	127	165·87 ± 0·32	5·42 ± 0·23	3·26 ± 0·14
Beheira	525	167·68 ± 0·17	5·74 ± 0·12	3·42 ± 0·07
Gharbia	1105	167·33 ± 0·12	5·94 ± 0·09	3·55 ± 0·05
Menufia	718	167·70 ± 0·16	6·25 ± 0·11	3·73 ± 0·07
Daqahlia	504	166·06 ± 0·18	6·00 ± 0·13	3·61 ± 0·08
Sharqia	516	165·54 ± 0·19	6·33 ± 0·13	3·82 ± 0·08
Qaliubia	295	166·24 ± 0·25	6·31 ± 0·18	3·80 ± 0·10
Giza...	326	167·80 ± 0·22	5·88 ± 0·16	3·50 ± 0·09
Fayum	413	167·20 ± 0·20	5·92 ± 0·14	3·54 ± 0·08
Beni Suef	384	166·23 ± 0·20	5·91 ± 0·14	3·55 ± 0·09
Minia	491	166·97 ± 0·17	5·66 ± 0·12	3·39 ± 0·07
Assiut	889	166·89 ± 0·14	6·03 ± 0·10	3·62 ± 0·06
Girga	610	167·77 ± 0·16	5·92 ± 0·11	3·53 ± 0·07
Qena	824	167·80 ± 0·14	5·90 ± 0·10	3·52 ± 0·06
(1) Aswan North	115	168·33 ± 0·39	6·23 ± 0·28	3·70 ± 0·16
(2) Aswan Town	52	163·64 ± 0·84	8·97 ± 0·59	5·48 ± 0·36
(3) Aswan South	95	165·06 ± 0·48	6·94 ± 0·34	4·21 ± 0·21
Aswan: (1+2+3) ...	262	166·21 ± 0·31	7·36 ± 0·22	4·43 ± 0·13

Cephalic Index.

Mudirias	n	$M \pm E_M$	$\sigma \pm E_\sigma$	$V \pm E_V$
		mm.	mm.	
Alexandria	643	76·29 ± 0·08	3·13 ± 0·06	4·10 ± 0·08
Cairo	799	75·87 ± 0·07	2·95 ± 0·05	3·89 ± 0·07
Canal	127	76·18 ± 0·18	3·05 ± 0·13	4·01 ± 0·17
Beheira	526	75·20 ± 0·09	3·04 ± 0·06	4·04 ± 0·08
Gharbia	1104	75·22 ± 0·06	2·91 ± 0·04	3·86 ± 0·06
Menufia	717	75·21 ± 0·08	3·02 ± 0·05	4·02 ± 0·07
Daqahlia	504	75·69 ± 0·10	3·29 ± 0·07	4·35 ± 0·09
Sharqia	515	75·39 ± 0·09	3·09 ± 0·06	4·10 ± 0·09
Qaliubia	295	75·36 ± 0·11	2·85 ± 0·08	3·78 ± 0·10
Giza[a]	326	74·75 ± 0·11	2·83 ± 0·08	3·79 ± 0·10
Fayum	413	74·24 ± 0·08	2·27 ± 0·05	3·06 ± 0·07
Beni Suef	384	74·47 ± 0·10	2·98 ± 0·07	4·00 ± 0·10
Minia	491	74·46 ± 0·09	2·83 ± 0·06	3·80 ± 0·08
Assiut	887	74·70 ± 0·06	2·74 ± 0·04	3·67 ± 0·06
Girga[b]	610	74·38 ± 0·08	2·84 ± 0·05	3·82 ± 0·07
Qena	824	74·48 ± 0·07	2·85 ± 0·05	3·83 ± 0·06
(1) Aswan North	115	75·14 ± 0·20	3·15 ± 0·14	4·19 ± 0·19
(2) Aswan Town	52	75·99 ± 0·31	3·31 ± 0·22	4·35 ± 0·29
(3) Aswan South[c, d, e] ...	95	76·57 ± 0·18	2·65 ± 0·13	3·46 ± 0·17
Aswan: A. M. (1+2+3)...	262	75·83 ± 0·13	3·08 ± 0·09	4·06 ± 0·12
♂ *Skull Measurements.*				
[a] (as above)	128	75·58 ± 0·16	2·66 ± 0·11	3·52 ± 0·15
[b] („)	50	70·31 ± 0·26	2·70 ± 0·18	3·84 ± 0·26
[c] („)	110	73·38 ± 0·20	3·17 ± 0·14	4·32 ± 0·20
[d] („)	41	72·98 ± 0·31	2·95 ± 0·22	4·05 ± 0·30
[e] („)	81	75·50 ± 0·31	4·13 ± 0·22	5·47 ± 0·29

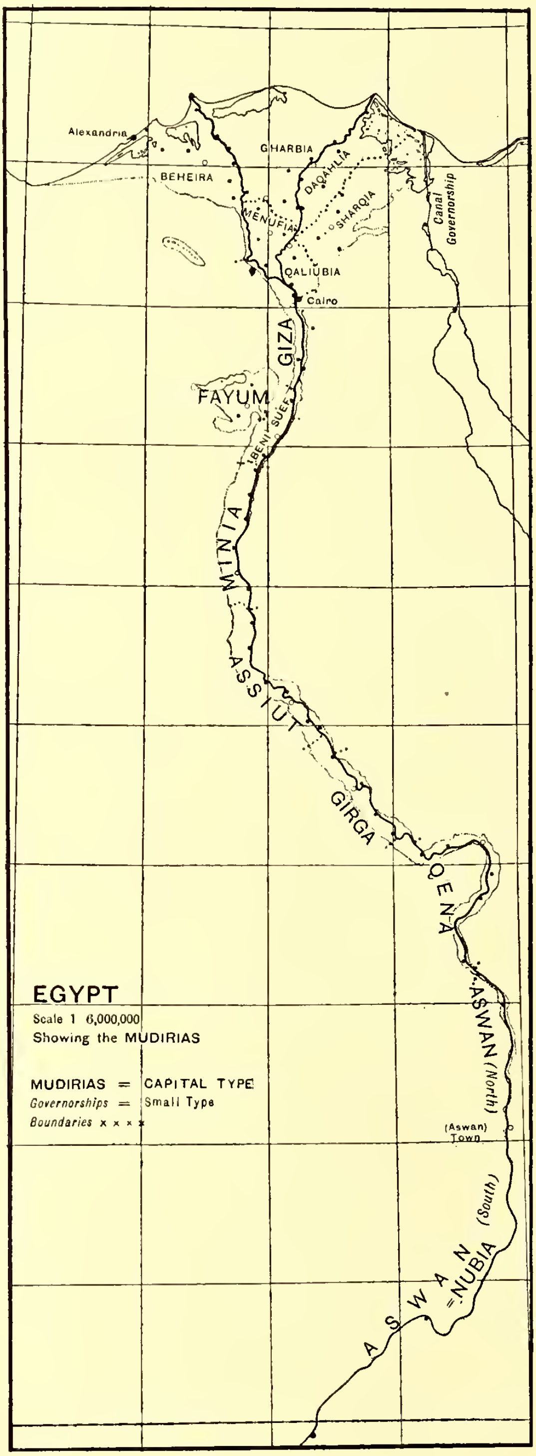

Alexandria
GHARBIA
BEHEIRA
DAQAHLIA
SHARQIA
MENUFIA
Canal Governorship
QALIUBIA
Cairo
GIZA
FAYUM
BENI SUEF
MINIA
ASSIUT
GIRGA
QENA
ASWAN (North)
(Aswan) Town
ASWAN = NUBIA (South)
EGYPT
Scale 1 6,000,000
Showing the MUDIRIAS
MUDIRIAS = CAPITAL TYPE
Governorships = Small Type
Boundaries x x x x

Fig. 2.

and Giza the difference is still more marked. Further, if the composition of the Cairene population is remembered (p. 68), the index for natives of the city must be still more increased, and the brachycephaly becomes more significant. If the population has been for some time composed as it now is, the cephalic index of native Cairenes must be corrected to 75·87 + 0·85 or 76·72, but the assumption made here is doubtful. The same tendency to relative brachycephaly is to be observed in the Canal Governorship, but the difference between the indices for this district and Daqahlia is not so significant as in the other cases. This perhaps is not to be wondered at, seeing how largely the towns of Port Saïd and Ismailia are the products of comparatively recent colonization, and that 17 per cent. of the present population of this district and probably at least 35 per cent. of the whole originally are from Daqahlia. The difference from Sharqia, which also adjoins the Canal district, but contributes much more sparsely to its population, is much more marked (0·79 ± 0·21).

2. The dwellers in towns are very markedly shorter in the arms than the rural populations. This holds for all four towns dealt with here. The difference is probably to be accounted for by the large amount of hard work, much of it lifting of water on to the land, that the fellahin undergo. This lengthening of the cubit increases towards the South, where the manual labour of cultivation is heaviest, but it *may* be connected with increase of negro blood.

3. There is some justification for the usual division of Egypt into Upper and Lower, as far as cephalic index goes. The cephalic indices for the six mudirias of the latter lie between 75·21 and 75·69, the weighted mean being 75·31. For Upper Egypt (excluding Aswan as being inhabited by a race largely composed of Barabra) the index varies between 74·24 and 74·75, with a weighted mean 74·51. The gap between the highest of the one and the lowest of the other is 0·46 ± 0·14, which is probably significant.

4. There is a slight increase of stature in moving from east to west across the Delta, and also a slight but less clearly marked increase in moving southward. The former may be due to an infusion of Bedawi blood, which is said to be more common in the east, and the latter has been attributed to an increased infusion of negro blood. There is not any sharp division between Upper and Lower Egypt.

5. The people of Upper Egypt are on the whole bigger boned than those of Lower Egypt. The former have longer but narrower heads, longer fingers, longer feet, and longer forearms, than the latter.

6. The measurements made by Professor Myers give distinct evidence of the selection already mentioned, as may be seen from the following comparative tables:

Head Length.

	Myers	Craig	M. – C.
	mm.	mm.	mm.
Beheira	196·82 ± 0·60	191·18 ± 0·17	5·64
Daqahlia ...	193·00 ± 0·39	190·35 ± 0·20	2·65
Sharqia	196·75 ± 0·73	190·79 ± 0·18	5·96
Giza	194·56 ± 0·54	191·66 ± 0·22	2·90
Girga	194·53 ± 0·43	191·51 ± 0·16	3·02
Qena	194·79 ± 0·54	191·19 ± 0·14	3·60

Head Breadth.

	Myers	Craig	M. – C.
	mm.	mm.	mm.
Beheira	144·38 ± 0·42	143·61 ± 0·14	0·77
Daqahlia ...	144·64 ± 0·28	143·99 ± 0·15	0·65
Sharqia	145·40 ± 0·74	143·63 ± 0·14	1·77
Giza	143·41 ± 0·37	143·16 ± 0·17	0·25
Girga	144·33 ± 0·34	142·28 ± 0·12	2·05
Qena	143·91 ± 0·35	142·32 ± 0·11	1·59

Cephalic Index.

	Myers	Craig	M. – C.
	mm.	mm.	mm.
Beheira	73·42 ± 0·2	75·20 ± 0·09	−1·78
Daqahlia ...	75·01 ± 0·1	75·69 ± 0·10	−0·58
Sharqia	73·94 ± 0·4	75·39 ± 0·09	−1·45
Giza	73·76 ± 0·2	74·75 ± 0·11	−0·99
Girga	74·25 ± 0·2	74·38 ± 0·08	−0·13
Qena	73·94 ± 0·2	74·48 ± 0·07	−0·54

It will be noticed that the differences are in every case systematic, and that in all cases but two they are so great that they can hardly be due to accident*.

Since Professor Myers' subjects are larger men than those measured here, but have smaller cephalic indices, there appears to be a negative correlation between cephalic index and stature, contrary to what has been found for Alexandrian criminals, but similar to the result obtained in Nubia (see p. 68, footnote).

---

* [Differences of head-spanner used, and exact method of measurement deserve to be fully considered. The correlation of stature and cephalic index within the race is very small and it is difficult to believe that the selection of the former could be the source of the systematic differences recorded of the latter. Editor.]

THE TEACHER'S ESTIMATION OF THE GENERAL INTELLIGENCE OF SCHOOL CHILDREN.

By H. WAITE, M.A., B.Sc.

(i) *Introductory.* A number of articles have recently appeared on the value of the teacher's estimate of the General Intelligence of School Children, and in several the writers have rather hastily come to the conclusion that those estimates are practically worthless. It is difficult to understand why the teacher's claim should be so lightly set aside when we consider that his life is spent in educating the young; he is daily in close touch with them, observing them constantly in their efforts to grasp new ideas and to grapple with new problems, and fortified at the outset with a preparation for his work in which a study of the child-mind has been by no means neglected, he can hardly fail to acquire the power of judging, with a good degree of accuracy, the mental capacity of those who pass through his hands. The teacher's judgment is usually given, too, after observations extending over a period of many weeks; he is thus able to take into account the effects of memory, perseverance, environment and other factors which in their results are closely connected with the intelligence. It is almost, if not quite, impossible to make due allowance for these when testing the intelligence by methods which are based on answers given by the child to a series of questions during an interview which may last only a few minutes. Anyone who seriously considers the question is bound to admit that few, if any, are better fitted to form an estimate of the intelligence of a child than that child's teachers. When such estimates have been collected and compared with the results of various tests of general intelligence, a high coefficient of correlation is obtained.

This paper has been written with a view of investigating the teacher's claim. The numbers dealt with are small, and the results would in consequence be of little value if it were not that they agree very closely with those of other recent investigations, and hence tend to confirm the view that the teacher's opinion is at least worthy of consideration in questions connected with the intelligence of children.

One of the best measures of the intelligence of a child is the position he takes in his form at school, both for the ordinary work of the term and also in examinations; the former, however, largely depends on powers of application, perseverance and general care, and is thus not so reliable a test of mental capacity as the latter.

(ii) *Material.* For the purpose of the present paper particulars have been taken from two Term Report-Sheets of a London Secondary School for Boys. These include (1) Age, (2) Form, (3) General Intelligence (as judged by the masters), (4) Place in Form, based on the term's marks, and, in one series also, (5) Examination Results. Tables have been compiled and correlations obtained (*a*) between General Intelligence and Examination Place from the one Report, and (*b*) between General Intelligence and Place in Form from both. Corrections have been made in each case for constant Age and constant Form.

The ages of the boys range from 10 to 18; the numbers investigated are 182 and 238 respectively, and comprise the whole of the pupils in attendance for the periods under review, with the exception of some three or four who were absent during the examination or for most of the term through illness.

The method adopted for estimating the General Intelligence was as follows:—The masters, most of whom are specialists and only teach certain subjects, were required to give their opinions of the mental capacity of each boy whom they had taught during the term, using numbers up to a maximum of 5. The numbers 5, 4, 3, 2, 1 correspond approximately to the terms Very Able, Distinctly Capable, Fairly Intelligent, Slow Intelligent, and Slow, respectively, in Professor Karl Pearson's Scale of Ability*. The entrance examination which each boy must pass before admission to the school makes it unnecessary to provide for the remaining two classes of that Scale, viz.:—Slow Dull and Mentally Defective. Every form is taught by four, five or six different masters whose judgments are roughly averaged by totalling the marks given for each boy and reducing to a possible maximum of 20. We thus have a numerical scale of intelligence, which however, must be considered chiefly as qualitative in character, ranging from about 4 to 20, and a measure of capacity which is in effect the mean of several judgments; moreover, the risk of a judgment formed by a single master on account of a boy's special aptitude or exceptional dullness in any particular subject and also the personal equation are largely eliminated.

At the time of the first estimation of intelligence the whole of the 182 boys had been in the school at least six months and many of them much longer but the second estimation was made when about 33 per cent. of the 238 had been less than three months in the school. To this shorter period of observation is probably due the lower correlation found as the result of the second investigation.

The examination was in all subjects of the curriculum, and was conducted in part by the teacher who made the estimate and in part by the Head Master and his colleagues. Place in Form is fixed by marks obtained during a complete term for work done in school and under the teachers' supervision. Marks gained for homework done under conditions which are not entirely controlled by the teachers are not included.

---

* See Appendix to this paper.

The determination of General Intelligence was made before the examination results and the order in form were known. The Place in Form and the Examination Place as used in the Tables were found by diminishing the actual position by ·5 and dividing by the number in the form. Using this method the mean Place in Form or in Examination is always ·5.

(iii) *General Intelligence and Place in Examination.* Tables I to IX which follow are taken from the first Report above mentioned, X to XV from the second. Tables I to VI are used for the correlation of General Intelligence and Place in Examination; IV to IX and X to XV for the correlation of General Intelligence and Place in Form in the two cases respectively.

TABLE I.

General Intelligence and Place in Examination.

General Intelligence.

Place in Examination.	*4—8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16—19*	Totals
·0+	—	—	1	1	3	2	5	4	4	20
·1	—	—	—	—	3	6	4	1	1	15
·2	—	—	—	5	4	3	4	2	2	20
·3	—	—	3	—	4	4	1	2	1	15
·4	—	—	4	—	6	5	4	1	—	20
·5	—	2	4	2	5	2	1	—	1	17
·6	—	2	8	3	4	1	1	1	—	20
7	5	2	6	3	—	1	2	—	1	20
·8	3	5	2	2	1	1	—	—	—	14
·9	10	4	4	1	—	1	—	1	—	21
Totals	18	15	32	17	30	26	22	12	10	182

TABLE II.

Age and Place in Examination.

Age.

Place in Examination.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
·0+	—	—	3	6	5	5	—	1	20
·1	—	1	—	7	2	3	2	—	15
·2	—	2	3	6	3	5	1	—	20
·3	—	1	2	6	5	1	—	—	15
·4	1	1	3	4	8	2	1	—	20
·5	—	3	2	5	4	2	—	1	17
·6	—	3	4	3	6	1	1	2	20
·7	2	1	3	4	2	5	1	2	20
·8	—	—	3	3	2	4	1	1	14
·9	—	2	2	3	8	2	2	2	21
Totals	3	14	25	47	45	30	9	9	182

TABLE III.

Form and Place in Examination.

Form.

Place in Examination.	III	IV*c*	IV*b*	IV*a*	V*b*	V*a*	L. VI	VI	Totals
·0+	2	3	3	3	3	3	2	1	20
·1	1	2	2	3	2	3	1	1	15
·2	2	3	3	3	3	3	2	1	20
·3	1	3	2	2	2	3	1	1	15
·4	2	2	4	3	3	3	2	1	20
·5	2	3	2	2	3	2	2	1	17
·6	1	3	4	4	2	4	1	1	20
·7	2	3	3	3	3	3	2	1	20
·8	1	2	2	3	2	3	—	1	14
·9	2	3	3	3	3	3	3	1	21
Totals	16	27	28	29	26	30	16	10	182

TABLE IV.

Age and General Intelligence.

Age.

General Intelligence.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
4—8	—	2	6	1	4	1	2	2	18
9	—	—	2	6	3	2	1	1	15
10	1	2	4	5	10	8	2	—	32
11	1	—	3	6	4	2	—	1	17
12	—	6	4	11	4	5	—	—	30
13	—	2	3	9	8	3	—	1	26
14	1	2	—	7	8	2	2	—	22
15	—	—	2	2	4	2	1	1	12
16—19	—	—	1	—	—	5	1	3	10
Totals	3	14	25	47	45	30	9	9	182

TABLE V.

Form and General Intelligence.

Form.

General Intelligence.	III	IV*c*	IV*b*	IV*a*	V*b*	V*a*	L. VI	VI	Totals
4—8	2	5	2	8	1	—	—	—	18
9	1	—	6	1	3	3	—	1	15
10	2	5	3	2	10	6	4	—	32
11	3	3	4	1	2	3	1	—	17
12	3	7	6	6	4	3	1	—	30
13	3	—	4	6	3	6	3	1	26
14	2	6	2	3	2	4	3	—	22
15	—	—	1	2	1	4	2	2	12
16—19	—	1	—	—	—	1	2	6	10
Totals	16	27	28	29	26	30	16	10	182

TABLE VI.

Age and Form.

Age.

		10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
Form.	III	3	4	3	4	2	—	—	—	16
	IV*c*	—	5	7	7	7	1	—	—	27
	IV*b*	—	3	3	14	5	1	2	—	28
	IV*a*	—	2	9	7	4	3	2	2	29
	V*b*	—	—	3	10	7	5	1	—	26
	V*a*	—	—	—	5	13	12	—	—	30
	L. VI	—	—	—	—	7	6	2	1	16
	VI	—	—	—	—	—	2	2	6	10
	Totals	3	14	25	47	45	30	9	9	182

Remarks on the Tables.

Table I. Intelligence and Examination Place. The correlation, worked by the correlation ratio method, is $\eta = -\cdot 689$. The negative sign is due to the method of fixing Examination Place, the lowest numerical value denoting the highest place in the examination. If this order were reversed the sign would be positive, but at the same time the signs of r_{13} and r_{14} of Tables II and III would be changed; finally the coefficient of partial correlation would be of the same numerical value but of opposite sign.

Table II. Age and Examination Place. The correlation ratio method gives $\eta = \cdot 233$; by the method of moments we obtain $r = \cdot 044$, while a fourfold table gives the value ·130. The results suggest that the real relationship between Age and Examination Place is in any case small*. The partial correlation has been worked out for each of these values, and the differences found are slight.

Table III. Form and Examination Place. The correlation ratio is $\eta = \cdot 011$, and since the probable error is ± ·05, it will be taken as zero.

Table IV. Age and Intelligence. The mean General Intelligence for different ages, based on the assumption that the scale adopted is roughly quantitative, is:—

Age	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
Intelligence ...	11·6	11·5	10·9	11·8	11·7	12·2	11·5	12·4

The mean for all ages is 11·7, so that the only considerable deviations are at the ages 12 and 17, and these are probably partially due to the small numbers investigated.

Worked by the correlation ratio method, $\eta = \cdot 343$, while a fourfold table gives $r = \cdot 189$. The regression is not linear, and it is probable that the smaller value is

---

* The material is a good deal selected as an examination of the arrays of boys in Table II. aged 14 and 15 will show; able boys stay on working for scholarships or competitive examinations, duller boys in the hope of completing their training. This accounts for the differences between η and r.

the more reliable. Correcting for constant Form we get $\eta = \cdot021$ and $r = -\cdot209$, Thus the correlation is either negligible or negative. The second alternative may be explained by the fact that a certain number of the duller boys remain longer at school because they are backward, and so reduce the average intelligence for the greater ages.

Table V. Form and Intelligence. Taken as a contingency table the correlation is ·547, which, corrected for age, becomes ·454. The mean General Intelligence for different forms is:—

Form ...	III	IV *c*	IV *b*	IV *a*	V *b*	V *a*	L. VI	VI
Intelligence ...	11·3	11·4	11·1	11·2	11·0	12·2	12·9	14·8

The rise in intelligence towards the end of the scale, which appears also in the high correlation is due to many of the duller boys not getting beyond form V *b* while only the sharpest ever reach the sixth form.

Table VI. Age and Form. The correlation for this Table, found by the correlation ratio method is not so high as might be expected. This is on account of the admission of rather old boys to the lower forms, while those who join the school between the ages of 10 and 12 have generally reached one of the fifth forms or even the lower sixth by about the age of 14.

The mean ages in the different forms are:—

Form ...	III	IV *c*	IV *b*	IV *a*	V *b*	V *a*	L. VI	VI
Age	11·9	12·7	13·1	13·4	13·7	14·2	14·8	16·4

Putting the above results together we have:—

Intelligence and Examination Place, $r_{12} = -\cdot689$

Age and Examination Place, ... $r_{13} = \cdot233\ (\eta)$

$= \cdot044\ (r)$

$= \cdot130$ (by fourfold table)

Form and Examination Place, ... $r_{14} = 0$

Age and Intelligence, ... $r_{23} = \cdot343\ (\eta)$

$= \cdot189$ (by fourfold table)

Form and Intelligence, ... $r_{24} = \cdot547$

Age and Form, ... $r_{34} = \cdot602$

Correlation of General Intelligence and Examination Place for constant Age and constant Form:—

(*a*) With correlation ratio values of r_{13} and r_{23} $\rho_{12} = -\cdot867 \pm \cdot012$.

(*b*) With smallest value of r_{13} ... ... $\rho_{12} = -\cdot825 \pm \cdot016$.

(*c*) With fourfold table values ... ... $\rho_{12} = -\cdot817 \pm \cdot017$.

(iv) GENERAL INTELLIGENCE AND PLACE IN FORM. FIRST INVESTIGATION.

TABLE VII. *General Intelligence and Place in Form.*

General Intelligence.

Place in Form.	*4—8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16—19*	Totals
·*0*+	—	—	3	1	4	3	4	1	4	20
·*1*	—	—	1	1	—	3	6	3	1	15
·*2*	—	1	2	1	3	5	3	3	2	20
·*3*	—	—	3	1	5	4	1	1	1	16
·*4*	—	—	2	2	4	6	3	2	—	19
·*5*	—	1	4	3	7	2	2	—	1	20
·*6*	2	2	5	1	3	1	1	1	—	16
·*7*	3	2	7	3	3	—	1	—	1	20
·*8*	6	3	3	1	1	1	—	1	—	16
·*9*	7	6	2	3	—	1	1	—	—	20
Totals	18	15	32	17	30	26	22	12	10	182

TABLE VIII. *Age and Place in Form.*

Age.

Place in Form.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
·*0*+	—	—	3	9	2	5	—	1	20
·*1*	—	2	—	6	3	1	3	—	15
·*2*	—	—	3	6	6	5	—	—	20
·*3*	—	4	2	4	4	2	—	—	16
·*4*	—	3	—	3	9	3	1	—	19
·*5*	—	1	2	7	6	2	1	1	20
·*6*	1	—	3	3	2	5	—	2	16
·*7*	2	2	4	4	3	2	2	1	20
·*8*	—	—	4	3	5	2	1	1	16
·*9*	—	2	4	2	5	3	1	3	20
Totals	3	14	25	47	45	30	9	9	182

TABLE IX. *Form and Place in Form.*

Form.

Place in Form.	III	IV*c*	IV*b*	IV*a*	V*b*	V*a*	L. VI	VI	Totals
·*0*+	2	3	3	3	3	3	2	1	20
·*1*	1	2	2	3	2	3	1	1	15
·*2*	2	3	3	3	3	3	2	1	20
·*3*	1	3	3	2	2	3	1	1	16
·*4*	2	2	3	3	3	3	2	1	19
·*5*	2	3	3	3	3	3	2	1	20
·*6*	1	3	2	3	2	3	1	1	16
·*7*	2	3	3	3	3	3	2	1	20
·*8*	1	2	3	3	2	3	1	1	16
·*9*	2	3	3	3	3	3	2	1	20
Totals	16	27	28	29	26	30	16	10	182

Table VII. Intelligence and Place in Form. The correlation here is lower than in the corresponding result for the examination. This difference also appears when corrections are made for constant Age and constant Form. The correlation, $\eta = -\cdot 624$, is found by the correlation ratio method.

Table VIII. Age and Place in Form. This correlation when obtained by the correlation ratio method is rather higher than that of Table II. This, if correct, may be due to greater care and perseverance which are expected from the older boys. The method of moments, however, gives a coefficient slightly smaller than that obtained from Table II by the same process.

Table IX. Form and Place in Form. The correlation ratio, $\eta = \cdot 009$, is negligible, and will be taken as zero. The slight difference in the distribution of Place in Form in these three Tables from that in Tables I, II and III is due to several cases of equality in the examination.

The coefficients for Tables IV to IX are:—

Intelligence and Place in Form, ...	$r_{12} = -\cdot 624$
Age and Place in Form, ...	$r_{13} = \cdot 294\ (\eta)$
	$= \cdot 033\ (r)$
Form and Place in Form, ...	$r_{14} = 0$
Age and Intelligence, ...	$r_{23} = \cdot 343$
Form and Intelligence, ...	$r_{24} = \cdot 547$
Age and Form, ...	$r_{34} = \cdot 602$

Correlation of General Intelligence and Place in Form for constant Age and constant Form:—

(*a*) With larger value of r_{13} ... ... $\rho_{12} = -\cdot 810 \pm \cdot 017$.

(*b*) „ smaller „ ... ... $\rho_{12} = -\cdot 747 \pm \cdot 021$.

(v) GENERAL INTELLIGENCE AND PLACE IN FORM. SECOND INVESTIGATION.

TABLE X.

General Intelligence and Place in Form.

General Intelligence.

Place in Form.	*4—8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16—19*	Totals
·0+	—	—	—	4	3	6	1	4	5	23
·1	2	—	1	4	6	4	2	2	4	25
·2	1	—	1	3	5	6	3	1	2	22
·3	—	—	2	9	5	1	6	—	2	25
·4	2	2	2	5	6	4	1	—	1	23
·5	—	1	3	8	5	6	2	—	—	25
·6	—	2	2	5	7	4	3	—	—	23
·7	2	4	4	6	4	2	2	—	—	24
·8	2	2	2	10	3	3	—	1	—	23
·9	5	3	6	7	2	1	1	—	—	25
Totals	14	14	23	61	46	37	21	8	14	238

TABLE XI.

Age and Place in Form.

	Age.								
Place in Form.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
·*0*+	—	1	3	10	8	—	—	1	23
·*1*	1	—	2	7	10	4	1	—	25
·*2*	—	1	4	8	6	1	1	1	22
·*3*	1	2	2	9	4	6	—	1	25
·*4*	1	1	3	10	5	1	1	1	23
·*5*	1	3	6	6	6	2	1	—	25
·*6*	1	2	5	7	5	2	1	—	23
·*7*	—	5	5	5	6	1	1	1	24
·*8*	—	2	7	4	5	4	1	—	23
·*9*	2	5	6	6	3	1	2	—	25
Totals	7	22	43	72	58	22	9	5	238

TABLE XII.

Form and Place in Form.

	Form.									
Place in Form.	III	IV*c*	IV*b*	IV*a*	V*c*	V*b*	V*a*	L. VI	VI	Totals
·*0*+	2	3	3	3	3	3	3	2	1	23
·*1*	3	3	3	3	3	3	3	3	1	25
·*2*	2	2	3	3	3	3	3	2	1	22
·*3*	3	3	3	3	3	3	3	3	1	25
·*4*	2	3	3	3	2	4	3	2	1	23
·*5*	3	3	3	3	3	3	3	3	1	25
·*6*	2	3	3	3	3	3	3	2	1	23
·*7*	3	2	3	3	3	3	3	3	1	24
·*8*	2	3	3	3	3	3	3	2	1	23
·*9*	3	3	3	3	3	3	3	3	1	25
Totals	25	28	30	30	29	31	30	25	10	238

TABLE XIII.

Age and General Intelligence.

	Age.								
General Intelligence.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
4—*8*	—	1	3	3	5	—	2	—	14
9	2	2	1	3	3	1	1	1	14
10	—	2	5	10	2	3	1	—	23
11	2	5	12	16	17	7	2	—	61
12	1	5	6	21	10	2	1	—	46
13	1	5	11	14	4	2	—	—	37
14	—	2	3	3	8	4	1	—	21
15	—	—	—	2	4	1	1	—	8
16—*19*	1	—	2	—	5	2	—	4	14
Totals	7	22	43	72	58	22	9	5	238

TABLE XIV.

Form and General Intelligence.

Form.

General Intelligence.	III	IV*c*	IV*b*	IV*a*	V*c*	V*b*	V*a*	L. VI	VI	Totals
4—8	5	5	—	1	3	—	—	—	—	14
9	4	2	2	1	2	1	—	2	—	14
10	—	2	5	6	6	4	—	—	—	23
11	7	9	6	6	5	11	8	9	—	61
12	5	6	5	5	6	5	9	5	—	46
13	3	4	7	7	4	4	6	2	—	37
14	—	—	5	3	—	4	4	1	4	21
15	—	—	—	—	2	1	2	2	1	8
16—19	1	—	—	1	1	1	1	4	5	14
Totals	25	28	30	30	29	31	30	25	10	238

TABLE XV.

Age and Form.

Age.

Form.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
III	6	4	5	6	4	—	—	—	25
IV*c*	1	6	3	12	6	—	—	—	28
IV*b*	—	7	9	7	3	4	—	—	30
IV*a*	—	3	12	8	5	—	2	—	30
V*c*	—	1	5	12	7	2	2	—	29
V*b*	—	—	6	15	4	4	1	1	31
V*a*	—	1	3	10	13	2	1	—	30
L. VI	—	—	—	2	13	9	1	—	25
VI	—	—	—	—	3	1	2	4	10
Totals	7	22	43	72	58	22	9	5	238

Tables X to XV. The same methods have been employed with these as with the corresponding Tables of the earlier sets, and many of the remarks on Tables I to IX apply equally to these.

Table XIII. Age and Intelligence. It is interesting to compare the mean intelligence at different ages in this Table and in Table IV.

Age	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Mean
Intelligence, Table IV.	11·6	11·5	10·9	11·8	11·7	12·2	11·5	12·4	11·77
„ „ XIII.	11·5	11·5	11·7	11·6	12·0	12·2	10·9	14·6	11·80

The drop at Age 16 which is specially marked in Table XIII is rather curious. Reference to the Table shows that of the 9 boys in the group several are near the lower end of the scale of intelligence, thus giving a low average to the group.

The correlation ratio is $\eta = \cdot 316$, while a fourfold table gives $r = \cdot 131$. These, corrected for Form become ·040, and $- \cdot 223$ respectively, either showing, as in Table IV, that age has no effect in *increasing* the teacher's estimate of capacity.

Table XIV. Form and Intelligence. Comparison of mean Intelligence with that in Table V.

Form	III	IV*c*	IV*b*	IV*a*	V*c**	V*b*	V*a*	L. VI	VI	Mean
Intelligence, Table V.	11·3	11·4	11·1	11·2	—	11·0	12·2	12·9	14·8	11·77
" " XIV.	10·7	10·8	11·8	11·7	11·3	11·9	12·5	12·4	15·1	11·80

Table XV. Age and Form. Comparison of mean ages in different Forms with those in Table VI:—

Form	III	IV*c*	IV*b*	IV*a*	V*c**	V*b*	V*a*	L. VI	VI
Age, Table VI.	11·9	12·7	13·1	13·4	—	13·7	14·2	14·8	16·4
" " XV.	11·9	12·6	12·6	12·8	13·3	13·4	13·5	14·3	15·7

The correlation coefficients for Tables X to XV are:—

Intelligence and Place in Form, ... $r_{12} = -·484$

Age and Place in Form, ... $r_{13} = -·292\ (\eta)$

$-·166\ (r)$

Form and Place in Form, ... $r_{14} = 0$

Age and Intelligence, ... $r_{23} = ·316\ (\eta)$

$·131$ (by fourfold table)

Form and Intelligence, ... $r_{24} = ·493$

Age and Form, ... $r_{34} = ·586$

Correlation of General Intelligence and Place in Form for constant Age and constant Form:—

(*a*) With the higher values of r_{13} and r_{23} ... $\rho_{12} = -·582 \pm ·029$.

(*b*) " lower " " ... $\rho_{12} = -·631 \pm ·026$.

This correlation is lower than in the first investigation for General Intelligence and Place in Form and, as was mentioned before, the result is most likely due, at least in part, to the fact that one-third of the boys had been under observation for a much shorter time than in the first case.

(vi) *General Conclusions.* It will be seen that although the individual correlations for the two series differ considerably, the partial correlations of General Intelligence and Place in Form for constant Age and constant Form are not widely separated. We see at the same time, however, that General Intelligence is more closely related to Examination Place than to Place in Form (·87 against ·81, taking the higher value in each case; or ·83 against 75, taking the lower values). These values agree well with those found by Miss H. G. Jones in her series from the Aberdeen Schools (see *Biometrika*, Vol. VII. p. 547). Place in Form depends largely on boldness and presence of mind in classwork; a shy boy

---

* Form V*c* did not exist at the time for which the first Report was made.

of considerable intelligence may take more nearly his true place under an examination test. Again, when the organisation of the school is such that each master has charge of a particular form only a small portion of the time, the most reliable estimate of intelligence is obtained after a boy has been several months in the school. It seems clear from these results that the teacher's appreciation of General Intelligence judged by a reasonably defined system of categories and with sufficient time for observation, if corrected for Class and Age gives at least a close idea of what a boy's Place in Form will be, and a very close measure of his examinational value. In the face of this, it is surely very unreasonable to assert, as a recent writer has done, that Place in Class is a much better measure of ability than the teacher's estimate of General Intelligence. If we use Examination Place and correct for Age and Form we reach a result which is in close agreement with the teacher's classification by General Intelligence. The present results were of course made in one school, by using a defined system of categories, and under the supervision of the Head Master, but they do not differ substantially from those considered by Miss Jones, in which the categories were far more loosely defined, the teachers were rather more numerous, and the children were younger. In each investigation the teachers were left to their own judgment as to intelligence, although in the present case a rough average was taken of their estimates. The point is very much more important than it appears, because there is a growing tendency to question examination results as a measure of General Intelligence. No single psychological test or complex of tests is in the least likely to replace our present methods of judging general efficiency for public or other service. These are the examination test, and the testimonial of the teacher, be he primary schoolmaster or university professor. If the teacher's opinion be given honestly and thoughtfully, as it usually is, there appears to be very little difference between the results that flow from it and from an examination classification. The real criticism of the examination test lies not so much in its giving results that diverge from an experienced teacher's estimate of General Intelligence, but in the fact that the stress of *competitive* examinations, upon success in which future prospects largely depend, may injure either the physical or mental capacity of the candidate.

The present numbers are peculiar to a school in which there is considerable selection with age, only the boys of about average intelligence not prolonging their education; thus Form and Age are quite sensibly correlated with Intelligence. The superficial observer might suggest that this condemned the system of intelligence categories, which should give an appreciation independent of age. But the partial correlation of Age and Intelligence for a constant Form in the first series is either ·021 or − ·209; for the second series the same partial correlation is either ·040 or − ·223, either value showing that greater age has not led the teachers to attribute greater intelligence to the boy. It is reasonable to suppose that this negative correlation is due to selection *within the form*, i.e. even within the form there is a tail of elder boys with something less, and a tail of younger boys with something more, than the average intelligence

The divergence between the η and r values, i.e. the absence of accurately linear regression is not, I believe, due to any inherent quality of the characters discussed. It results from selection according to age and ability peculiar to this school and to the arrangement of boys in their forms. It would probably disappear entirely were a number of schools with a far larger range of pupils investigated. For this reason it seems likely that the values of r, as found by the fourfold table, express the fundamental relations more closely than those found for η.

Note. As the masters are specialists they judge the capacity of the boys from somewhat different points of view. Under these circumstances it is worth while to compare the estimates which were made quite independently, and also without any knowledge, on the part of the masters, of the object for which they were required.

In connection with the first of the above Reports the total number of pairs of judgments given was 1409 ; of these, 662 or 47 per cent. were in exact agreement; 684 or 48·5 per cent. differed by one place; 62 or 4·4 per cent. differed by two places while in only one case was there a divergence amounting to three places. For the second Report 2018 pairs of judgments were given of which 976 or 48·4 per cent. agreed exactly; 944 or 46·7 per cent. differed by one place, 92 or 4·6 per cent. by two places, and 6 by three places. Thus, in less than 5 per cent. of the whole of the 3427 pairs of judgments was there a difference of opinion amounting to more than one place in the scale adopted, although in all cases the judgments were based on ability shown in *different* subjects or groups of subjects.

Again in estimating General Intelligence for the second Report, it happened that a master was called upon to give his opinion a second time on the same boy in 355 cases. The interval which elapsed was about nine months, and no record was kept by the masters themselves of the first set of records. In 202 cases the same opinions were given as on the previous occasion; in 81 cases there was a rise, and in 68 a fall, of one place in the scale; in only 4 cases was there a difference of two places, three of these being a rise.

The above figures are an additional proof that the teacher's judgment of mental capacity is formed with careful discrimination and an intimate knowledge of his subject.

The pairs of judgments discussed above may be examined by means of contingency tables. I have arranged judgments given by First Master in rows against those by Second Master in columns, the "First" and "Second" being fixed by the order of the masters in an arbitrary list; e.g., each case judged by a First Master as C and by a Second as D will be included in the $[C, D]$ cell, but since in any pair of judgments either master may be considered the First Master, I have added rows to columns, thereby doubling the number of pairs and at the same time producing a symmetrical table.

TABLE XVI.

Paired Judgments by different Masters. (First Report.)

Judgment by First Master.

Judgment by Second Master.	*C*	*D*	*E*	*F*	*G*	Totals
C	26	55	29	1	—	111
D	55	246	326	24	—	651
E	29	326	842	253	17	1467
F	1	24	253	214	31	523
G	—	—	17	31	10	58
Totals	111	651	1467	523	58	2810

TABLE XVII.

Paired Judgments by different Masters. (Second Report.)

Judgment by First Master.

Judgment by Second Master.	*C*	*D*	*E*	*F*	*G*	Totals
C	34	51	17	1	—	103
D	51	258	479	46	5	839
E	17	479	1400	343	28	2267
F	1	46	343	218	72	680
G	—	5	28	72	42	147
Totals	103	839	2267	680	147	4036

The correlation coefficient for the first Report containing 1405 pairs of judgments is $r = \cdot 47$; that for the second which includes 2018 pairs is $r = \cdot 50$. This result should be looked upon as a measurement of the intelligence exhibited by boys in different subjects rather than as a measurement of the personal equation between different masters. I have also employed a contingency table to examine the pairs of judgments by the same master at different times.

TABLE XVIII.

Paired Judgments by the same Master. (With an interval of nine months.)

First Judgment.

Second Judgment.	*C*	*D*	*E*	*F*	*G*	Totals
C	2	3	—	—	—	5
D	8	37	35	1	—	81
E	3	35	120	28	—	186
F	—	—	30	35	2	67
G	—	—	—	8	8	16
Totals	13	75	185	72	10	355

The coefficient of mean square contingency obtained from this table is $r = \cdot 66$.

The agreement here is considerably closer than in Tables XVI and XVII as might have been expected; for a particular master's opinion of a boy's mental capacity given on two occasions separated by several months is less likely to vary than the opinions given by two masters about the same time from the point of view of the boy's achievements in their respective subjects. The defect of perfect correlation shown in Table XVIII may be due to (i) variation in the teacher's judgment, (ii) longer knowledge of the boy, or (iii) to development in the boy himself. The last seems excluded by the small correlation of age and intelligence and the general character of the total frequencies of first and second judgments. The influence of longer knowledge on judgment has already been shown to be of importance, but the precise part played by (i) and (ii) cannot be fully evaluated until further experiments on the influence of the time factor on the teacher's judgment have been made.

APPENDIX.

Professor Karl Pearson's Scale of Ability.

A. Mentally Defective. Capable of holding in the mind only the simplest facts, and incapable of perceiving or reasoning about the relationship between facts.

B. Slow Dull. Capable of perceiving relationship between facts in some few fields with long and continuous effort; but not generally nor without much assistance.

C. Slow. Very slow in thought generally, but with time understanding is reached.

D. Slow Intelligent. Slow generally, although possibly more rapid in certain fields; quite sure of knowledge when once acquired.

E. Fairly Intelligent. Ready to grasp, and capable of perceiving facts in most fields; capable of understanding without much effort.

F. Distinctly Capable. A mind quick in perception and in reasoning rightly about the perceived.

G. Very Able. Quite exceptionally able intellectually, as evidenced either by the person's career or by consensus of opinion of acquaintances, or by school record in case of children.

ON THE SIGNIFICANCE OF THE TEACHER'S APPRECIATION OF GENERAL INTELLIGENCE.

BY WALTER H. GILBY, B.Sc. with the assistance of KARL PEARSON, F.R.S.

(1) *Introductory.* The object of the present investigation is to ascertain whether the judgment of the teacher made on a fairly long experience of his class has or has not a significance of its own. A recent writer has stated that it seems to him "that, to a great extent, and unless they are very carefully controlled, the teachers' judgments are *relatively* as well as absolutely valueless—i.e. that we cannot attach any great confidence even to the classification of one teacher of a single class. His judgment is probably affected to some extent by the age of his pupils*." This is one of those sweeping judgments, given apparently without any close study of the subject, with which one is only too familiar just now. It is perfectly possible to test the extent to which age affects the teacher's judgment of intelligence, and this has already been done several times. Although controls have been previously made of the accuracy of teachers' judgments by comparing two or three teachers' independent opinions, it seemed desirable in view of such dogmatic assertions as the above to deal with the matter at some length and *de novo*. The method adopted in the present instance was to test the judgment of the class teacher, i.e. a single teacher, who had before him written definitions of each category of general intelligence, against the place and percentage of marks obtained by the same boy in an examination conducted by another and independent man. In order that there might be some approach to the conditions involved in the mass data provided by school inspections and surveys, data were obtained from eight schools and from 36 teachers who were willing to spend time and care upon the observations.

(2) *Class of Schools.* The schools are primary schools of the usual county-council type. The characters of the different schools are as follows:

School No. 1. This school is a comparatively new one. The pupils are children of fairly well-to-do tradespeople, civil servants, clerks, etc. There are very few poor children, and no children attending the school have free dinners.

---

* G. U. Yule, *Journal of the Royal Statistical Society*, Vol. LXXIII. p. 550.

School No. 2. Whilst the children attending this school are poorer than those in No. 1, yet none are really poor. No free dinners are provided.

School No. 3. This school is of the same type as No. 2.

School No. 4. The children attending this school are still poorer than those above, about 1 °/₀ having free dinners in the winter months. No free dinners are required in summer.

School No. 5. This school is attended by a very mixed class of children. While a few are very poor, the greater proportion are well clad and well fed. Between 1 °/₀ and 2 °/₀ of the children have free dinners in the winter months, but none during the summer.

School No. 6. This school is in a poor neighbourhood and the children are decidedly worse clothed and fed than those in the former schools. About 7 °/₀ of the children have free dinners in the winter, and 1 °/₀ in the summer.

School No. 7. This school is of a similar type to the one above. Between 7 °/₀ and 8 °/₀ have free dinners in the winter.

School No. 8. This school is situated in a very poor district. The children are much worse clothed than in any of the other schools, and the school is of a much poorer type than the others. The number of free dinners could not be ascertained; it was considered as confidential.

The total number of children from all schools—all boys—about whom particulars were obtained was 1725.

(3) *Categories used and Particulars recorded.* The standard in which each child was working, the age on a given date, the position occupied in class as determined by examination, and the percentage of marks gained in the last term's examination conducted by the headmaster were obtained for each child. These particulars were abstracted from the school records. An estimate of each child's "general intelligence" was made by the class teacher according to the categories provided by Professor Pearson: see *Biometrika*, Vol. VIII. p. 93. Each teacher had been in charge of his class for nearly 12 months when the estimate was made; hence ample opportunity had been afforded for observation, and each teacher had sufficient personal knowledge of the children in his class to form a just estimate of their intelligence*. Instructions were given that age, standard, etc., were not to be considered in forming the estimate of general intelligence, but that the teachers were to proceed from the verbal definitions of the categories. The results show that age and standard had little influence on the teachers' judgments.

So few children were placed in Class A, that of the mentally defective, that those occurring (7 altogether) were for statistical purposes included in Class B. At the present time nearly all really mentally deficient children are removed from the ordinary schools and attend special centres.

---

* This point is of very great importance. No real appreciation of intelligence can be formed by placing the child under novel conditions with an unfamiliar inquisitor.

(4) *Clothing.* The children were classified in addition to their general intelligence by their clothing. This was judged in accordance with the subjoined scale provided by Professor Pearson. The grades are distinguished by Roman numerals, I to V:

I = Very well clad. II. Well clad, stuff suit, good boots; sufficient, even if poor. III. Clothing poor but passable; an old and, perhaps, ragged suit with some attempt at proper underclothing. IV. Clothing insufficient; boots bad and leaking. V. Clothing the worst; no boots or makeshift substitutes for them.

Here again so few children occurred in Class V—five in all—that for statistical purposes they were included in Class IV.

A boy's "order in class" was taken to be his place less ·5 divided by the number of boys in the class. Thus if a boy was mth out of n boys, his class order was $(m - 0{\cdot}5)/n$. The advantage of this method is that the mean order in each class is $\frac{1}{2}$ and independent of the number in the class.

The numbers in each school are given in Table I. It will be seen that the boys in Schools Nos. 3 and 7 were very few in number. They belonged to special classes whose teachers were interested in the work. In certain cases those teachers may take the higher divisions of certain standards, and thus we find a considerable relationship between intelligence and school*. When intelligence was correlated with age in the eight schools independently the relationship was found to be positive in three schools and negative in five schools, but the latter group included the two schools, Nos. 3 and 7, with under 50 boys dealt with in each. Similarly age and clothing had positive correlation in three and negative correlation in five schools, the latter including Nos. 3 and 7 again. Intelligence and clothing had inappreciable correlation in two schools, and quite sensible correlation in six. This was still true, if the correlation of clothing and intelligence was taken for a constant age.

The ultimate relation of clothing to intelligence is an extremely interesting and important one, for the state of the clothing is often taken as a measure of home conditions and the intelligence of the children is thus asserted to depend on environment. The average correlation of the present data between clothes and intelligence based on the individual values for the eight schools is ·21 when corrected for age. The correlation between clothing and intelligence for constant age and constant standard† for the 1725 boys is ·22. Either method leads us to the same conclusion: the intelligence is related to the condition of the clothing, the more intelligent child having the better clothing. Now there

---

* When the headmasters take up an investigation of this kind and the whole schools fall into the record the contingency between intelligence and school is low. But it is very usual in schools to divide the standard into two parts, one the section with progressive children, the other the inert group; naturally the grade of intelligence is quite different in the two sections, and the casual critic talks about the personal equation of the teacher within the same school to explain a result which a little knowledge of the actual conditions would have rendered quite clear.

† The partial correlation for constant standard was taken to correct as far as possible any personal equation in the clothing estimates of the teachers.

are several fallacies that may arise in the interpretation of this correlation. It might be asserted that the more intelligent child will look after his clothing better, but although this may produce some effect on general tidy appearance, it cannot contribute largely to influencing an estimate based on the existence of sufficient or insufficient underclothing and the presence of boots, etc. There can we think be little doubt that the evidence of clothing is roughly a measure of home conditions. But it is none the less fallacious to assert that the inferior intelligence evidenced by the poorer clothing is necessarily a product of bad home environment. It will be clear that intelligent parents who have usually higher wages will provide better clothing and will look after their children better. Hence the problem turns—as most such problems do—on the relative intensities of nature and nurture. Is the lower intelligence of the children due to the poorer home environment evidenced by the worse clothing, or is the worse clothing only a mark of the lower intelligence of the parents, which is naturally reproduced in their children? If we look into this point algebraically, we may write the subscripts 1 = intelligence of the children, 2 = intelligence of the parents, 3 = clothing of the children. Then what we really want is:

$$ {}_2\rho_{13} = \frac{r_{13} - r_{12}r_{23}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{23}^2}}, $$

i.e. the correlation between clothing and intelligence of the children *for constant intelligence of the parents.* If this correlation be substantial, then the proposition that the intelligence of the children is directly influenced by their environment will receive some support. The problem then turns on whether $r_{12} \times r_{23}$ is of sensibly the same order or not as r_{13}. We may safely say that r_{12} lies between ·4 and ·5; what value are we to give to r_{23}? No direct estimate of the relation of parental intelligence to the clothing of the offspring is at present available, but we think few would be hardy enough to assert that it would be unreasonable to consider it as lying between ·4 and ·6 at the least. The commonest experience seems to show that a tidy child followed home will disclose a careful intelligent mother, and a father, whose intelligence is measured by adequate wages. Until, however, this point has been definitely examined statistically, it is futile to dogmatise about clothing being a standard of parental neglect, and that such neglect is producing poor intelligence in the offspring. A better argument might be deduced, if it could be shown that adequately fed and clothed pauper and asylum children are of superior intelligence to the children of the public primary schools. Many other tests of the presumed influence of environment are of the like superficial character to the clothing test as evidence of the influence of home environment; they are fallacious until they have been modified by correction for the hereditary factor*.

---

* It is not unusual for the school medical officer to find a correlation between intelligence and evidences of parental neglect, dirtiness, poor clothing or inadequate nutrition. It does not follow that this relationship which lies between ·2 and ·3 is the source in whole or even part of the poor intelligence. It may be, but the evidence given, which wholly neglects the hereditary factor, is quite insufficient to prove that it is.

(5) *Graphic Exhibition of the Relationship of Clothes and Intelligence.* The value of the crude correlation between intelligence and clothing found by mean square contingency corrected for number of cells was ·29. Found by a two-rowed Table*, the first row containing Class I of clothing and the second row Classes II—V, the correlation ratio was ·30, a result very close to the contingency value. It appeared worth while investigating the whole problem of the linearity of the regression in this case of clothing and intelligence as a justification of the use of the correlation methods employed. Accordingly the intelligence categories were plotted on a normal scale, the standard deviation of intelligence being taken as the unit. At the means of the intelligence groups was set up the means of the clothing of such groups, the range of Class II of clothing being taken as the unit, and all means measured in terms of this unit from the boundary between Classes I and II. In obtaining the mean clothing for any given array of intelligence, we had first to express the mean in terms of the standard deviation of the array and then these means in terms of h the range of Class II of clothing. The following numerical results were reached. $\bar{x}$ is measured from upper $\bar{x}'$ from lower limit of Class II, σ_I, and $\sigma_{cl.}$ are standard deviations.

$$\begin{array}{lllll}
\bar{x}_B/\sigma_B = {\cdot}6624, & \bar{x}_B'/\sigma_B = {\cdot}1744, & h/\sigma_B = {\cdot}8363, & \sigma_B/\sigma_{cl.} = 1{\cdot}4239, & \bar{x}_B/h = {\cdot}7916, \\
x_C/\sigma_C = {\cdot}7750, & \bar{x}_C'/\sigma_C = {\cdot}4560, & h/\sigma_C = 1{\cdot}2310, & \sigma_C/\sigma_{cl.} = {\cdot}9679, & \bar{x}_C/h = {\cdot}6296, \\
\bar{x}_D/\sigma_D = {\cdot}5899, & \bar{x}_D'/\sigma_D = {\cdot}7519, & h/\sigma_D = 1{\cdot}3418, & \sigma_D/\sigma_{cl.} = {\cdot}8880, & \bar{x}_D/h = {\cdot}4396, \\
\bar{x}_E/\sigma_E = {\cdot}2776, & \bar{x}_E'/\sigma_E = 1{\cdot}1137, & h/\sigma_E = 1{\cdot}3913, & \sigma_E/\sigma_{cl.} = {\cdot}8564, & \bar{x}_E/h = {\cdot}1995, \\
\bar{x}_F/\sigma_F = -{\cdot}0435, & \bar{x}_F'/\sigma_F = 1{\cdot}2021, & h/\sigma_F = 1{\cdot}1586, & \sigma_F/\sigma_{cl.} = 1{\cdot}0284, & \bar{x}_F/h = -{\cdot}0376, \\
\bar{x}_G/\sigma_G = -{\cdot}4152, & \bar{x}_G'/\sigma_G = 1{\cdot}3739, & h/\sigma_G = {\cdot}9587, & \sigma_G/\sigma_{cl.} = 1{\cdot}2428, & \bar{x}_G/h = -{\cdot}4835, \\
\hline
\bar{x}_{cl.}/\sigma_{cl.} = {\cdot}3353, & \bar{x}_{cl.}/\sigma_{cl.} = {\cdot}8562, & h/\sigma_{cl.} = 1{\cdot}1915, & \text{—} & \bar{x}_{cl.}/\sigma_{cl.} = {\cdot}2806.
\end{array}$$

For absolute normality the ratio of the S. D. of the array to the S. D. of the population should be a constant; it is clearly rather variable running up at the terminal arrays, so that the distribution is not truly homoscedastic. The η's found by the new method† are

$$\eta = {\cdot}343, \qquad \eta' = {\cdot}340.$$

The slope of the regression line

$$= \eta \times \sigma_{cl.}/\sigma_I = \frac{\eta}{1{\cdot}1915} \times \frac{h}{\sigma_I} = {\cdot}2882,$$

since b and σ_I have been taken as our units of clothing and intelligence respectively.

In this manner the graph has been constructed and it shows, in a remarkable manner, how very closely the regression, even with our qualitative scales, is truly linear. The general method of plotting characters on normal scales and then testing the linearity of the resulting regression deserves fuller recognition; for besides conveying results effectively to the eye, the continual reappearance of linear regression when dealing with these qualitative characters is a feature which must give greater confidence in the methods applied.

---

* Method given in *Biometrika*, Vol. VII. Formula (iii.), p. 250.

† *Biometrika*, Vol. VII. Formula (ii.), p. 249, and illustration, p. 257.

The reader should bear in mind that the graph, and the values of the correlation obtained are the *crude* values, and that the latter are reduced 30 °/₀, i.e. to ·22, when correction is made for age and standard.

(6) *Correlations determined.* The following is a list of the correlations found:

(1) Standard and Age, ·853 (Greater Age, higher standard).

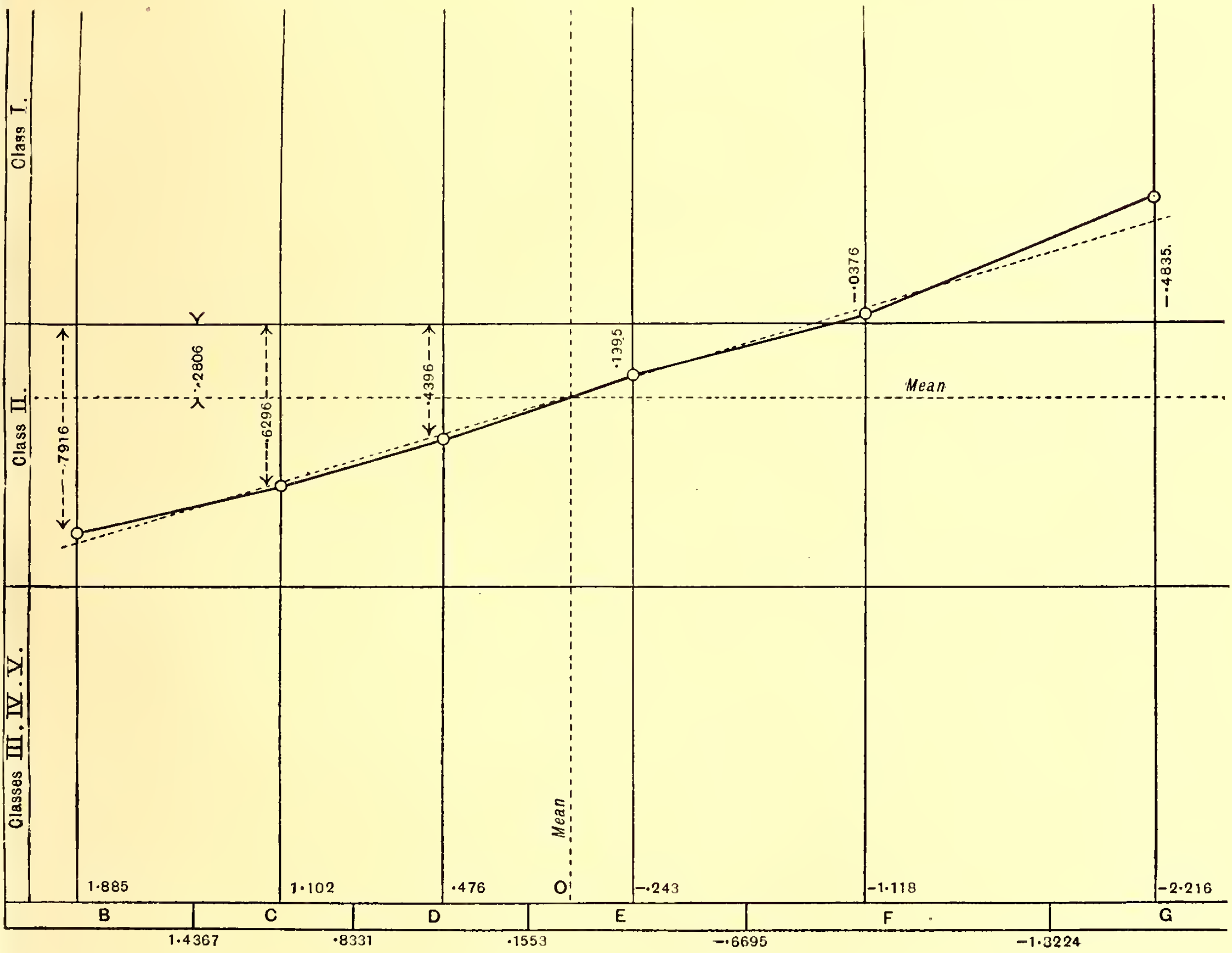
(2) Standard and Intelligence, ·185 (Higher Standard, better intelligence).

(3) Standard and Order in Examination, ·002.

(4) Standard and Percentage of Marks, ·254 (Higher Standard, higher percentage).

(5) Standard and Clothing, ·258 (Higher Standard, better clothing).

(6) Age and Intelligence, ·079 (Higher Age, higher intelligence).



Intelligence.

(7) Age and Order in Examination, – ·059 (Higher Age, lower place).

(8) Age and Percentage of Marks, ·119 (Higher Percentage, greater age).

(9) Age and Clothing, ·053 (Better Clothing, higher age).

(10) Intelligence and Order in Examination, ·679 (Higher Intelligence, higher place).

(11) Intelligence and Percentage of Marks, ·694 (Higher Intelligence, higher percentage).

(12) Intelligence and Clothing, ·291 (Higher Intelligence, better clothing).

(13) Order in Examination and Percentage of Marks, ·798.

(14) Order in Examination and Clothing, ·207 (Higher Place, better clothing).

(15) Percentage of Marks and Clothing, ·306 (Higher Percentage, better clothing).

(16) School and Intelligence, ·308 (Better School*, better intelligence).

(17) School and Clothing, ·362 (Better Clothing, better school).

Of these correlations: (1), (3), (4), (6), (9), (10), (11), (14), (15) were found by the correlation ratio for the arrays of the quantitative variable; (2), (5), (12), (16), and (17) were found by mean square contingency, corrected for the number of cells; (7), (8), and (13), both variables being quantitative, were found by the fundamental product-moment method.

As we might anticipate, there is no relation between order in class and standard. There is very little relation between age and order in class, or between age and intelligence. The really significant correlations are those between order in class and percentage in marks with grade of general intelligence. If we take the partial correlations for constant age and constant standard we find:

Correlation of General Intelligence and Order in Examination for constant age and constant standard = ·686.

Correlation of General Intelligence and Percentage of Marks for constant age and constant standard = ·671.

Here it must be remembered that we are using eight different schools and the judgment of thirty-six different teachers to determine the general intelligence; further the percentage of marks and the places in class were settled by eight different headmasters examining their schools independently of the class teachers.

It will we think be evident from this that there is a very marked correlation between the teacher's estimate of general intelligence and the examination value of his pupil. The teacher's judgment of general intelligence will give at least an estimate of this value, and we believe is of even more importance. It is possible and we believe reasonable to hold that the lack of still higher correlation is not

---

* The schools were arranged in order of poverty of school population, estimated chiefly by the percentage of free dinners.

necessarily owing to personal equation in the teacher, but is largely due to the fact that the experienced teacher in his estimate of intelligence gives us something of more importance than examination value, and not wholly measurable by examination value. At any rate the results reached are sufficient to condemn all *a priori* sweeping judgments as to the futility of the teacher's estimate of "general intelligence."

TABLE I.

Age and Standard.

Standard.

Age.	I	II	III	IV	V	VI	VII	VIII	Totals
7½	14	2	—	—	—	—	—	—	16
8	39	10	6	—	—	—	—	—	55
8½	52	38	20	1	—	—	—	—	111
9	27	70	57	1	1	—	—	—	156
9½	19	43	82	18	3	—	—	—	165
10	7	20	66	36	5	—	—	—	134
10½	4	15	43	37	22	2	—	—	123
11	1	10	37	40	33	9	1	—	131
11½	3	7	23	30	55	27	4	—	149
12	1	3	10	25	52	35	11	—	137
12½	—	—	6	9	23	63	22	3	126
13	—	2	1	11	26	71	23	1	135
13½	—	—	1	5	18	54	48	6	132
14	—	—	—	8	8	39	34	15	104
14½	—	1	—	—	3	7	13	12	36
15	—	—	—	—	1	4	4	6	15
Totals	167	221	352	221	250	311	160	43	1725
Means	8·70	9·42	9·98	11·11	11·91	12·85	13·30	14·08	11·14

Found from vertical arrays r taken $= \eta =$ ·853.

TABLE II.

Standard and Intelligence.

Standard.

Intelligence.	I	II	III	IV	V	VI	VII	VIII	Totals
B	17	23	42	16	18	10	4	—	130
C	27	34	42	25	38	32	19	2	219
D	45	61	69	41	66	73	39	13	407
E	50	66	117	75	77	80	52	18	535
F	27	36	72	53	45	98	35	9	375
G	1	1	10	11	6	18	11	1	59
Totals	167	221	352	221	250	311	160	43	1725

$\phi^2 =$ ·0556, corrected $\phi^2 =$ ·0353. Correlation + ·185.

TABLE III. *Order in Examination and Standard.*

Standard.

Order in Examination.	I	II	III	IV	V	VI	VII	VIII	Totals
*·00— ·05**	8	13	16	10	11	15	8	2	83
·05— ·10	7	7	21	11	12	17	7	2	84
·10— ·15	7	11	12	13	14	13	8	2	80
·15— ·20	13	14	21	10	13	19	10	3	103
·20— ·25	8	9	16	9	12	13	7	2	76
·25— ·30	5	15	20	14	13	14	9	1	91
·30— ·35	10	6	17	7	12	19	7	4	82
·35— ·40	10	16	17	15	15	12	6	2	93
·40— ·45	7	8	18	10	10	19	11	2	85
·45— ·50	7	11	17	13	11	11	7	—	77
·50— ·55	9	10	18	9	15	19	10	3	93
·55— ·60	10	13	21	7	11	14	4	3	83
·60— ·65	8	8	14	16	12	17	8	3	86
·65— ·70	6	10	17	11	14	17	10	1	86
·70— ·75	12	14	18	9	11	12	5	2	83
·75— ·80	8	12	20	13	13	17	11	1	95
·80— ·85	7	10	17	10	14	15	9	4	86
·85— ·90	8	11	15	13	14	18	8	1	88
·90— ·95	9	11	19	10	10	15	7	3	84
·95—1·00	8	12	18	11	13	15	8	2	87
Totals	167	221	352	221	250	311	160	43	1725

Found from vertical arrays r taken $= \eta = \cdot 002$.

TABLE IV. *Standard and Percentages.*

Standard.

Percentages.	I	II	III	IV	V	VI	VII	VIII	Totals
5	—	—	1	—	—	—	—	—	1
10	2	2	1	1	—	—	—	—	6
15	2	3	2	1	—	—	—	—	8
20	2	3	1	3	—	1	—	—	10
25	3	2	4	3	1	—	—	—	13
30	3	8	5	1	4	4	—	—	25
35	10	4	7	5	7	6	—	—	39
40	9	13	14	10	9	7	2	—	64
45	8	13	18	14	17	13	—	—	83
50	11	22	29	10	25	22	6	—	125
55	13	29	23	14	24	23	9	4	139
60	18	21	45	24	45	43	11	—	207
65	24	28	39	30	33	49	22	5	230
70	19	37	53	27	28	47	25	5	241
75	11	17	53	25	24	40	34	11	215
80	15	15	27	24	14	24	31	8	158
85	9	4	21	14	13	23	12	8	104
90	8	—	7	12	5	9	8	1	50
95	—	—	2	3	1	—	—	1	7
Totals	167	221	352	221	250	311	160	43	1725

Found from vertical arrays r taken $= \eta = + \cdot 254$.

*** In grouping Order in Examination such groups as ·00—·05 contain everything up to and *under* ·05.**

TABLE V.

Standard and Clothing.

Standard.

Clothing.	I	II	III	IV	V	VI	VII	VIII	Totals
I	20	71	157	82	101	127	59	19	636
II	87	88	134	77	117	145	81	22	751
III	56	42	41	45	29	32	18	2	265
IV, V	4	20	20	17	3	7	2	—	73
Totals	167	221	352	221	250	311	160	43	1725

$\phi^2 = \cdot 0832$, corrected $\phi^2 = \cdot 0710$, $r = +\cdot 258$.

Found from mean square contingency $r = +\cdot 258$.

TABLE VI.

Age and Intelligence.

Intelligence.

Age.	*B*	*C*	*D*	*E*	*F*	*G*	Totals
7½	—	1	2	8	5	—	16
8	7	5	15	16	12	—	55
8½	6	11	26	47	20	1	111
9	23	26	38	36	28	5	156
9½	10	20	30	55	42	8	165
10	9	13	23	52	34	3	134
10½	12	15	38	32	22	4	123
11	12	16	34	36	26	7	131
11½	14	13	33	48	31	10	149
12	10	21	27	42	33	4	137
12½	4	10	28	45	32	7	126
13	6	21	29	43	33	3	135
13½	8	16	30	40	33	5	132
14	4	20	40	24	14	2	104
14½	1	10	13	6	6	—	36
15	4	1	1	5	4	—	15
Totals	130	219	407	535	375	59	1725
Means	10·80	11·34	11·27	11·03	11·15	11·25	11·14

Correlation $r = \eta = +\cdot 079$.

TABLE VII.

Order in Examination and Age.

Age.

Order in Examination.	7½	8	8½	9	9½	10	10½	11	11½	12	12½	13	13½	14	14½	15	Totals
·00— ·05	1	1	5	8	9	6	9	3	11	3	7	5	6	6	1	2	83
·05— ·10	—	3	2	5	14	6	8	8	10	7	5	1	5	6	3	1	84
·10— ·15	2	2	1	11	9	8	9	8	5	3	4	6	8	3	1	—	80
·15— ·20	1	4	9	7	13	14	4	7	6	5	5	10	8	5	2	3	103
·20— ·25	—	5	2	6	9	7	6	10	3	6	4	8	5	4	—	1	76
·25— ·30	—	1	7	9	6	12	8	5	8	8	10	3	7	6	1	—	91
·30— ·35	1	3	5	6	14	6	3	3	10	3	7	7	7	3	3	1	82
·35— ·40	2	4	7	11	10	4	7	7	8	10	5	6	3	7	2	—	93
·40— ·45	1	4	5	5	9	6	5	6	8	11	5	10	5	4	1	—	85
·45— ·50	—	2	4	5	10	10	6	8	5	10	3	4	8	1	1	—	77
·50— ·55	1	1	13	5	8	3	6	9	10	5	11	6	7	4	3	1	93
·55— ·60	1	6	5	7	10	8	3	4	8	7	7	9	4	3	1	—	83
·60— ·65	2	2	4	8	4	5	7	8	6	9	7	11	7	4	2	—	86
·65— ·70	—	2	6	8	4	6	9	4	8	6	7	7	9	7	2	1	86
·70— ·75	2	6	5	6	11	6	5	6	5	8	1	10	5	4	2	1	83
·75— ·80	1	2	7	8	5	8	4	9	8	7	10	7	6	6	6	1	95
·80— ·85	—	1	8	10	4	4	5	9	4	11	4	11	4	9	1	1	86
·85— ·90	—	1	7	10	6	2	3	6	11	11	7	6	12	6	—	—	88
·90— ·95	1	3	4	8	7	5	8	6	9	4	5	2	8	9	3	2	84
·95—1·00	—	2	5	13	3	8	8	5	6	3	12	6	8	7	1	—	87
Totals	16	55	111	156	165	134	123	131	149	137	126	135	132	104	36	15	1725

Found by product-moment method: $r = -·059$.

TABLE VIII.

Ages and Percentages of Marks.

Age.

Percentages.	7½	8	8½	9	9½	10	10½	11	11½	12	12½	13	13½	14	14½	15	15½	Totals
5	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
10	—	—	1	3	—	—	1	—	—	—	—	—	—	1	—	—	—	6
15	—	1	1	2	1	—	2	—	1	—	—	—	—	—	—	—	—	8
20	—	1	1	—	2	1	3	1	—	—	1	—	—	—	—	—	—	10
25	—	1	—	—	3	2	1	1	2	1	1	1	—	—	—	—	—	13
30	—	—	3	3	3	1	4	2	1	2	2	1	1	2	—	—	—	25
35	—	1	3	9	3	1	1	1	4	4	2	3	3	3	—	1	—	39
40	1	2	8	8	4	7	4	6	4	6	1	3	9	1	—	—	—	64
45	1	2	5	6	12	8	8	4	11	4	5	3	8	6	—	—	—	83
50	2	3	12	15	9	9	7	14	9	7	11	11	6	4	3	3	—	125
55	1	2	6	21	6	6	14	16	10	20	10	11	7	7	2	—	—	139
60	4	8	10	12	24	18	13	16	20	14	19	23	12	9	5	—	—	207
65	1	11	19	17	14	21	15	14	17	24	20	20	16	16	3	2	—	230
70	2	13	17	25	28	15	16	20	22	18	13	21	11	13	4	3	—	241
75	—	2	12	17	27	18	10	11	20	18	16	16	23	17	8	—	—	215
80	—	4	8	9	15	20	10	11	9	6	12	10	22	14	6	2	—	158
85	3	2	2	4	12	3	8	9	14	8	8	8	8	9	3	2	1	104
90	1	2	2	5	1	4	3	4	5	4	5	4	6	2	2	—	—	50
95	—	—	1	—	1	—	2	1	—	1	—	—	—	—	—	—	1	7
Totals	16	55	111	156	165	134	123	131	149	137	126	135	132	104	36	13	2	1725

Found by product-moment method: $r = +·119$.

TABLE IX. *Age and Clothing.*

Clothing.

Age.	I	II	III	IV	Totals
7½	1	11	4	—	16
8	9	29	16	1	55
8½	28	57	23	3	111
9	57	64	27	8	156
9½	68	69	22	6	165
10	57	54	16	7	134
10½	42	57	17	7	123
11	55	45	23	8	131
11½	67	50	21	11	149
12	55	53	23	6	137
12½	52	59	12	3	126
13	48	64	20	3	135
13½	44	68	18	2	132
14	33	46	20	5	104
14½	8	22	3	3	36
15	12	3	—	—	15
Totals	636	751	265	73	1725
Means	11·23	11·15	10·94	11·09	11·14

Found from vertical array $r = \eta = +{\cdot}053$.

TABLE X. *Order in Examination and Intelligence.*

Intelligence.

Order in Examination.	*B*	*C*	*D*	*E*	*F*	*G*	Totals
·00— ·05	—	—	1	14	48	20	83
·05— ·10	—	—	4	22	45	13	84
·10— ·15	—	—	7	13	50	10	80
·15— ·20	—	2	7	33	55	6	103
·20— ·25	—	3	8	35	27	3	76
·25— ·30	—	2	12	37	38	2	91
·30— ·35	—	2	17	41	18	4	82
·35— ·40	2	5	25	44	17	—	93
·40— ·45	1	3	21	40	20	—	85
·45— ·50	2	10	17	37	11	—	77
·50— ·55	3	10	31	37	12	—	93
·55— ·60	5	12	29	29	8	—	83
·60— ·65	4	11	29	35	6	1	86
·65— ·70	6	17	36	23	4	—	86
·70— ·75	4	18	36	22	3	—	83
·75— ·80	13	29	27	20	6	—	95
·80— ·85	13	26	27	19	1	—	86
·85— ·90	23	22	31	8	4	—	88
·90— ·95	23	24	23	13	1	—	84
·95—1·00	31	23	19	13	1	—	87
Totals ...	130	219	407	535	375	59	1725
Means ...	·835	·738	·618	·456	·249	·117	·501

Found by means of vertical arrays $r = \eta = +{\cdot}679$.

TABLE XI.

Percentage of Marks and Intelligence.

Intelligence.

Percentage of Marks.	*B*	*C*	*D*	*E*	*F*	*G*	Totals
4·95— 9·95	—	—	1	—	—	—	1
9·95—14·95	6	—	—	—	—	—	6
14·95—19·95	5	2	1	—	—	—	8
19·95—24·95	7	2	1	—	—	—	10
24·95—29·95	7	5	—	1	—	—	13
29·95—34·95	10	9	5	—	1	—	25
34·95—39·95	15	8	10	5	1	—	39
39·95—44·95	14	22	18	10	—	—	64
44·95—49·95	21	27	23	11	1	—	83
49·95—54·95	19	30	47	25	4	—	125
54·95—59·95	13	37	60	24	5	—	139
59·95—64·95	11	30	77	69	19	1	207
64·95—69·95	1	28	71	101	28	1	230
69·95—74·95	1	8	48	130	53	1	241
74·95—79·95	—	7	32	82	83	11	215
79·95—84·95	—	4	7	47	88	12	158
84·95—89·95	—	—	6	21	56	21	104
89·95—94·95	—	—	—	8	33	9	50
94·95—99·95	—	—	—	1	3	3	7
Totals ...	130	219	407	535	375	59	1725
Means ...	42·75	54·50	61·45	69·80	78·65	84·80	66·45

Found by means of vertical arrays, $r=\eta=+\cdot694$.

TABLE XII.

Intelligence and Clothing.

Intelligence.

Clothing.	*B*	*C*	*D*	*E*	*F*	*G*	Totals
I	33	48	113	209	194	39	636
II	41	100	202	255	138	15	751
III	39	58	70	61	33	4	265
IV and V	17	13	22	10	10	1	73
Totals	130	219	407	535	375	59	1725

$\phi^2=\cdot1014$, corrected $\phi^2=\cdot0927$, $r=+\cdot291$.

TABLE XIII. *Order in Examination and Percentages of Marks.*

Percentages.

Order in Examination.	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	Totals
·00— ·05	—	—	—	—	—	—	—	—	—	—	—	2	1	2	4	18	24	26	6	83
·05— ·10	—	—	—	—	—	—	—	—	—	—	—	—	1	9	12	22	28	11	1	84
·10— ·15	—	—	—	—	—	—	—	—	—	—	—	1	6	7	15	24	23	4	—	80
·15— ·20	—	—	—	—	—	—	—	—	—	—	—	7	6	19	21	33	14	3	—	103
·20— ·25	—	—	—	—	—	—	—	—	—	—	—	7	7	12	25	20	4	1	—	76
·25— ·30	—	—	—	—	—	—	—	—	—	—	4	—	12	22	36	12	1	4	—	91
·30— ·35	—	—	—	—	—	—	—	—	—	—	4	7	9	16	39	6	—	1	—	82
·35— ·40	—	—	—	—	—	—	—	—	—	3	6	8	19	33	18	4	2	—	—	93
·40— ·45	—	—	—	—	—	—	—	—	—	4	4	7	19	31	15	4	1	—	—	85
·45 — ·50	—	—	—	—	—	—	—	—	3	3	5	7	24	24	4	2	5	—	—	77
·50— ·55	—	—	—	—	—	—	—	—	6	7	2	17	27	26	5	2	1	—	—	93
·55— ·60	—	—	—	—	—	—	—	3	3	7	3	21	31	8	6	—	1	—	—	83
·60— ·65	—	—	—	—	—	—	1	2	3	8	7	29	16	14	4	2	—	—	—	86
·65— ·70	—	—	—	—	—	—	2	3	8	4	19	24	14	5	3	4	—	—	—	86
·70— ·75	—	—	—	—	—	5	—	5	9	10	14	21	13	3	3	—	—	—	—	83
·75— ·80	—	—	—	—	—	1	1	10	7	15	21	25	6	4	2	3	—	—	—	95
·80— ·85	—	—	—	—	2	1	9	10	8	14	24	6	6	4	1	1	—	—	—	86
·85— ·90	—	—	—	2	1	2	12	9	14	20	12	8	6	1	—	1	—	—	—	88
·90— ·95	—	—	2	3	7	6	8	8	6	23	8	8	3	1	1	—	—	—	—	84
·95—1·00	1	6	6	5	3	10	6	14	16	7	6	2	4	—	1	—	—	—	—	87
Totals	1	6	8	10	13	25	39	64	83	125	139	207	230	241	215	158	104	50	7	1725

Found by product-moment method: $r = +{\cdot}798$.

TABLE XIV. *Order in Examination and Clothing.*

Clothing.

Order in Examination.	I	II	III	IV & V	Totals
·00— ·05	46	28	6	3	83
·05— ·10	49	25	8	2	84
·10— ·15	41	26	11	2	80
·15— ·20	43	45	13	2	103
·20— ·25	31	34	10	1	76
·25— ·30	36	37	15	3	91
·30— ·35	38	33	10	1	82
·35— ·40	32	43	15	3	93
·40— ·45	40	37	6	2	85
·45— ·50	30	36	5	6	77
·50— ·55	33	44	12	4	93
·55— ·60	22	48	9	4	83
·60— ·65	26	41	17	2	86
·65— ·70	26	41	17	2	86
·70— ·75	25	39	13	6	83
·75— ·80	26	44	20	5	95
·80— ·85	27	40	15	4	86
·85— ·90	25	32	26	5	88
·90— ·95	22	36	20	6	84
·95—1·00	18	42	17	10	87
Totals	636	751	265	73	1725
Means	·431	·523	·579	·620	·501

Found by means of vertical arrays, $r = \eta = +{\cdot}207$

TABLE XV.

Clothing and Percentages of Marks.

Clothing.

Percentages.	I	II	III	IV & V	Totals
4·95— 9·95	—	—	—	1	1
9·95—14·95	2	2	—	2	6
14·95—19·95	1	3	2	2	8
19·95—24·95	—	4	3	3	10
24·95—29·95	2	5	3	3	13
29·95—34·95	7	8	7	3	25
34·95—39·95	7	12	18	2	39
39·95—44·95	14	26	17	7	64
44·95—49·95	21	37	15	10	83
49·95—54·95	34	63	21	7	125
54·95—59·95	35	72	29	3	139
59·95—64·95	56	100	44	7	207
64·95—69·95	87	107	25	11	230
69·95—74·95	96	110	33	2	241
74·95—79·95	86	102	20	7	215
79·95—84·95	83	57	16	2	158
84·95—89·95	63	33	8	—	104
89·95—94·95	37	9	3	1	50
94·95—99·95	5	1	1	—	7
Totals	636	751	265	73	1725
Means	71·10	65·46	60·60	52·40	66·45

Found by means of vertical arrays, $r=\eta=+\cdot306$.

TABLE XVI.

School and Intelligence.

Intelligence.

School.	*B*	*C*	*D*	*E*	*F*	*G*	Totals
No. *1*	6	18	36	43	39	4	146
No. *2*	14	25	52	87	54	13	245
No. *3*	—	—	1	8	37	—	46
No. *4*	14	19	60	94	73	12	272
No. *5*	33	69	132	187	85	14	520
No. *6*	45	50	69	72	66	15	317
No. *7*	—	6	20	8	9	1	44
No. *8*	18	32	37	36	12	—	135
Totals	130	219	407	535	375	59	1725

$\phi^2=\cdot125$, corrected $\phi^2=\cdot105$, $r=+\cdot308$.

TABLE XVII.

School and Clothing.

Clothing.

School.	I	II	III	IV & V	Totals
No. *1*	86	49	10	1	146
No. *2*	102	116	24	3	245
No. *3*	25	19	2	—	46
No. *4*	137	98	33	4	272
No. *5*	209	222	73	16	520
No. *6*	65	154	71	27	317
No. *7*	9	33	1	1	44
No. *8*	3	60	51	21	135
Totals	636	751	265	73	1725

$\phi^2=\cdot162$, corrected $\phi^2=\cdot150$, $r=+\cdot362$.

THE DANGER OF CERTAIN FORMULAE SUGGESTED AS SUBSTITUTES FOR THE CORRELATION COEFFICIENT.

By DAVID HERON, D.Sc.

Statistical Theory has suffered much in the past from the illegitimate application of processes which, when applied to appropriate data, are perfectly sound; but the introduction, without a single word of warning, of methods which in no circumstances can give correct results is much more dangerous. Especially is this the case when the methods claim to shorten the labour of the calculation of statistical constants, since they are invariably adopted by those who, unable or unwilling to examine critically their claims to validity, are dependent on any formula that is offered to them.

A Text-Book of Statistical Theory should above all be free from such blunders and it is therefore much to be regretted that in Mr G. Udny Yule's recent text-book[*], greater care has not been taken to ensure that the processes described there have a sound theoretical foundation.

On the present occasion, attention will only be directed to a single point, the methods suggested by Mr Yule for the measurement of the degree of association between characters which are classified alternatively.

Two distinct methods are given by Mr Yule to meet this case which arises so frequently in practical statistical work. The first is the Coefficient of Association[†], which in Mr Yule's somewhat cumbersome notation is

$$Q=\frac{(AB)(\alpha\beta)-(A\beta)(\alpha B)}{(AB)(\alpha\beta)+(A\beta)(\alpha B)},$$

in which A and B are used to denote the number of objects or individuals with the qualities A, B, while the corresponding Greek letters are used to denote the number of objects or individuals who are "not A," "not B," and AB denotes the number of those who are both A and B, and so on.

The second method suggested is to use a formula given by him in his text-book[‡],

$$Q'=\frac{N\delta}{\sqrt{(A)(\alpha)(B)(\beta)}},$$

---

[*] *An Introduction to the Theory of Statistics*, Griffin and Co. Ltd., 1911.

[†] *Phil. Trans.* A. Vol. 258, p. 272.

[‡] p. 213.

where N is the total number of observations and $\delta = (AB) - (A)(B)/N$. This latter formula is said in some cases to give a theoretical value of the correlation coefficient*, and it is suggested that it might be used instead of the Coefficient of Association, but no indication is given of any limitation to the application of the method, nor is any example given of its use.

The most cursory examination of these two methods makes it quite clear that they lead to results which differ very widely indeed, and to illustrate this point a few examples may be given.

Thus Mr Yule has discussed† the relationship between deaf-mutism and imbecility and finds that the Coefficient of Association, Q, is ·91. If, however, the degree of association be calculated by Mr Yule's second method, we find that Q', Mr Yule's "theoretical value of the correlation coefficient," is ·02!

To take a further example, the most extensive use of the Coefficient of Association has been made by Mr Yule in his discussion of the Association of Defects in Children and Adults‡, based on material collected by Dr Warner. I have already§ given fully my reasons for considering these data to be of little value, and, if there be any doubt on that point, there is certainly none as to the unsoundness of the methods adopted by Mr Yule in his analysis of those data. The Coefficient of Association has been used throughout to determine the relationship between the various forms of defect, mental and physical. Most stress has, however, been laid on the relationship between Low Nutrition and Mental Dullness. It is therefore of some interest to calculate Mr Yule's "theoretical value of the correlation coefficient" and to compare the results with the Coefficients of Association already given by Mr Yule.

The results are as follows:

TABLE I. *The Relationship between Low Nutrition and Mental Dullness as determined by* (1) *Mr Yule's Coefficient of Association, and* (2) *Mr Yule's "Theoretical Value of the Correlation Coefficient."*

	Boys		Girls	
	Coefficient of Association	"Theoretical Value of r"	Coefficient of Association	"Theoretical Value of r"
Irrespective of the Presence of other Defects ...	·82 ± ·01	·22	·84 ± ·01	·24
Where no other Defect is present	·95 ± ·01	·23	·94 ± ·01	·21
Nerve Defects also present	·08 ± ·05	·02	·01 ± ·06	·00
Nerve and Developmental Defects also present	·20 ± ·06	·07	− ·03 ± ·06	− ·01

---

[*] *The Correlation-coefficient for a two-× two-fold Table.* ..."In some cases, however, a theoretical value is obtainable for the coefficient, which holds good even for the limiting cases when there are only two rows and two columns," p. 212.

[†] *Text-book*, pp. 33 and 38.

[‡] *Phil. Trans.* A. Vol. 258, pp. 257 *et seq.*

[§] *Mental Defect, Mal-Nutrition, and the Teacher's Appreciation of Intelligence. Questions of the Day and the Fray*, No. 2, Dulau and Co.

These results are very striking. We see that Mr Yule's "theoretical value of r" gives values which are roughly about a quarter of those given by his Coefficient of Association. It is difficult to see what sound conclusions can be based on such coefficients.

One further example must be given. Mr Yule has devoted much attention to, and has given tables and diagrams in illustration of the decrease in the intensity of association with advancing age between blindness and mental derangement*. I have calculated on the same material for males only his "theoretical value of r" and the results are as follows:

TABLE II.

The Relationship between Blindness and Mental Derangement for different Age Groups.

Age Group	Coefficient of Association	"Theoretical Value of r"
5—	·921 ± ·013	·011
15—	·753 ± ·034	·006
25—	·607 ± ·049	·005
35—	·572 ± ·041	·006
45—	·459 ± ·048	·005
55—	·412 ± ·049	·006
65—	·198 ± ·070	·003
75—	− ·126 ± ·111	− ·003

We see therefore that while the Coefficient of Association decreases steadily from ·921 to − ·126, the "theoretical value of r" ranges between ·011 and − ·003. I have expressed those results graphically in Fig. 1 and it will be noticed that the line of the "theoretical value of r" can hardly be distinguished from the zero line.

It will at once be admitted that these results justify a detailed examination of Mr Yule's methods. It is quite clear that he is unaware of the limitations of the processes which he recommends and that he has failed to apply the most obvious test of their validity, i.e. to compare the results obtained by the two methods when applied to the same data.

I shall deal first of all with the Coefficient of Association. This expressed in the more usual notation is

$$Q = \frac{ad - bc}{ad + bc},$$

where a, b, c, d are the frequencies in the four quadrants of a four-fold table. Mr Yule has given in his original paper a table showing the relative values of the Coefficient of Association and the actual correlation (not his "theoretical value of r") for the special case in which the divisions are taken through the means, i.e. $a + b = c + d$ and $a + c = b + d$. He found that "Q is always slightly in excess of r, the greatest difference being rather more than ·1 for $Q = ·7$." Now this is a fairly

---

* This case is also given as an example in his *Text-Book*, p. 41.

good agreement and had the Coefficient of Association been used solely in cases where the divisions were taken through the means or very nearly so, no very serious objection could have been taken to his results as approximations. But this is very far from being the case. Instead of restricting the use of the method to cases where $\frac{a+b}{N}$, $\frac{a+c}{N}$ were nearly ·5, he has applied it to cases where $\frac{a+b}{N}$ and $\frac{a+c}{N}$ are as high as ·99978 and has failed to investigate the effect of these extreme divisions.

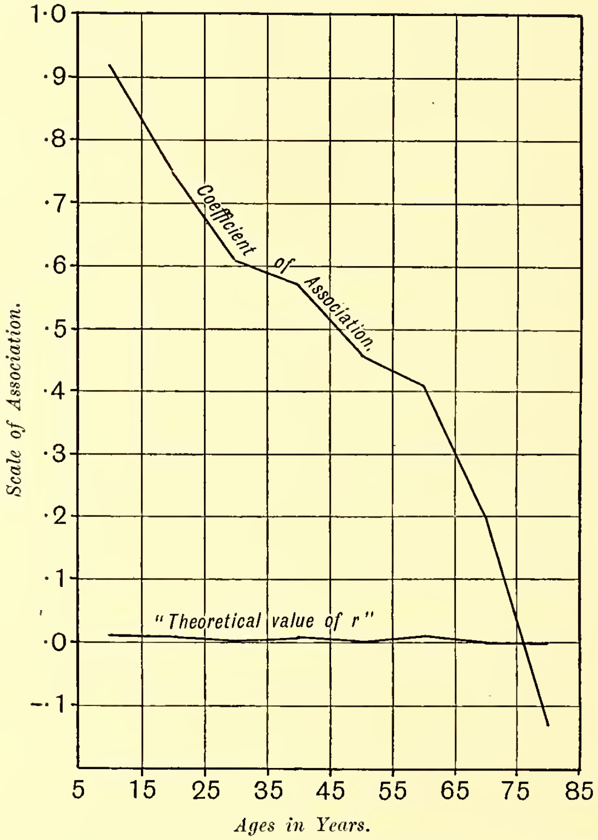


Fig. 1. Diagram to show the relation between Blindness and Mental Derangement at different ages as determined (1) by Mr Yule's Coefficient of Association and (2) by Mr Yule's "Theoretical value of r."

I propose therefore to consider a series of frequency distributions which are Gaussian* and to compare the values of the Coefficient of Association found for

---

* In doing this I am far from assuming all distributions to be Gaussian. But many are very approximately so, and if Mr Yule's methods lead to absurd results in this case, with what confidence can they be applied to others of which we know nothing as to the frequency-distribution? Mr Yule does not, however, warn us that his coefficients are not to be used when the frequency is approximately Gaussian but actually tests them on such material.

these distributions with the actual value of the Coefficient of Correlation, without however restricting myself to median divisions.

It is clear that, if we take a Gaussian frequency surface of given correlation and divide it by two planes parallel to the axes of x and y at distances h and k from the origin, h and k being measured in terms of the standard deviations of their respective variables, then the value of r is independent of h and k. The point to be determined is this: To what extent is the Coefficient of Association dependent on the values of h and k?

If a, b, c, d are the frequencies in the four quadrants of the frequency surface, it has been shown* that

$$\frac{d}{N}=\frac{b+d}{N}\cdot\frac{c+d}{N}+\overset{\infty}{\underset{1}{S}}\left(\frac{r^n}{n!}HK\bar{u}_{n-1}\bar{w}_{n-1}\right)$$

$$=\frac{b+d}{N}\cdot\frac{c+d}{N}+\overset{\infty}{\underset{1}{S}}\,.\,HK\left(r+\frac{hkr^2}{2!}+\frac{(h^2-1)(k^2-1)}{3!}r^3+\text{etc.}\right).$$

Now the coefficients of the r's on the right have been determined by Mr Everitt for values of h and k from 0 to 3·09, in his most valuable "Tables of the Tetrachoric Functions for Fourfold Correlation Tables†." It is clear then that in a normal frequency surface of definite correlation, $\frac{d}{N}$, and hence a, b, c, d can be determined by summing the series on the right, and we can at once compare Q and r for any values of h and k. I have throughout made $h=k$, and it should be noted that this will reduce rather than accentuate the differences between Q and r. I have calculated Q for a series of values of $h=k$ from 0 to 3·09 in some cases extending this to $h=k=5$ in order to make certain of the form of the curve. The results are given in Table III, but I shall delay their consideration until I have discussed Mr Yule's "theoretical value of the correlation coefficient."

This coefficient

$$Q'=\frac{N\delta}{\sqrt{(A)(\alpha)(B)(\beta)}}$$

is reached by Mr Yule in his *Text-Book*‡ by a process which is nothing short of extraordinary. It would be interesting to know what justification he can give for the assumptions he has made in the proof.

If we express Q' in the more usual notation we obtain

$$Q'=\frac{ad-bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}}.$$

Now this formula was given ten years ago by Professor Karl Pearson§, not as the correlation between the two characters under consideration, but as r_{hk}, the correlation between errors in the position of their means when each is measured

---

* Pearson, *Phil. Trans.* A. 262, p. 6.

† *Biometrika*, Vol. VII, No. 4, p. 437.

‡ p. 213.

§ *Phil. Trans.* A. 262, p. 12.

in terms of the standard deviation of its own distribution; now had Mr Yule used the more familiar notation, there is little doubt that he would have recognised it, and this furnishes an excellent example of the danger of inventing new notations.

It is interesting to note that Professor Boas*, using still another notation and relying on a method of proof which is of the same degree of validity as Mr Yule's, reached this same formula in the form

$$r = \frac{p_{1,2} - p_1 p_2}{\sqrt{p_1(1-p_1)(p_2)(1-p_2)}},$$

and, like Mr Yule, failed to recognise it as an old friend. Professor Pearson subsequently pointed out† that of the formulae given by Professor Boas, those which were valid, had been in constant use for many years and remarked that "there is some danger, unless we see how new values for the correlation coefficient are related to the old values, in a multitude of formulae leading to divergent and possibly inconsistent results." He added that this coefficient (Mr Yule's "theoretical value of r") differs in the simplest cases from the true coefficient of correlation and often differs considerably. In the bulk of cases it does not approach r nearly as closely as the Q_5‡ coefficient of association, and its use is liable to be misleading, especially if compared with the value of the true coefficient of correlation calculated by other processes.

Now if we make $h = k$ and therefore $b = c$, the formula for Q' can be simplified considerably, reducing to

$$Q' = \frac{ad - c^2}{(a+c)(d+c)},$$

and if we express it in the form

$$Q' = \frac{ad - c^2}{ad + c^2 + c(a+d)},$$

we see that Q' is always less than Q, and generally very markedly so.

I have calculated the values of Q' for the same cases for which Q has been calculated and the results are given together in Table III, while in Figs. 2, 3, 4 and 5, the comparative values are shown graphically. It is clear that while Q asymptotes to unity as $h = k$ is increased, Q' asymptotes to zero.

Now Mr Yule has used his coefficient of association in cases where h and k are as high as 3·5, and we see that, with this value of $h = k$, when the actual value of the correlation, r, is ·1, his coefficient of association, Q, is ·57; when r is actually ·3, Q is ·93; when r is actually ·5, Q is ·99; when r is actually ·9, Q is greater than ·99. On the other hand when r is actually ·1, Mr Yule's "theoretical value of r," Q' is ·00; when r is actually ·3, Q' is ·01, when r is actually ·5, Q' is ·03; when r is actually ·9, Q' is ·40.

---

* *Science*, May 21, 1909.

† *Science*, July 2, 1909.

‡ For this approximation, see below, p. 120.

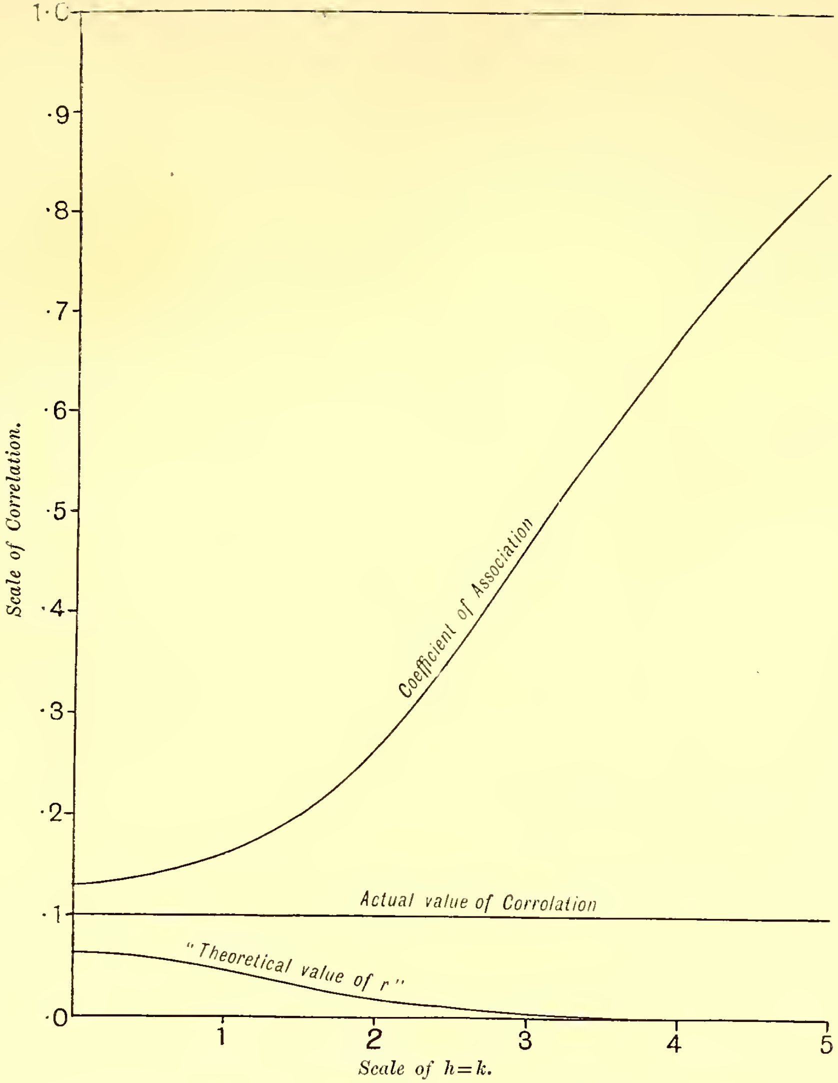


FIG. 2. Diagram to show the changes in value of (1) Mr Yule's Coefficient of Association and (2) Mr Yule's "Theoretical value of r" for different values of $h=k$ in a Gaussian Frequency Surface in which $r=·1$.

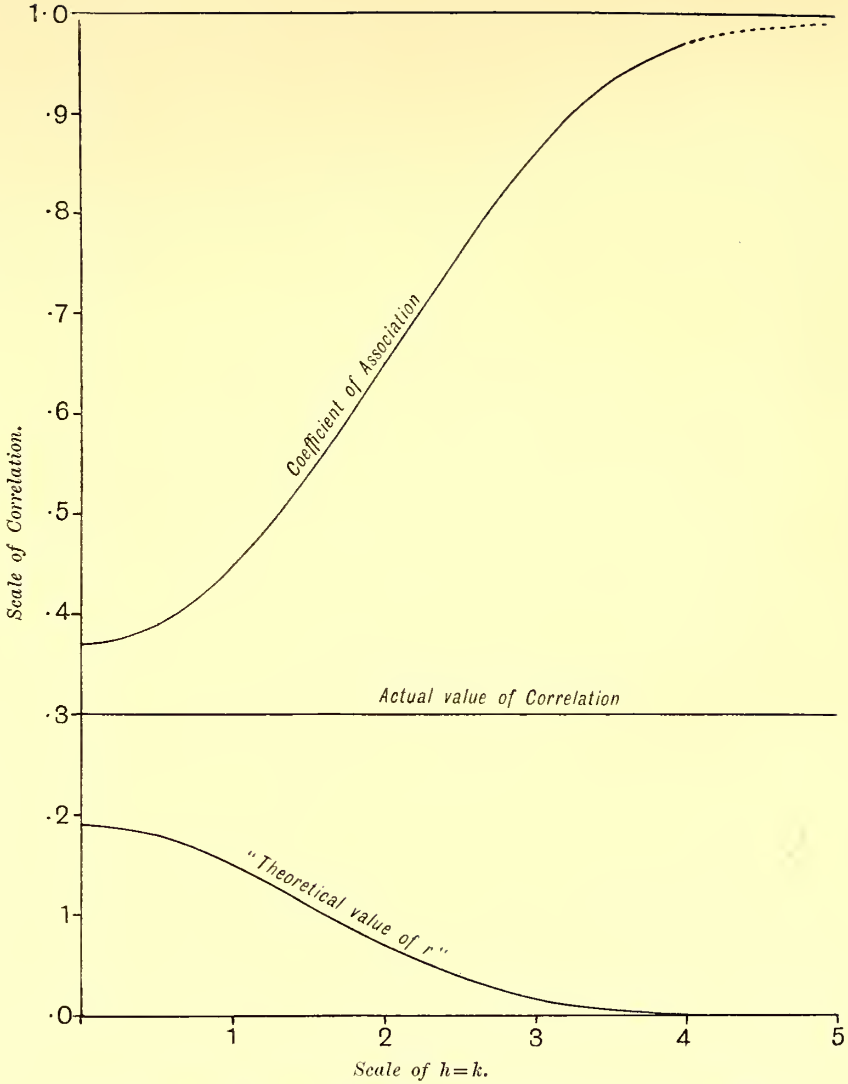


FIG. 3. Diagram to show the changes in value of (1) Mr Yule's Coefficient of Association and (2) Mr Yule's "Theoretical value of r" for different values of $h=k$, in a Gaussian Frequency Surface in which $r=·3$.

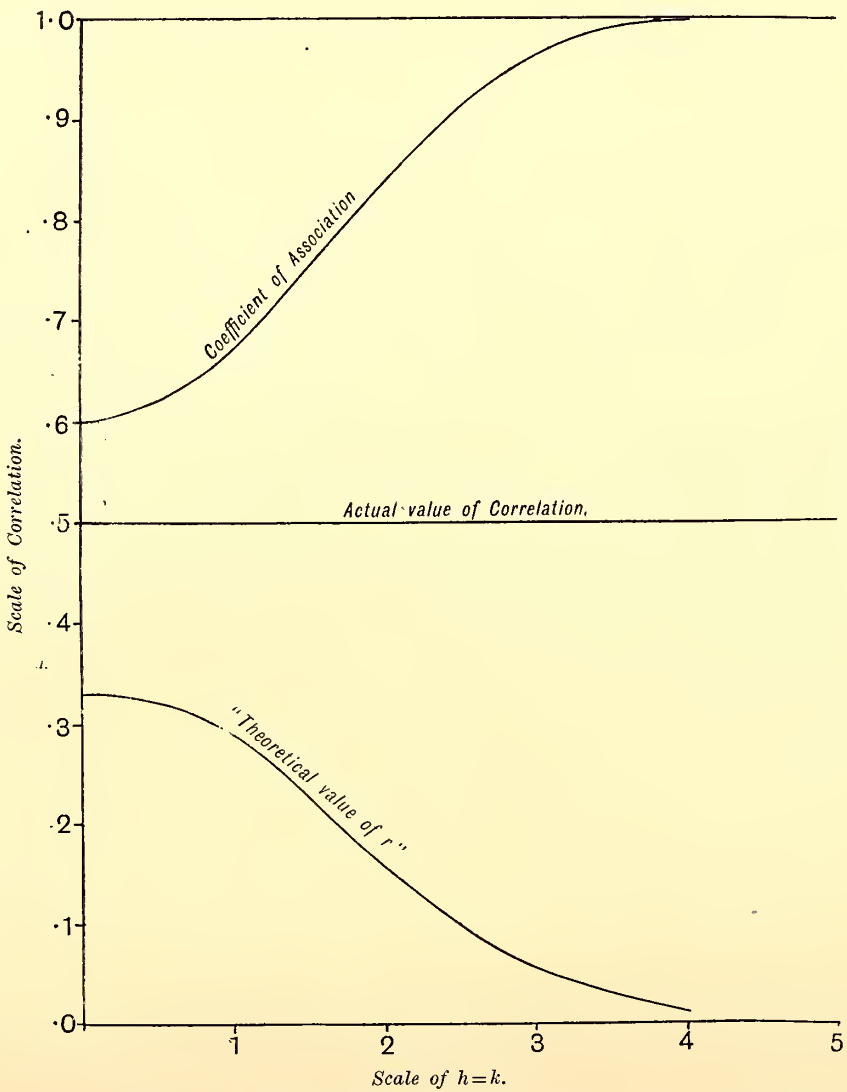


FIG. 4. Diagram to show the changes in value of (1) Mr Yule's Coefficient of Association and (2) Mr Yule's "Theoretical value of r" for different values of $h=k$, in a Gaussian Frequency Surface in which $r=\cdot 5$.

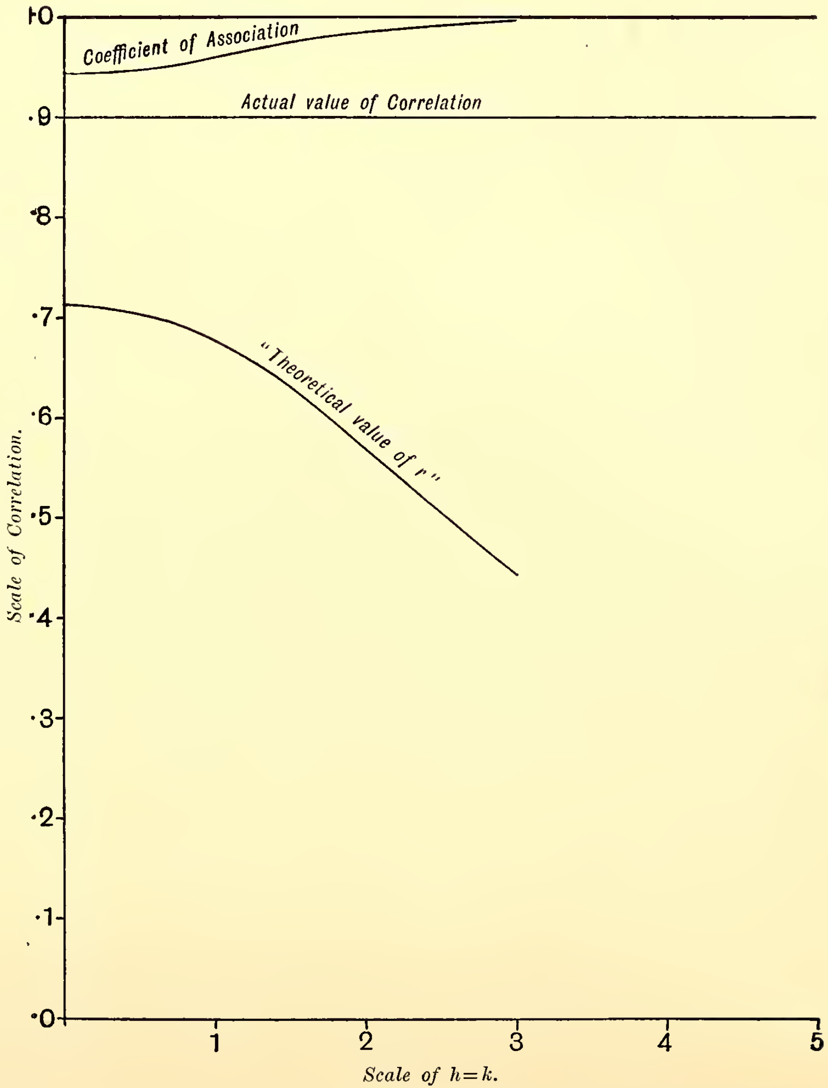


FIG. 5. Diagram to show the changes in value of (1) Mr Yule's Coefficient of Association and (2) Mr Yule's "Theoretical value of r" for different values of $h=k$ in a Gaussian Frequency Surface in which $r=\cdot 9$.

TABLE III.

Showing the Comparative Values of Q, the Coefficient of Association, r, the Actual Correlation and Q', Mr Yule's "Theoretical Value of r," for Various Values of $h=k$.

$h=k$	Q	r	Q'	Q	r	Q'	Q	r	Q
0	·13	·10	·06	·37	·30	·19	·60	·50	·33
·522	·14	·10	·06	·39	·30	·18	·62	·50	·32
1·032	·16	·10	·05	·45	·30	·15	·67	·50	·28
1·635	·22	·10	·03	·56	·30	·10	·77	·50	·21
2·512	·36	·10	·01	·76	·30	·04	·91	·50	·10
3·090	·48	·10	·00	·88	·30	·01	·97	·50	·05
3·5	·57	·10	·00	·93	·30	·01	·99	·50	·03
4	·67	·10	·00	·97	·30	·00	1·00	·50	·01
5	·84	·10	·00	—	—	—	—	—	—

$h=k$	Q	r	Q'
0	·95	·90	·71
·60	·95	·90	·70
1·00	·96	·90	·68
1·50	·97	·90	·63
2	·99	·90	·58
3	1·00	·90	·44

As a further example of the differences between Mr Yule's coefficient of association, his "theoretical value of r" and the actual value of r, I have taken a constant value of $h = k = 3·09$, and have plotted the curves $Qx = ry$ and $Q'x = ry$ for all values of r. Now it is clear that if R be any coefficient which is taken as a substitute for the correlation coefficient, then the more closely the curve $Rx = ry$ approaches the line $x = y$ the better will be the approximation to the true value of the correlation. I have given these curves in Fig. 6, and it is obvious that Q and Q' are the same as r only when $r = -1, 0$, and $+1$, and over the whole range they differ so widely from r as to be absolutely useless as approximations.

Further we see from the examples given that the curves for the coefficient of association for different values of $h = k$ asymptote to the line $y = 1$, whatever be the value of r; while the curves for Mr Yule's "theoretical value of r" asymptote to the line $y = 0$. Thus we have only to take sufficiently high values of h and k to get either 1 or 0 as our measure of the intensity of correlation. The first example given, in which either ·92 or ·02 could be obtained as the measure of the closeness of association between deaf-mutism and imbecility is a striking illustration of this fact.

It is unfortunate that Mr Yule has refrained from giving any example of the use of the second coefficient, Q', but he has made the most extensive use of the first, the coefficient of association. His paper "On the Association of Attributes in Statistics" is based on nearly 500 coefficients of association. Yet such coefficients are, as we have just seen, *relatively* as well as *absolutely* worthless, unless the material is always divided in the same proportions, and even in this case a number of coefficients of association are not strictly comparable since a small increase in the actual correlation will be followed by a large increase in the value of the coefficient of association when r is small and by a small increase when r is large.

It is not difficult to give some striking instances of the effect of neglecting the fact that the value of the coefficient of association is entirely dependent on the choice of axes. Thus in dealing with Dr Warner's data in the *Phil. Trans.* paper already cited, Mr Yule gravely discusses the "apparent law that associations were on the whole higher where populations were healthier or less defective." But we

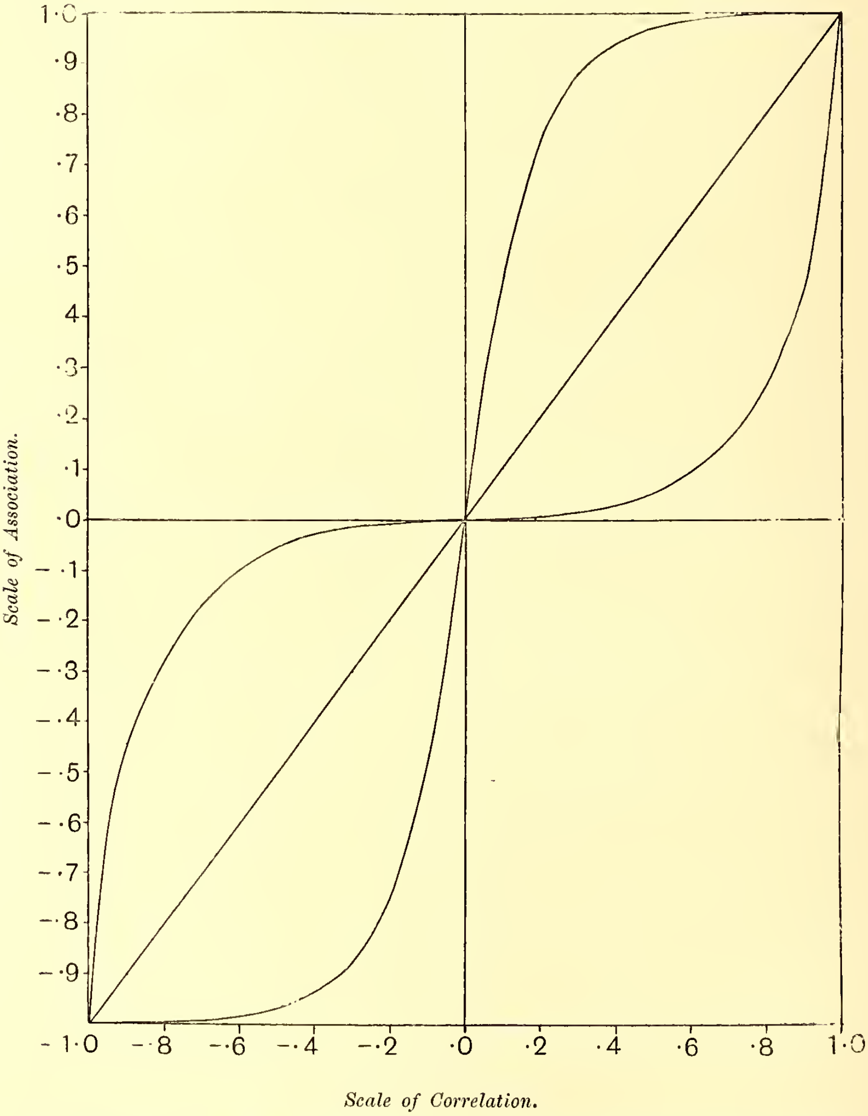


FIG. 6. Diagram to show the relative values of (1) Mr Yule's Coefficient of Association, (2) Mr Yule's "Theoretical value of r" and (3) the actual value of r, when $h=k=3{\cdot}09$.

have just seen that the further away from the mean the dividing line is taken, i.e. the higher the values of h and k and the fewer the number of defectives, then the higher the value of the coefficient of association; so that the "apparent law" arises solely from his faulty method. Had Mr Yule used his "theoretical value of r" instead of the coefficient of association, it would have been suggested that the associations were on the whole *lower* where populations were healthier or less defective. Neither coefficient enables any light at all to be thrown on the question at issue.

For precisely similar reasons his discussion of the change of association with age must be dismissed as entirely fallacious. There may be and probably is, some decrease of association with advancing age, but the enunciation of such a law on the basis of a number of coefficients of association is purely idle. In the example that has already been given, Mr Yule has stated that the association between blindness and mental derangement decreased steadily from ·921 at the age group 5—15 to $-$·126 at the age group 75 and upwards. But the proportions of the blind increase steadily from 26 per 100,000 to 1051 per 100,000, while the proportions of the mentally deranged increase steadily from 85 per 100,000 to 679 per 100,000, and this increase in the number of defectives will in itself produce a large decrease in the value of the coefficients of association. Until the amount of association has been expressed in terms of coefficients of correlation, no conclusions can be drawn from the data.

In the same way Mr Yule's statements that "the differences exhibited by the sexes as regards association are so marked that they can hardly have failed to have struck the reader of the foregoing tables," and that "in an immense majority of cases, the associations are greater for females than for males" must be rejected. The apparent difference arises from the fact that "besides being more highly associated, women are also in general less defective."

That so much labour should have been spent on the analysis of data of very doubtful value by methods which can throw no light on any problem whatsoever is deeply to be regretted. As a warning of the danger of basing definite and far-reaching conclusions on results obtained by methods which have not been adequately tested, Mr Yule's memoir may in fact be commended, but it must be stated emphatically that not a single one of the conclusions reached by Mr Yule can be justified by the data or the methods he has used.

One further point is of some interest. The process of finding the actual value of the coefficient of correlation from a four-fold table involves somewhat lengthy arithmetic and hence various formulae have been suggested by Professor Pearson by which an approximate value of r might be obtained without excessive labour. Since the publication of Everitt's Tables, however, these approximations are of considerably less importance owing to the very great reduction in the labour involved. I have, however, tested the comparative accuracy of the various approximations suggested for the special case where r is actually ·5.

The various approximations are as follows:

$$Q_1 = \sin\frac{\pi}{2}\frac{ad-bc}{(a+b)(b+d)},$$

$$Q_2 = \frac{ad-bc}{ad+bc} \text{ (Mr Yule's Coefficient of Association, } Q\text{)},$$

$$Q_3 = \sin\frac{\pi}{2}\frac{\sqrt{ad}-\sqrt{bc}}{\sqrt{ad}+\sqrt{bc}},$$

$$Q_4 = \sin\frac{\pi}{2}\frac{1}{1+\dfrac{2bcN}{(ad-bc)(b+c)}},\quad ad > bc,$$

$$Q_5 = \sin\frac{\pi}{2}\frac{1}{\sqrt{1+k^2}},$$

where

$$k^2 = \frac{4abcdN^2}{(ad-bc)^2(a+d)(b+c)},$$

$$Q_6 = \sin\frac{\pi}{2}(r_{hk}) = \sin\frac{\pi}{2}(Q'),$$

and

$$Q' = \frac{ad-bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}},$$

Mr Yule's "theoretical value of r."

Now in the work already done, I have always taken $h = k$ and hence $b = c$. With this restriction, some of the formulae are considerably simplified. Thus $Q_1 = Q_4 = Q_6$.

I have given in Table IV the values of these coefficients for various values of $h = k$ in the case of the frequency surface for which $r = \cdot 5$.

TABLE IV.

Showing the Comparative Degree of Accuracy of Various Approximations to the Coefficient of Correlation, ·50.

h	$Q_1 = Q_4 = Q_6$	Q_2	Q_3	Q_5	Q'
0	·50	·60	·50	·50	·33
·52	·48	·62	·52	·51	·32
1·03	·42	·67	·57	·50	·28
1·63	·32	·77	·67	·46	·21
2·51	·16	·91	·85	·35	·10
3·09	·08	·97	·94	·24	·05

These results are also given graphically in Fig. 7. It will at once be seen that Mr Yule's coefficients, Q and Q' are very poor approximations, whatever be the values of h and k. The other approximations suggested by Professor Pearson give fairly good results if we restrict their use to small values of h and k, while Q_5 may be used with safety for values of h and k as high as 1·5.

It would be comparatively easy from the values given of the various approximations to obtain a formula* which would give still better results and could be used over a much larger range of values of h and k, but I have purposely refrained from doing so. In any work of importance, it is always best to calculate at once the actual coefficient of correlation with the help of Everitt's Tables.

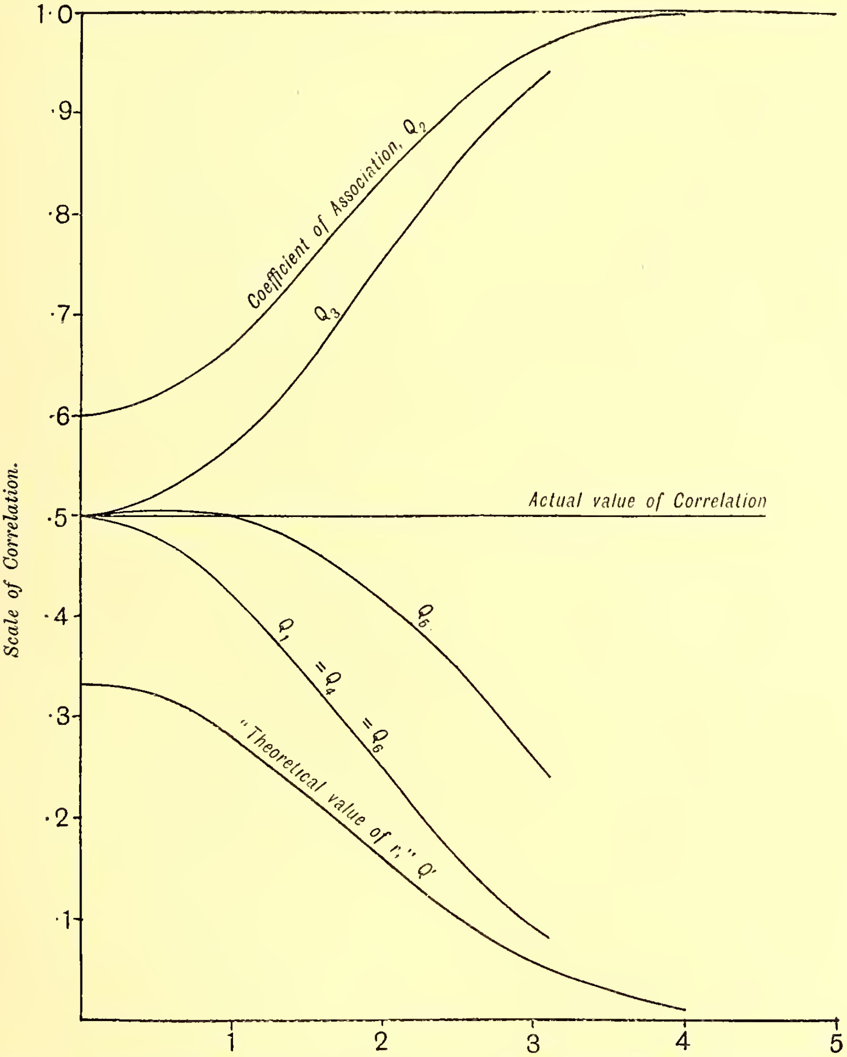


FIG. 7. Diagram to show the relative degrees of accuracy of various approximations to the correlation coefficient.

---

* Thus the harmonic mean of Q_3 and Q_5 (for $r = ·5$) gives quite good results, and the arithmetic mean of Q_2 and Q' also. Best of all (for the same special case) appears the arithmetic mean of Q_1 and Q_3, which cannot be adequately shown as differing from r on the scale of Fig. 7.

This discussion of the danger of Mr Yule's correlation formulae is by no means idle since the Coefficient of Association at least has recently been adopted by certain Italian writers. Thus Niceforo ("Contributo allo Studio della Variabilità di Alcuni Caratteri Anthropologici": *Atti della Società Romana di Anthropologia,* Vol. XVI. Fasc. 1) discusses the correlation between the mean and the coefficient of variability in twenty series of anthropological measurements and finds the Coefficient of Association to be − ·909 while the actual value of the correlation coefficient, found by the product moment method from his data, is − ·616. I have however been unable to reconstruct the fourfold table from which this coefficient, − ·909, has been deduced so that there may be some slip in Niceforo's arithmetic. This is the more possible since out of the coefficients of variation given, very few as printed can be correct. Niceforo also states that the Coefficient of Association has been extensively used by himself in a paper on "Corrélations économiques" in the *Journal de la Société de Statistique de Paris,* 1911, but I have not been able to find his paper in that journal. There is very little doubt that Mr Yule's "theoretical value of the correlation coefficient," owing to its apparently simple form, will also be seized upon by those who have not sufficient theoretical knowledge to judge of its real value. In this way much harm will arise to statistical science, owing to the postulating of false conclusions resulting from the use of inadequate formulae.

CRANIAL TYPE-CONTOURS.

By the late R. CREWDSON BENINGTON, M.D.

Prepared for Press by KARL PEARSON, F.R.S.

[This paper is accompanied by copies of the XXXII text-figures printed on tissue.]

(1) *Introductory.* During the last two years of his life Dr Crewdson Benington was a most active worker in the Biometric Laboratory at University College, London. He was concerned with anthropometric, more especially craniometric, problems. Had he been spared another year to complete his investigations (he died in the autumn of 1909), there is little doubt that several valuable papers would have been issued under his name. He had three distinct pieces of work in hand, all unfortunately lacking the final measurements, reductions and verbal descriptions. These were:

(i) The completion of a lengthy investigation of the long bones of the English skeleton by a full system of measurements on the humerus and the radius*.

(ii) A study of crania from the Congo, and a comparison with measurements on other negro races†.

(iii) A study of the three chief contours of the human skull with special reference to the relation of man to the anthropoid apes.

It is a side branch of this latter study with which the present paper is concerned. Dr Crewdson Benington, by aid of the Klaatsch contour tracer belonging to the Biometric Laboratory, drew the three contours of many hundreds of crania of men and apes, partly in the collections of University College, partly in the Royal College of Surgeons, and partly at the British Museum (Natural History). But when he came to discuss his material, it seemed to him that the chief advantage of the graphical method was lost, if the contours were used solely to obtain the mean values of a few characters and these characters only were compared for the different races. In such a treatment the graphical representations of the individual crania are used only as a link between purely numerical investigations. Almost every anatomist lays special weight on qualitative characters of the crania

---

* Material kindly placed at Dr Benington's disposal by Professor Thane.

† Dr Benington by the kindness of Professor A. Keith measured most of the collection of Congo crania at the Royal College of Surgeons. The series was afterwards completed by Miss E. V. Thompson. He also measured two further series of crania from the same district in the British Museum. This study is now at press and will shortly be published.

which he asserts are not represented by numerical estimates, but which form essential factors of his appreciation of both sex and race. There seemed a need accordingly to construct from the contours *average or type contours*, which might serve as a first stage in the determination of a true type skull. If a skull can be found in a given series which agrees practically with the three chief type contours in all the details of its three individual contours, then that skull may, till a better method is forthcoming, be taken as the racial type. If we remember that the parts of the skull without being very highly correlated are sensibly correlated, we see at once that the mean skull is not a purely fictitious entity, it will approach closely to the actual modal skull characteristic of the biggest class in the community. It is the skull possessing the maximum frequency, or corresponds to that of the individuals near the apex of the multiple correlation surface. Our object therefore should be to obtain from the collection of individual contour tracings mean contour tracings, and by comparison with the individual contours to reach the type skull. This skull can then be perpetuated by photography, and, if accompanied by the mean contour tracings, serves, until a better method is devised, to represent the type with all those qualitative features on which the anatomist lays special stress. The same mean contour tracings will also serve to differentiate the subtle differences due to sex and possibly to cross-breeding*.

(2) *Construction of Type Contours.* It remains of course to develop some method of obtaining an approach to a mean or type contour from the original individual contours. Several methods may be conceived, but after considerable thought and some experimental work the following scheme was settled upon and found to work effectively in practice.

The usual three contours are taken:

(*a*) *The transverse vertical or auricular coronal section.* The skull being placed on the craniophor and adjusted in the usual manner to the Frankfurt horizontal plane† the horizontal rod of the craniophor determines the "apex" of the skull. The plane through the apex of the skull (which is marked with a pencil stroke) and the "auricular points" is the transverse vertical plane of the skull. The skull being now transferred to the Klaatsch apparatus this vertical plane is made horizontal and the contour traced from "auricular point" to "auricular point" through the apex. The line joining auricular point to auricular point is the "auricular line" of the contour‡, and the line perpendicular to it

---

* For example, apart from true negroes, a certain number of skulls are put aside as "negroid." Let us suppose this collection contoured, reduced to the mean contours and the closest individual to these mean contours selected. This individual will then represent the average "negroid" type, and this skull can be compared by photography or directly with those of the true negro and of the pure race selected in the same manner.

† See Fawcett and Lee: *Biometrika*, Vol. I. p. 414.

‡ The "auricular line" is the transverse straight line tangential to the *upper* border of the ear openings; it does not correspond to the *centre* of the auricular passages, but is represented by the line at the top of the auricular plugs of the craniophor. Most cephalometers used on the living head, not only measure the flesh at the apex, but also measure from the centre of the auricular passages and thus give an excess of about 8 or 9 mm. over the true cranial auricular height.

through its mid-point is the "vertical axis" of the transverse vertical contour. This vertical axis is divided into ten equal parts by aid of the proportional compasses, and lines parallel to the auricular line are drawn through the points of division; these lines will not necessarily be bisected by the vertical axis owing to the asymmetry of the skull. They are numbered from below upwards: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, where 1 represents the lengths on the auricular axis. An eleventh line is taken one quarter of the last section from the apex[*]; the distances to right and left of the vertical axis up to the contour are measured off along these eleven horizontals. Also the length of the vertical axis is measured. Thus 23 separate measurements are made on the contour. Since the skull is placed with the occiput downwards on the Klaatsch, the right side of the skull is to the observer's left when viewing the contour, and the left of the skull to his right; this is a point which it is needful to bear in mind.

We have now 23 measurements which are entered in the recording book as

V.A.	1*R*	1*L*	2*R*	2*L*	3*R*	3*L*	4*R*	4*L*	5*R*	5*L*	6*R*	6*L*	7*R*	7*L*	8*R*	8*L*

9*R*	9*L*	10*R*	10*L*	$A\frac{1}{4}R$	$A\frac{1}{4}L$

From these measurements we can plot 23 points on the section, and if the section be drawn in with a spline between these points, we found it could be reproduced with an accuracy well within the instrumental errors of the contour tracer.

(*b*) *The sagittal or median section.* The skull being again placed on the craniophor and adjusted to the Frankfurt horizontal plane, the point γ[†] on the same horizontal level as the nasion is marked off on the median plane of the skull by means of the scriber. The skull is now removed to the Klaatsch, and adjusted so that the nasion, the bregma and the lambda lie in the horizontal plane. The contour is then drawn starting from the alveolar point and giving as much of the nose as possible. The alveolar point, the nasion, the bregma, the apex, the lambda, the gamma, the inion, the opisthion, the basion, and the auricular point are all marked on this contour drawing. The advantage of having all these points marked on the sagittal sections will be obvious; their mean positions can afterwards be at once inserted on the type contour.

---

[*] The "apex" is the highest point of the transverse vertical section in our terminology. The additional ordinates are necessary to obtain the true curvature in the neighbourhood of the apex; we term this the apex quarter line (A$\frac{1}{4}$).

[†] We use this Greek letter for the posterior "horizontal point" of the sagittal contour, and term it the "gamma."

We now took the horizontal line through the nasion, i.e. the line linking the nasion to the gamma as our "horizontal axis" and divided it into ten equal pieces by aid of the proportional compasses. The nasion counting as 0, and the others from nasion to gamma as 1, 2, 3, 4...9, lines are drawn through these ten points perpendicular to the horizontal axis; a further upright is taken near the gamma at one quarter the distance from gamma to 9; this for brevity is termed the $\gamma\frac{1}{4}$ line. At 0, i.e. vertically through the nasion, there will, in most male skulls, be a sensible ordinate, roughly of about 2 cm. In order to get the curvature in the neighbourhood of the glabella a *vertical* tangent was drawn by aid of the set-square to the most projecting part of the glabella; the point in which this tangent meets the prolonged gamma-nasion line being T and the point of contact of the tangent being G, GT and TN represented by the letters y and x respectively were measured, they are practically the coordinates of the glabella, and average roundly about 10 mm. and 3 mm. respectively for male skulls. Occasionally, but rarely, the glabella tangent cuts the horizontal axis between the nasion and gamma; in such cases x is to be reckoned as negative. Lastly, to complete the drawing of the frontal part of the sagittal contour, an extra ordinate was measured at $\frac{1}{4}$ the distance from 0 to 1; this line is spoken of as the $N\frac{1}{4}$ line. We have thus three points, the tangent at one of them and the nasion defining the nature of the contour in the glabellar part of the skull, and these were found amply sufficient to reproduce the individual contour as closely as it could be redrawn by the Klaatsch. A knowledge of $N\gamma$ and of the coordinates of the thirteen points sufficed to reproduce the whole contour by aid of a spline. Our record book had therefore the following entries:

$N\gamma$	x	y	0	$N\frac{1}{4}$	1	2	3	4	5	6	7	8	9	$\gamma\frac{1}{4}$

Of course in addition to these we had all the typical points of the sagittal section, and the measurements, lengths and angles advocated by Schwalbe (see *Zeitschrift für Morphologie*, Bd. I. S. 38). It should be at once noted that nasion to gamma is not any recognised length on the head or the skull; it is not the maximum length, nor the length from lambda to nasion, nor the horizontal length (see *Biometrika*, Vol. I. p. 414). It is chosen simply as a convenient horizontal base line by which to fix our system of axes, and measure our ordinates. If in taking the height of a man, the level of his eyes be marked off, and he be asked to look at this mark from several yards' distance, the head will be very nearly in a horizontal position (e.g. the horizontal position of Broca). A horizontal plane through the nasion will now very closely determine the gamma on the sagittal section, and this can be marked off on the flexible lead strip* which at the suggestion of Dr Charles Goring was used to give the sagittal section of the living head. The calliper measure of $N\gamma$ suffices to test the accuracy with which the

---

* 1 cm. deep by 1 mm. thick.

contour is drawn from the lead strip. Thus the nasion-gamma line enables us to compare with reasonable approximation the living head with the skull—a process which is much assisted if the distances of the auricular point from nasion and gamma have been determined for both skull and living head. This nasion-gamma line is the basis of the method developed by Dr Charles Goring for the determination of the sagittal contour of criminals' heads, and it has been applied by Dr Crewdson Benington in reconstructing a type English head from the three contours obtained from 106 non-commissioned officers and men of the Royal Engineers.

(c) *The glabellar horizontal section.* This is obtained by placing the skull in the craniophor and marking by aid of the scriber points on the same horizontal plane as the nasion and the gamma, in the region above both auricular passages. The skull is now adjusted in the Klaatsch holder, apex downwards, until these four points are shewn to be in the same horizontal plane. The horizontal contour is now drawn parallel to this plane through the glabella and it will pass below the lambda and does not pass far from the occipital point. The points in the median plane immediately above the nasion and below the lambda on this contour are especially marked with the Klaatsch tracer. They will be spoken of as F and O, the glabellar and occipital points. FO is the glabella-occipital line of the horizontal section. This fronto-occipital line being found, it is divided into ten equal parts by the proportional compasses, and lines perpendicular to the fronto-occipital line are drawn through the points of division, and the lengths up to the contour measured right and left. To obtain the frontal and occipital curvatures, parallel lines at $F\frac{1}{4}$ and $O\frac{1}{4}$ were taken as in the other cases. Thus 24 points are known, and they suffice by aid of a spline to reconstruct the horizontal section. Actually the skull being placed apex downwards on the Klaatsch support the right of the skull is on the right of our contour when we look from the frontal towards the occipital point. But in actually plotting the mean diagrams the measured lengths have been reversed.

Thus when we look at the horizontal section with the frontal point towards us the right of the skull is on our left, as it would be if we were facing the actual skull the right way up. This corresponds with the transverse vertical section which is also drawn with its right on our left, i.e. as it would appear in the actual skull, if facing us.

In our record book we have accordingly the following entries:

FO	$F\frac{1}{4}R$	$F\frac{1}{4}L$	$2R$	$2L$	$3R$	$3L$	$4R$	$4L$	$5R$	$5L$	$6R$	$6L$	$7R$	$7L$	$8R$	$8L$

$9R$	$9L$	$10R$	$10L$	$O\frac{1}{4}R$	$O\frac{1}{4}L$

Thus the three contours require 60 measurements to be made in order that they may be effectively reproduced from the plotted points.

Illustrations of the complete system of measurements made on the individual sagittal contour in the Biometric Laboratory are given in Figs. I and II. Fig. I represents the fine head of Thurtell the murderer from the Royal College of Surgeons. Fig. II the head of one of the long series (2000 crania) of Egyptians (600—300 B.C.) in the Biometric Laboratory. N = Nasion, Gl = Glabellar point, B = Bregma, λ = Lambda, γ = Gamma, the occipital horizontal point, I = Inion, *Op.* = Opisthion, *Aur.* = Auricular point, *Bas.* = Basion, *Alv.* = Alveolar point, *Suborb.* = infra-orbital point. *Suborb.* to *Aur.*, if joined by a line, gives the Frankfurt Horizontal Plane. $N\gamma$ is the horizontal plane through the nasion. The angles N, B and A are the nasal, basilar and alveolar angles of the fundamental triangle $NBAlv$. P is the profile angle measured from the Frankfurt horizontal. ϕ is the angle between the glabella-inion and glabella-bregma lines, ϕ' the angle between the nasion-inion and nasion-bregma lines. There are three interesting subtenses s, x and y. y is the maximum subtense or height of the calvaria measured from the glabella-inion line. A subtense may also be taken between the nasion-lambda line and the most distant point of the calvaria (in the median contour). It does not, however, differ very widely from the bregmatic subtense x, the perpendicular from the bregma on the nasion-lambda line; it is drawn but not lettered on Figs. I and II. On the other hand the bregmatic subtense s, or the perpendicular from the bregma on the glabella-inion line, differs widely from the maximum subtense y, and this difference is of interest.

(3) These sixty measurements, having been made on each of 50 to 100 sets of contour drawings, were arranged in columns, one for each measurement. These columns were added by an adding machine and their means determined. In the case of a considerable number of such sets the standard deviations were also found, in order that the probable error of the means might be determined.

The type contour was now found by plotting the mean vertical axis, the mean nasion-gamma line or the mean glabella-occipital line as the case might be. This was divided into ten equal parts by the proportional compasses, and the perpendicular lines, as well as the terminal quarter lines drawn. Along these were plotted right and left the mean lengths, and in the case of the sagittal section the mean glabella tangent and its mean point of contact. From these points by aid of a spline the three type sections of each series of crania were determined. These form the cranial types of Dr Crewdson Benington's investigation. Upon them the mean position of bregma, lambda, inion, basion, etc., the auricular point and other cranial mean characters, calvarial height and the Schwalbe angles, can be introduced, these positions being determined from their mean coordinates*.

---

* We have found close agreement, whether these points be put in by mean coordinates or inserted on the contour by mean distances from nasion, or, where necessary (e.g. auricular and alveolar points), by mean distances from two points (e.g. nasion and gamma).

In comparing the cranial type contours of two races, it is desirable to have some measure of whether they diverge more than the limits of the probable error of random sampling. This point is not a very easy problem to solve because of course the measurements made are highly correlated*. We considered that some appreciation of significance of divergence might be obtained in the following manner:

In the transverse vertical section we accepted the auricular line as a fundamental line which in position, not in length, might be considered as a fixed base. From the mean value of the vertical axis we then added and subtracted twice its probable error. On the lesser vertical axis at the tenth points found in the usual way were then plotted up the mean right and left ordinates less twice their probable errors, and on the larger vertical axis divided in the same manner were plotted up the mean right and left points plus twice their probable errors. We thus obtained a diagram with a type transverse vertical section and on either side of it a second diagram, the outer one rather larger than the type, the inner rather less than the type. Within the space between these two contours practically fell the great bulk of the individual contours. We will speak of this space as the type zone. If two type zones for two races do not fall wholly or at definite points well outside each other, we suggest that the races represented by these types are either undifferentiated or closely related. As it was difficult to appreciate clearly the type zones on the scale of the actual contours, the type zone diagrams were drawn to double the scale on relatively thin paper. The two corresponding type zone contours of two races were then placed on a large glass plate below which was diffused electric lighting, and the correspondence or divergence of the zones, when the auricular lines and the vertical axes were superimposed were noted.

The horizontal section was treated in exactly the same manner. Twice the probable error was added and subtracted at each end from the mean fronto-occipital axis, and these two greater and smaller fronto-occipital axes used as the bases for plotting the ordinates increased or decreased as the case might be by twice their probable errors. Thus a type zone was obtained in the case of the horizontal section.

The same method was adopted in the case of the sagittal section, but it seems in this case somewhat less conclusive, because the nasion-gamma line, although close to the nasio-occipital line, is not necessarily coincident with it, and may therefore be itself variable and less a standard for comparison. While the line from nasion to lambda might form a more natural base, it could not be even approximately determined in the living head, and the nasion-inion line is also nearly as difficult of true determination. One of the points of Dr Benington's investigation was a comparison of crania and living head contours, and therefore the nasion-gamma line was preserved as base for the type zone of the sagittal contour.

---

* And, further of course, the same number of skulls may not be used in every case.

Illustrations of the type zone contours for the three sections of English crania of the 17th century are given in Diagrams III, IV, V reduced to half working scale*.

(4) It is not the purpose of the present paper to reproduce type contours for the anthropoid apes and the different races of man, but rather to illustrate the possible value of the method in craniological studies. Accordingly special illustrations of the method have been selected in order to bring out its possibilities. The following series have been selected†. All represent male crania.

I *a*, *b*, *c*, the three contours of the Cro-magnon skull. These are not type contours, but are individual contours reproduced for comparison (Figs. VI—VIII).

II *a*, *b*, *c*. Congo. Skulls brought in 1864 from Fernand Vaz. Contours taken of 50 ♂ crania (Figs. IX—XI).

III *a*, *b*, *c*. Congo. Skulls brought in 1880 from Fernand Vaz. Contours taken of 20 ♂ crania (Figs. XII—XIV).

These two series, II and III, were brought from the Gaboon by Du Chaillu, and are in the British Museum, Natural History. Both series have been fully measured by Dr Benington, and his measurements will shortly be published.

IV *a*, *b*, *c*. Type contours of 41 ♂ Congo crania drawn by the kind permission of Professor Keith at the Royal College of Surgeons. They came from the Batetelu race (Figs. XV—XVII).

V *a*, *b*, *c*. Type contours of 100 ♂ Egyptian crania. From Series E in the Biometric Laboratory, University College. This series of nearly 2000 crania was brought by Professor Flinders Petrie from excavations in a single cemetery south of the Gizeh pyramids, and belongs to the XXVI—XXX Dynasties (B.C. 600 to 200) (Figs. XVIII—XX)‡. The measurements are in progress and have not yet been published.

VI *a*, *b*, *c*. Contours of 31 ♂ Eskimo crania at the Royal College of Surgeons (Figs XXI—XXIII).

VII *a*, *b*, *c*. Contours of 14 ♂ Guanche crania at the Royal College of Surgeons (Figs. XXIV—XXVI).

VIII *a*, *b*, *c*. Contours of 100 ♂ English crania from the collection of 17th century English skulls in the possession of Professor Thane at University College. Measurements on these series were published by Macdonell: see *Biometrika*, Vol. III, p. 191 (Figs. XXVII—XXIX).

IX *a*, *b*, *c*. Living head contours of 106 Royal Engineers (Figs. XXX—XXXII).

---

* The size of the scale is most important; it is impossible to work from true scale drawings, they are far too small; nor is it really feasible to work from photographic reproductions like the illustrations to this paper, the photographer, however careful, never gets in all cases the *exact* size of the originals, and is liable to introduce sensible distortions.

† Throughout (*a*) denotes the sagittal, (*b*) the transverse vertical, and (*c*) the horizontal contour.

‡ See Flinders Petrie: *Gizeh and Rifeh*, p. 29.

(5) *Living English Head and English Cranial Type, 17th century.* The idea at the bottom of this investigation was to test the Whitechapel Plague Pit crania (see *Biometrika,* Vol. III, p. 191) against the living English head in order to ascertain whether the results obtained for those crania may be considered as approximately true for modern crania. We were able by the aid of small gifts to induce 118 non-commissioned officers and men of the Royal Engineers to have their heads measured. Of these men 103 were of English parentage, English born; Three were of Scottish parentage, born in Scotland; two of Scottish parentage, born in London; five of Irish parentage, born in Ireland; two were of mixed Scottish and English parentage, one of mixed Scottish and colonial parentage; one of mixed Irish and Scottish, and one of mixed Irish and English parentage. It was not thought necessary to exclude these Scottish and Irish contingents; the head contours were taken in 106 cases.

The actual observations made on the 118 non-commissioned officers and men were as follows: (i) age, (ii) weight, (iii) height, (iv) previous occupation, (v) length of service, (vi) distinctions, (vii) birthplace of father, (viii) of mother, (ix) of subject, (x) nature of hair (wavy, curly, straight, crisp, soft), (xi) type of nose, (xii) Flower's greatest head length, (xiii) glabellar greatest head length, (xiv) nasion greatest head length, (xv) maximum breadth, (xvi) auricular height, (xvii) naso-mental height, (xviii) bi-malar width, (xix) nasal height, (xx) nasal width, (xxi) nasal index, (xxii) cephalic index from (xii), (xxiii) cephalic index from (xiii), (xxiv) minimum temporal breadth. These results will be discussed later elsewhere, but the mean values may be given here: (i) 26·1 yrs., (ii) 11 st. 2·8 lbs., (iii) 68·5″, (xii) 193·6 mm., (xiii) 194·9 mm., (xiv) 193·3 mm., (xv) 151·1 mm., (xvi) 125·9 mm. (?), (xvii) 121·6 mm., (xviii) 133·9 mm., (xix) 54·9 mm., (xx) 31·6 mm., (xxi) 57·8 mm., (xxii) 78·1, (xxiii) 77·5, (xxiv) 121·4 mm. Certain of these results may be profitably compared with those given by Mr Schuster for the same characters in this number of *Biometrika,* pp. 49—51.

Comparison of Oxford Undergraduates and Royal Engineers.

	Age	Weight	Stature	Head Length	Breadth	Auricular Height	Cephalic Index
Undergraduates	20·2	10 st. 12 lbs.	1765	196·1	152·8	136·6	78·02
Royal Engineers	26·1	11 st. 3 lbs.	1740	194·9	151·1	125·9 (?)	77·50

Most of the differences here are precisely those we are familiar with when different social classes are investigated. The outstanding point is the extreme difference in auricular height. I presume Mr Schuster measured from the centre of the auricular passage. His value is in excess of other measurements on the living head (see p. 50 of this volume of *Biometrika*), but the excess is nothing like so great as in the case of these Royal Engineers. It is not easy to suppose

Dr Crewdson Benington made a slip in reading his head-spanner*, but if we presume him to have read from the end of the **V**-block instead of the reading edge, his value of the auricular height would be 135·9 instead of 125·9, a value far closer to other observations. If we take, however, the auricular height as obtained for 100 Whitechapel crania, 111·5, it seems possible to reconcile it with Dr Crewdson Benington's result of 125·9 mm. For the cranial measurement is from the *top* of the auricular passage, and the living head from the centre line. The difference may amount to 5 to 6 mm. If we give 6 to 7 mm. flesh thickness at the apex of the cranium, we should expect the living head to show *at a maximum* 13 mm. more auricular height than the skull. This would give the English of the 17th century an auricular height of 111·5 + 13 = 124·5 mm., which is very close to Dr Crewdson Benington's observed result on the Royal Engineers. But such a large difference in auricular height as that between the Oxford men and the Engineers has not been hitherto noted between social classes. If it were confirmed, it would be a matter of very considerable interest. There seems, however, some chance of an error. It would be a most useful anatomical investigation to determine with some exactness the difference between the auricular height measured on the flesh-covered head and on the cranium. We see that the Engineers are heavier (because they are older), they are slightly shorter, and have smaller and slightly more dolichocephalic heads than Oxford undergraduates.

If we now compare the living head of the Royal Engineers with those of the 17th century English, we are met by the question of what is the correct allowance to be made for flesh (and of course hair) in taking the contours. If the reader will place Fig. XXVIII in the tissue series upon Fig. XXXI of the text series so that their median verticals coincide, and further make their No. 6 horizontals agree, he will, I think, be surprised at the correspondence of the two contours. An even more striking result is obtained if the No. 7 horizontals be made to agree. In the latter case we have an almost uniform distribution of flesh of 6 mm. thickness, but there is a greater divergence of the auricular lines (9 mm.) than I consider reasonable. In the former case we have flesh thicknesses varying from 5 to 7 mm. with a divergence of 7 mm. between the auricular lines, which is just possible. Now let the reader put down Fig. XXXI in tissue on the Congo skull, Fig. X, or the nearest approach to the English in this series, the Guanche, Fig. XXV, and he will find the accordance by no means so good; the "general parallelism" is wanting, and in one part or another—often where in actuality the flesh is thinnest—he will have to suppose 9 to 10 mm. thickness. On the whole it appears to me that the 5 to 7 mm. of flesh thickness† demanded when we

---

* Dr Benington worked with a Pearson spanner and I personally showed him how to use it, and where to read it.

† It must be remembered that this 5 to 7 mm. thickness covers also the hair thickness, which must come under the lead tape. Welcker has given 6 to 7 mm. for the thickness of the cranial flesh, Merkel 6 mm., Gladstone 4 to 5 mm., Derry 3·5 to 4 mm., but these values do not include the hair: see *Phil. Trans.* Vol. 196 A, 1901, p. 250; *Biometrika*, Vol. IV. p. 110, and Vol. VIII. p. 69. It is possible to avoid taking the hair into account with a well-designed head-spanner. It is of course impossible in the case of the contours taken by lead tapes. See Note added p. 137.

superpose the transverse vertical sections of the Royal Engineers and 17th century English heads do not indicate any *substantial* difference between the English head of to-day, and our plague pit crania, which we have been told are not typical "English."*

If now we superpose the tissue Fig. XXIX, 17th century English on Fig. XXXII living English (on solid paper), so that the median lines coincide and No. 6 horizontals are placed in agreement, we find in the frontal and occipital regions 5 mm. of thickness; nor, allowing for the hair, is the thickness anywhere excessive till we reach the temporal regions, where we find about 8·5 mm. I think that probably some of this excess is due to the difficulty of giving the lead tape a *slightly concave* form. The remainder is probably accounted for by the temporal muscle. The cranial contours drawn with the Klaatsch craniograph always show this concavity at the horizontal No. 3 on the horizontal contour; it is rare to find any but the slightest traces of it on contours of the living head.

Lastly, let tissue Fig. XXVII of the 17th century English skull be superposed on Fig. XXX (text) of the living English head. In this case the $N\gamma$ lines must be superposed and not the line NI of the living head with the line $N\gamma$ of the skull†. The best position is, I think, that in which the foot of No. 5 vertical of Fig. XXVII coincides with the point of intersection of No. 5 vertical and the $N\gamma$ line of Fig. XXX. This gives 4·5 mm. flesh thickness at the nasion and at the gamma. There is possibly some excess of thickness (8 mm.) a little forward of the bregma, but this is the very point where the lead tape leaves the flesh of the forehead and rises above the hair. Of course it may be argued, that the two nasions ought not to be placed on the same horizontal, but that the cranial nasion should be placed on the normal to the living head contour at its nasion, the two $N\gamma$ lines being then made parallel. If this be done, the parallelism is certainly improved and the two heads seem to be of much the same height with possibly, but not quite demonstrably, a slight metopic development. In order to appreciate the degree of resemblance the reader should superpose Fig. I, Thurtell, the murderer's sagittal contour, in tissue first on Fig. XXVII, the sagittal contour of the 17th century English and then on Fig. XXX, the sagittal contour of the living English head. He will find any fitting impossible, the murderer's *cranial* contour would contain the average *living* head contour inside itself. It differs about as much as can be conceived both in shape and size from the average English head.

We may conclude that in the class from which the Royal Engineers are drawn there is no *marked* change in the shape of the English head from that of the 17th century crania discussed by Macdonell and contoured by Benington.

---

* If 135·9 mm. and not 125·9 mm. were the true reading of the auricular height, we must add 10 mm. to the contour of Fig. XXXI below the line R. A. to L. A. It is impossible then to fit Fig. XXVIII in tissue to Fig. XXXI, nor indeed is it possible to fit any transverse *cranial* contour to such a living figure, without assuming 15 to 20 mm. flesh.

† There is a difficulty here in Dr Benington's notes. He states the mean value of $N\gamma$, but has apparently plotted his ordinates to the nasion-inion line.

(6) *Comparison of English and other Cranial Types.* Before wider comparisons have been made it might legitimately be argued that the parallelism exhibited by the English cranial and living head types is not greater than we should find by comparing average heads for other series together. We will therefore look into some of the points of agreement and disagreement with other racial types.

(*a*) I take out first the three series of Congo skulls determined by Dr Benington.

Let the reader place the tissues of Figs. XII, XIII and XIV, the Du Chaillu, Fernand Vaz, crania of 1880 on Figs. III, IV, V, like contour to like contour, and he will see at once that the differences are far in excess of anything coverable by probable errors. Now place the same tissues of the 1880 Congo crania on Figs. IX, X, XI, the 1864 Du Chaillu, Fernand Vaz, crania. The 1880 type crania is slightly larger, but the differences are precisely such as we have learnt to consider from Figs. III, IV and V to be of the probable error order*. If now we superpose the tissues of Figs. XV, XVI and XVII of the Batetelu Congo crania first on Figs. IX, X and XI and then on Figs. XII, XIII and XIV we find them in most respects intermediate between the two series. In most cases the differences are not greater than apparently is introduced by the asymmetry of the skull—there may be agreement on one side and divergence on the other. It would be hard to say whether the differences are of the order of probable error or are significant of intertribal variations—far larger numbers in a variety of local races must first be dealt with before this point can be answered; but there is no doubt of the general agreement of these contours *inter se* and their substantial disagreement from the English contours.

But it will be said that we all knew beforehand that the English and the negro cranium even in its least negro form were widely divergent. The contours tell us nothing new. That is quite true, but they do bring home to us the exact nature of this divergence as far as concerns these principal sections in a marked graphic manner. Note how much more vertical the negro cranium is in the gamma region and its deficiency of frontal and occipital development. If the bregmas be brought into the same vertical, the $N\gamma$ lines being coincident, it will be seen that the subtense, the "calotte height," is at least equal to, in the Batetelu Congo crania greater than, that of the English skull. Again in the transverse vertical sections it is by no means a defect of height by which the negro cranium differs from the English, there is, if anything, an excess of auricular height. It is contraction of the bi-auricular breadth which is for all three negro contours the persistent and marked feature. Lastly, in the horizontal contours, it is not only the lesser occipital and frontal development but the marked concavity of the temporal region (abont ordinate No. 4) which is the noteworthy characteristic.

(*b*) *English and Guanche Cranial Types.* If we now superpose Figs. XXIV, XXV and XXVI in tissue representing the Guanche cranial type upon Figs.

---

* All the differences are really less than the probable errors, because the two 1864 and 1880 Congo series are short series compared to those of Figs. III—V.

XXVII, XXVIII and XXIX giving the English 17th century head in solid, we find an extraordinary agreement in the sagittal and horizontal contours. The nasions and gammas are almost coincident, the lambdas and bregmas hardly 2 mm. apart; the deviations in either series are not beyond what we have found for probable errors, and distinctly less than those between the three series of Congo crania. The maximum divergence appears when the transverse vertical sections are superposed; the Guanche falls inside the English at all points by amounts varying from 0·5 to 3·5 mm.; the concavity to right and left of the vertex and the contraction at No. 2 of the breadth above the auricular passages being the most important differences. But even here the differences are not greater than the reader will find by superposing the transverse section Fig. XIII on Fig. XVI, i.e. are local rather than racial differences, while the differences of the sagittal and horizontal sections of English and Guanche are far less. I think we may safely conclude that the divergences between the English and Guanche cranial contour types are not as great as we find between two local races of Congo negroes.

(*c*) *English and Eskimo Cranial Types.* To test how far we are dealing with slight differences peculiar to our method of averaging, let us place Figs. XXVII, XXVIII and XXIX in tissue upon Figs. XXI, XXII and XXIII. We mark essential differences at once; in the sagittal section the Eskimo encloses entirely the English skull with 3 to 9 mm. to spare. In the horizontal section, the English skull, if 2 mm. shorter, has in its broadest part some 4 mm. to spare on both sides. In the transverse vertical section the Eskimo shows excess at the vertex and considerable defect just below, in the region of ordinates Nos. 8 and 9; but its bi-auricular breadth is again greater. The deviations throughout are quite significant and markedly in excess of the Guanche-English differences.

(*d*) *English, Negro and Egyptian Cranial Types.* It will not be without interest to compare these types by superposing Figs. XVIII, XIX and XX in tissue upon Figs. XXVII, XXVIII and XXIX, and, say, Figs. XV, XVI and XVII. Taking first the sagittal sections there is extraordinary agreement between English and Egyptian, if the nasions are superposed, not only as far as the bregma but right up to the region of ordinate No. 9. Then the Egyptian shows defect of occipital development, with the same tendency but less marked than in the negro, to verticality in the neighbourhood of γ. If now we superpose XVIII on XV, we find that the Egyptian differs far more from the Negro, than from the English type. It is now the Negro who is more vertical at the gamma, and who shows occipital deficiency. Nor is it possible to say that the Egyptian is nearer in sagittal contour to the Negro, because the Egyptian and English agree in differing by absolutely the same amount from Negro from nasion to within one centimetre of the lambda. From these onwards the Egyptian deviates towards the Negro but is always closer to the English. The reader will see these points excellently by placing Figs. XVIII and XXVII in tissue both together upon Fig. XV.

Turning now to the transverse sections, we may superpose XXVIII and XIX in tissue at the same time on XVI. The Egyptian is throughout smaller than the Congo Negro; if now the English be compared it will be found closer to the Negro than the Egyptian is in the upper $\frac{4}{10}$th of the section; after that Negro and Egyptian approach, while the English show greater bi-auricular width.

Lastly, if we superpose XX and XXIX in tissue on XVII, we note, making the frontals coincide, that both Egyptian and English have excess of occipital development, but that the Egyptian is nearer to the English than to the Negro; in the temporal regions the English is nearer to the Negro than the Egyptian, while from ordinates Nos. 4 to 10, the Negro and Egyptian are close together. The contours therefore do not suggest that the Egyptian of circa 400 B.C. differs from the European of the 17th century by being everywhere nearer to the negro.

(*e*) *Relative nearness of Negro, Egyptian and European.* Finally, if the reader wishes to realise how little is the evolutionary gap between Negro, Egyptian and European, let him place the tissues of Negro, Egyptian and 17th century English upon those of a prehistoric skull, like the Cro-magnon. Thus for the sagittal section, he places XV, XVIII and XXVII on VI, and realises at once how enormous is the gap between prehistoric man and Englishman compared to that between Negro and Englishman. XVI, XIX and XXVIII superposed on VII tell the same tale for the transverse vertical section; it is noteworthy that the English skull differs from Negro and Egyptian (or Guanche) in preserving the bi-auricular breadth of the Cro-magnon, while the Eskimo, smaller in every other respect, actually exceeds it. The same lesson may be drawn from the horizontal sections when XVII, XX and XXIX in tissue are placed on VIII, although we must leave it to the reader to place the superposed contours to suit his own views on the relative proportions of occipital and frontal change.

(7) The object of this paper has been solely to suggest a method, possibly of service to the craniologist. It is, I think, desirable that type cranial contours should be formed of modern and ancient crania. If possible 100 crania of both sexes* should be dealt with, and the final type forms should include at least the chief cranial points below the nasion-gamma line as in our Fig. I. We have taken the $N\gamma$ line as an axis of reference because it seemed possible to compare in that way cranial and living head contours, but this is by no means needful. Further, when it has been done, the final type contour may be considered as quite independent of this line. Thus interesting results may be obtained, if we compare different crania by making their nasion-bregma lines coincide; it is singular how little divergence will be found in cranial contours, if we make the nasio-bregmatic lines coincide†. Others may have their own views to emphasise from glabella-inion or nasion-inion bases, while not a few may prefer to adjust to coincidence the nasion-basilar line—the value of this is indicated by placing the tissue of Fig. II, the Egyptian, upon Fig. I, Thurtell's contour. All these points could be followed out in detail on the contours once drawn.

---

* As a first series it might be well to confine attention to adult male skulls, the sexing in this case being relatively easy.

† Cf. for example the Eskimo XXI superposed on the English XXVII and we see how much of the difference between the two types lies below the lambda.

I believe that type contours constructed for all the chief collections of homogeneous crania would be of very great service. These type contours placed in order of divergence would give us many suggestions as to racial affinities, and craniologists would be able to detect points of divergence and of similarity which it is hard to extract from tables of figures, and which would partially replace the need for the difficult or often impossible comparison of two long series placed side by side. Is it too much to hope for, if we suggest that craniologists by a concordat should agree to prepare type contours of the large homogeneous cranial series in their keeping, and that these contours should be issued together in a standard form—solid print and tissue for superposing? A work of this kind would ultimately be the standard book of each craniologist's library, especially if it were accompanied by good photographs of the skull nearest to the type contours. The present paper with its incomplete contours is not offered in any way as a contribution to such a work, it is intended solely to suggest that type contours may be of considerable value. If that point be recognised the many long days of labour which Dr Crewdson Benington gave to drawing individual contours and determining type contours will not have been wasted.

Note to p. 131. Owing to the kindness of Dr D. E. Derry, I have since writing the above been able to examine three sections of the external auditory meatus, showing the relation of the part of the cranial wall which rests on the horizontal knife edge of the craniophor to the position of the ear-plug of the head spanner, if *in situ*. I find it difficult to believe that the surface of the ear-plug when the latter is properly inserted can be more than 3 to 4 mm. below the bone, or that measuring from the axis of the ear-plug we ought to allow, as far as the lower extremity of the auricular height is concerned for more than 7 or 8 mm. in all. If we take 5 to 6 mm. at a maximum for the thickness of flesh at the vertex, this gives 12 to 14 mm. difference at a maximum between living head and cranial auricular heights. If the average auricular height on the living head be taken as 133 to 135 mm. we deduce 121 mm. for the average cranial auricular height, a result much in excess of most cranial experience. I only know three exceptions e.g. Ranke's "Altbayerische" peasants with slightly under 121 mm.; and Schuster's Long and Round Barrow skulls with 121 and 124 respectively. Otherwise we have: Ancient Etruscans, 116; Modern Copts, 116; North African Negroes, 115; Naqadas, 115; Thebans, 114; Eskimo, 114; English 17th century, 114; Modern Würtemberger, 115; Modern Badenser, 113; Modern French, 113; Modern Guanche, 110; Congo Negroes, 112, etc. On the other hand my own measurements on the University College Staff gave 135, those on the members of the Anatomical Congress, 133, and those on members of the British Association, 131 mm.—all values much in excess of Dr Benington's 126 mm., which was, however, on the living head of a different social class. I admit that Dr Benington gives us a low value compared to those just cited, but I cannot convince myself that he has made an error; his results are in keeping with the English skulls, and an auricular cranial height of 122 mm. has yet to be demonstrated for a series of modern English crania. Parsons gives the auricular height of the Hythe

Crania ♂ as 120 mm. (*Journal of the R. Anthrop. Institute*, Vol. XXXVIII. p. 430) and of the Rothwell Crania ♂ as 120 mm. also (*Ibid.* Vol. XL. p. 490). But I can nowhere find a statement of how he measured the auricular height, i.e. from the top or centre of the auricular passage, or how he made the skull horizontal. I sent my assistant Miss E. V. Thompson with our craniophor down to the Royal College of Surgeons and she measured the auricular height of the 33 available modern English crania, kindly placed at our disposal by Professor Keith. The result was 114·6, in perfect agreement with our Whitechapel and Liverpool Street crania, and wholly different from the measures given for Rothwell and Hythe. Ranke's Bavarian 121 mm. was probably due to the fact that he measured in the field, and thus used a craniometer measuring from the centre of the auricular passage. I suspect Parsons has done the same, without recognising that such measurements are wholly incompatible with our standardised Frankfurt concordat measurements. Anderson in a paper just issued (*Journal, R. A. Institute*, Vol. XL. p. 274) gives 7·2 mm. for the height of head minus height of skulls for 34 ♂s average age 69. But he has measured apparently from the *centre* of the auricular passage and this would give a further correction of 5—6 mm. at least, just the 13—14 mm. required by Benington, but not the 21 mm. needed, if he is in error. Thus Lee's difference of 11 mm. for the auricular height is rather too small than, as Anderson suggests, too large.

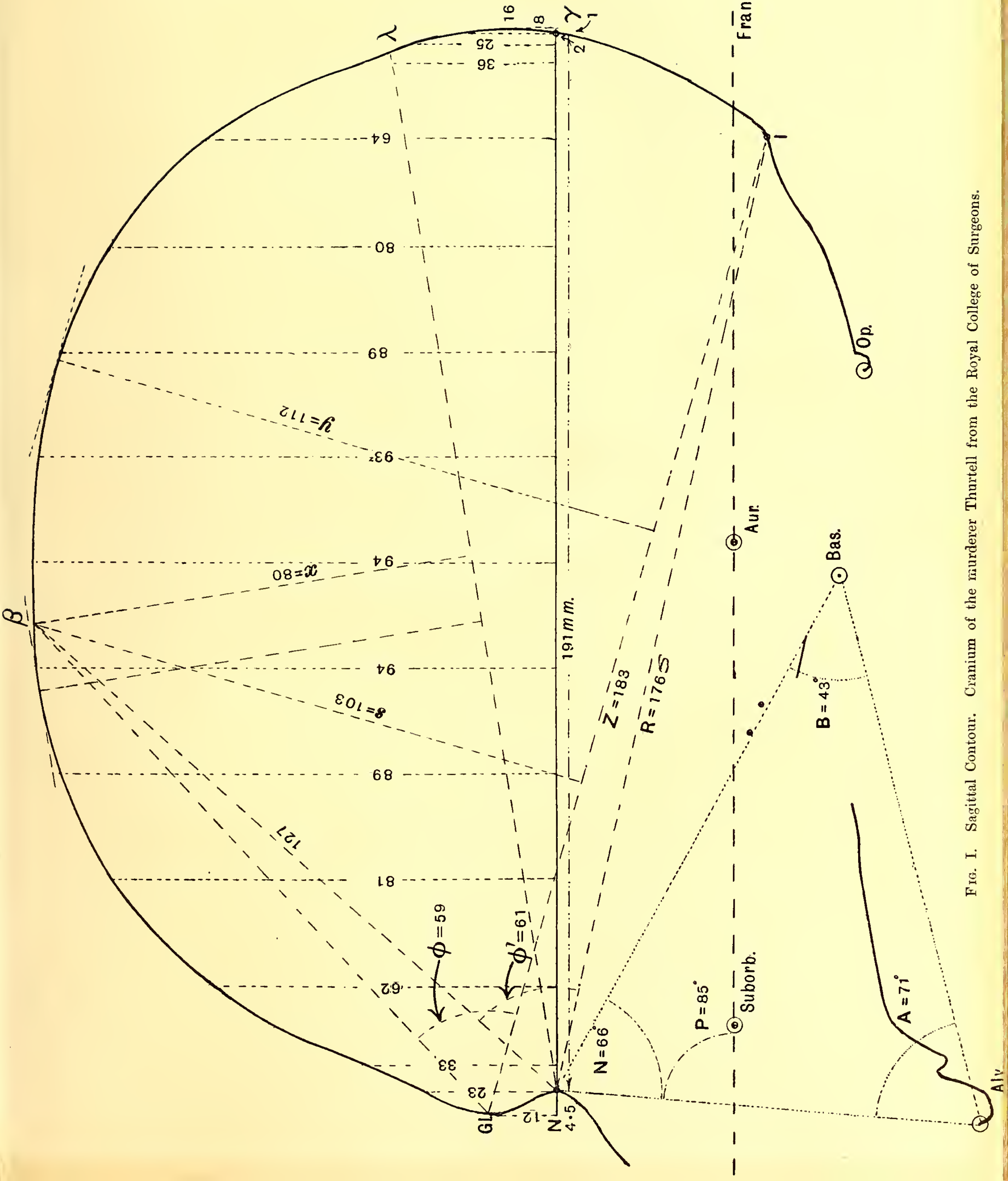


Fig. I. Sagittal Contour. Cranium of the murderer Thurtell from the Royal College of Surgeons.

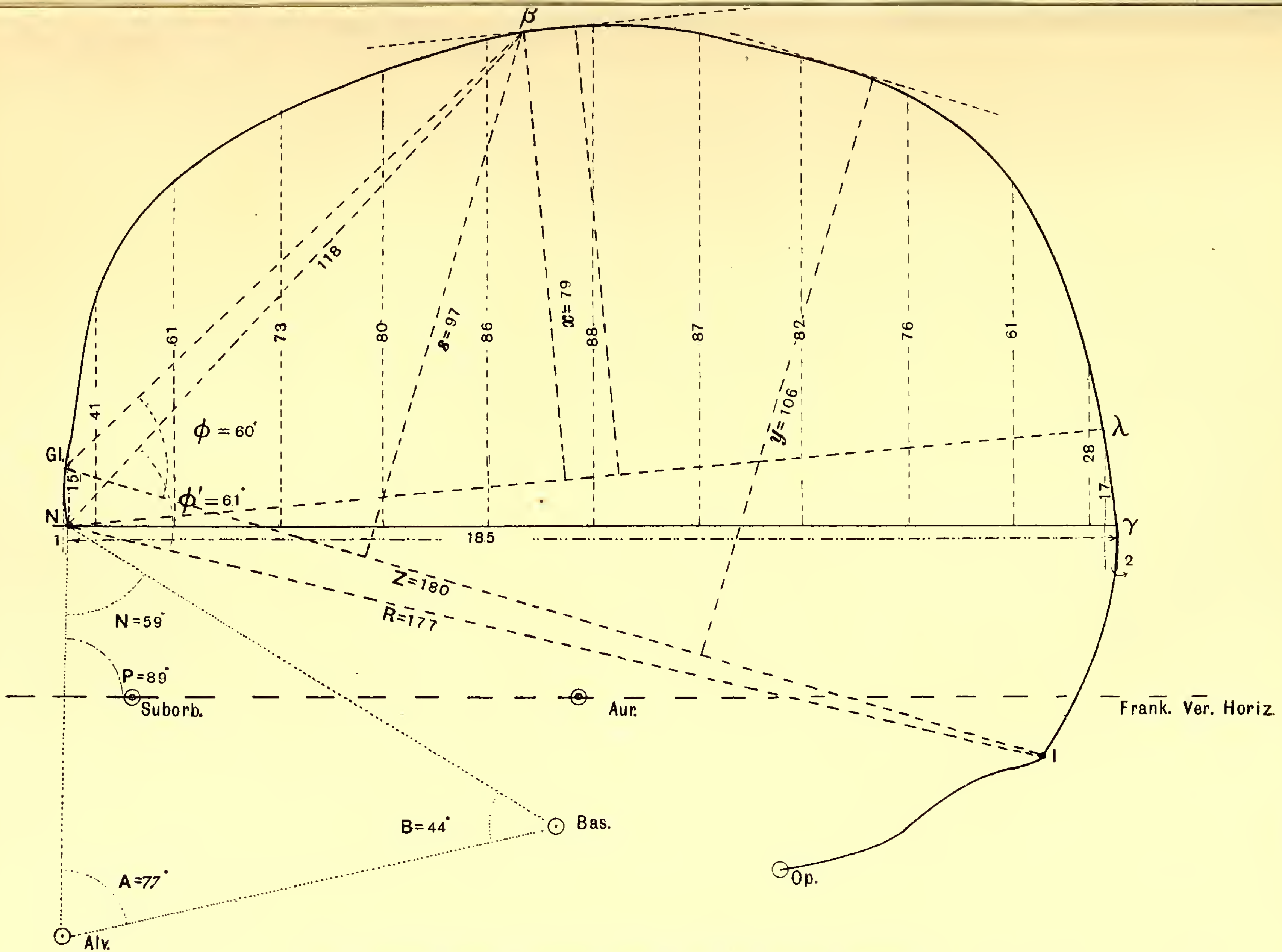


Fig. II. Sagittal Contour. Egyptian Cranium, E. 530 from Biometric Laboratory.

N.B. In splining in the contour the draughtsman has placed the protuberance *below* the true glabella of the original drawing.

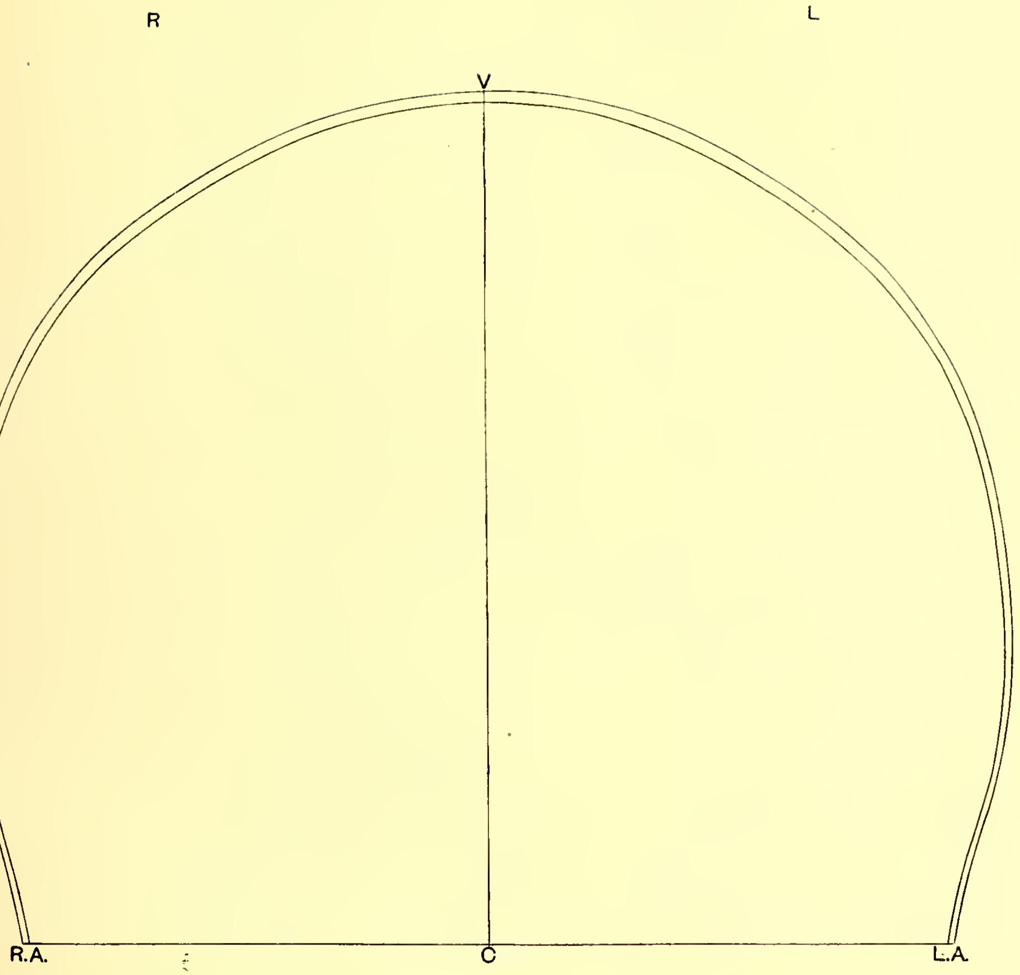


Fig. III. English Type Vertical Contour, 17th Century, showing range of variation.

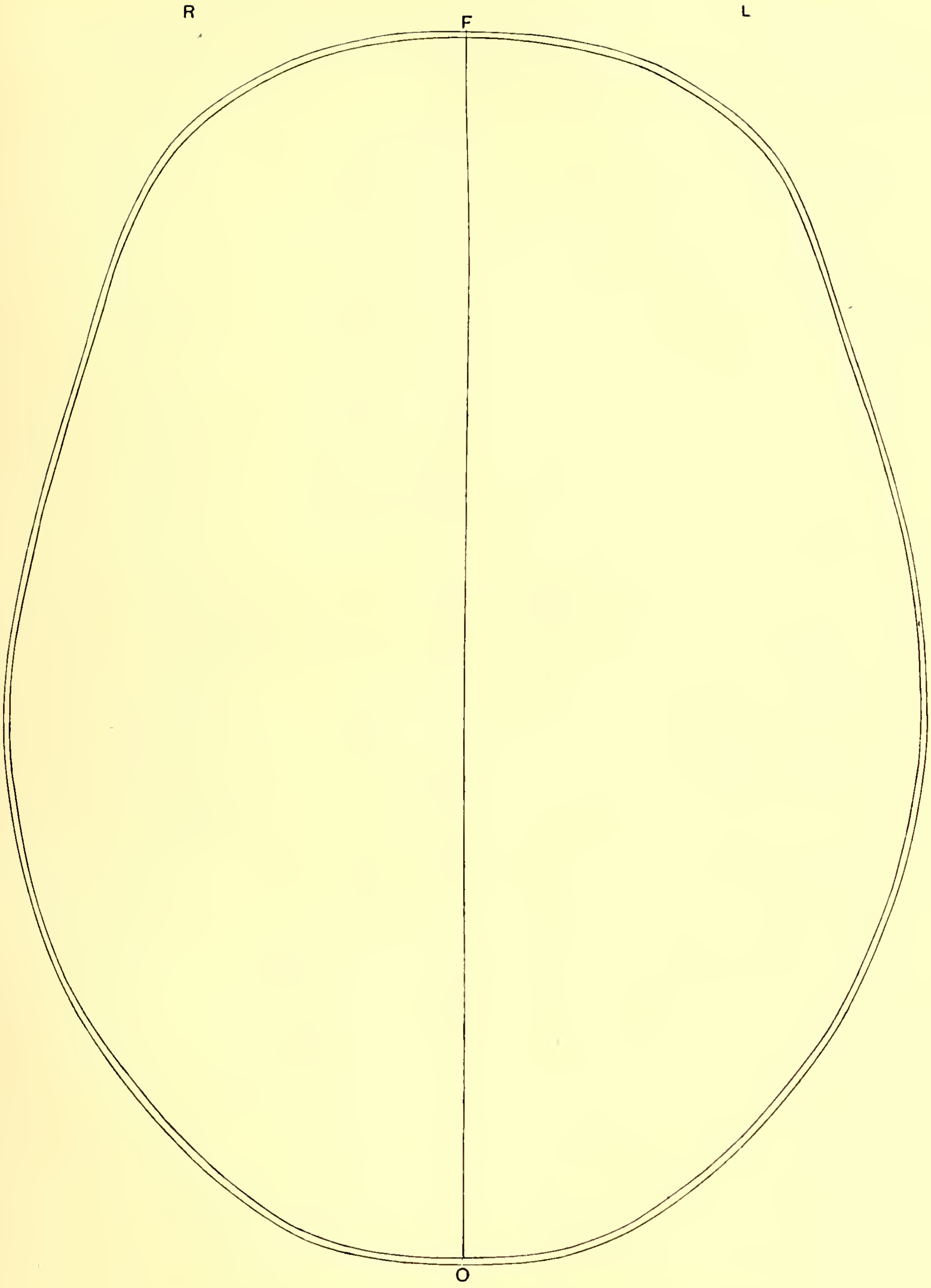


Fig. IV. English Type Horizontal Contour, 17th century, showing range of variation.

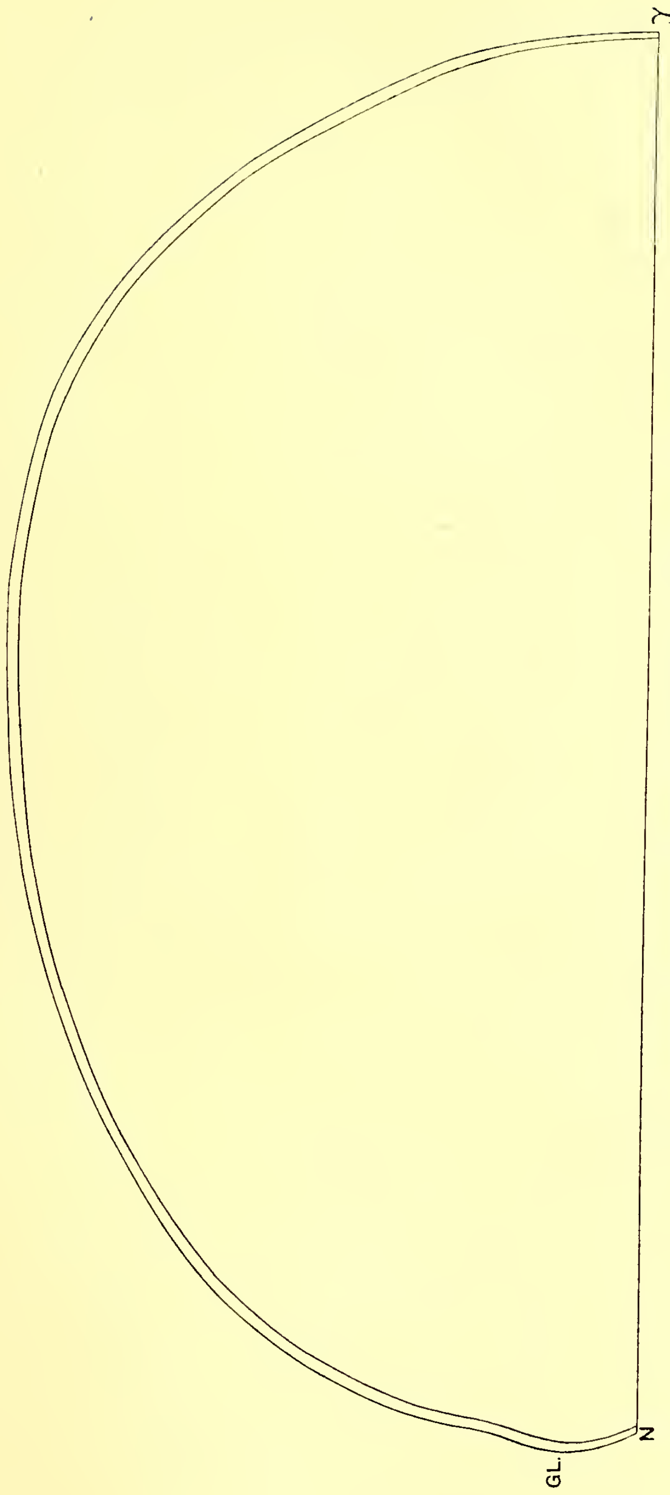


Fig. V. English Type Sagittal Contour, 17th century, showing range of variation.

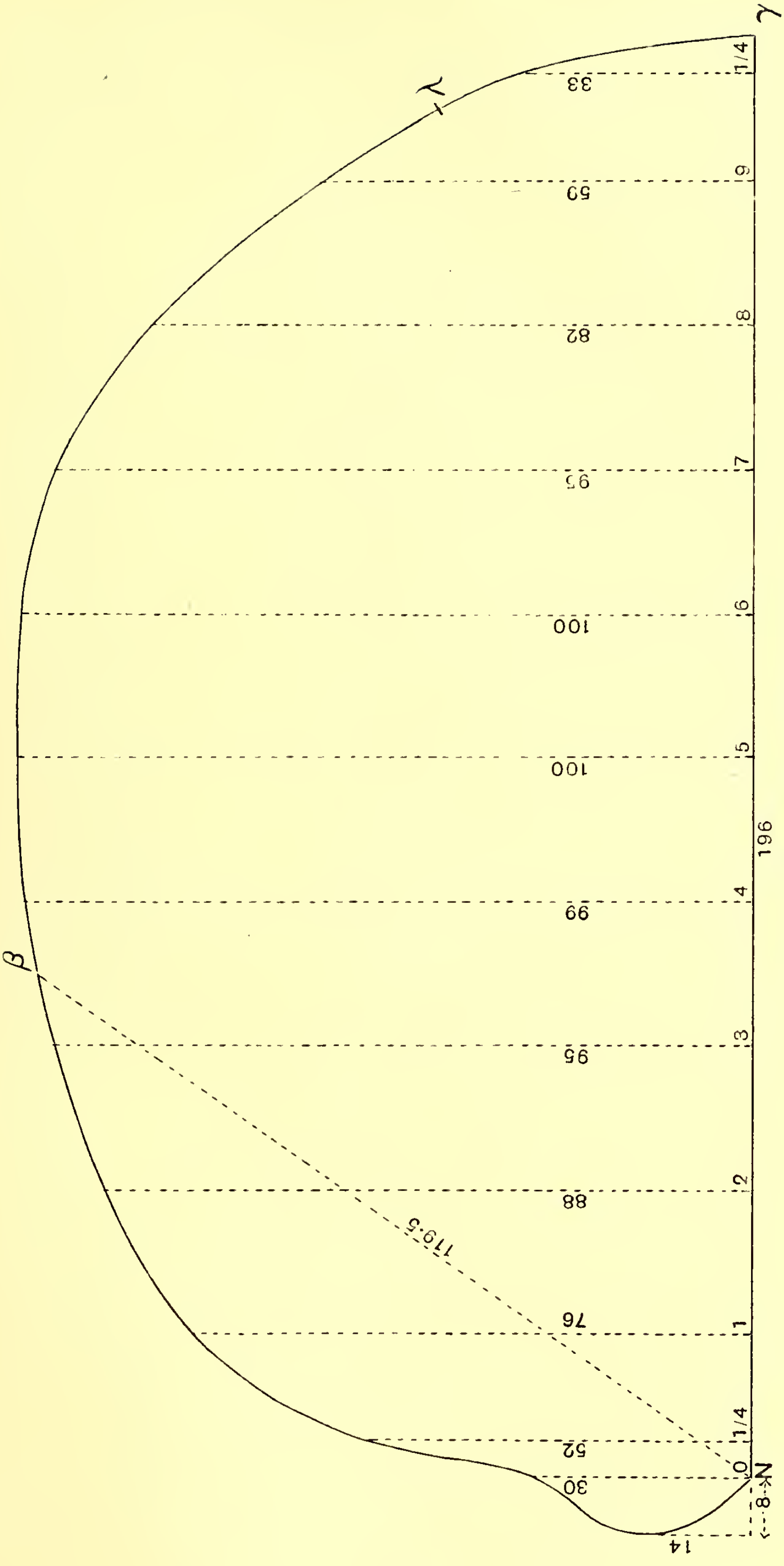


FIG. VI. Cro-Magnon. Sagittal Contour.

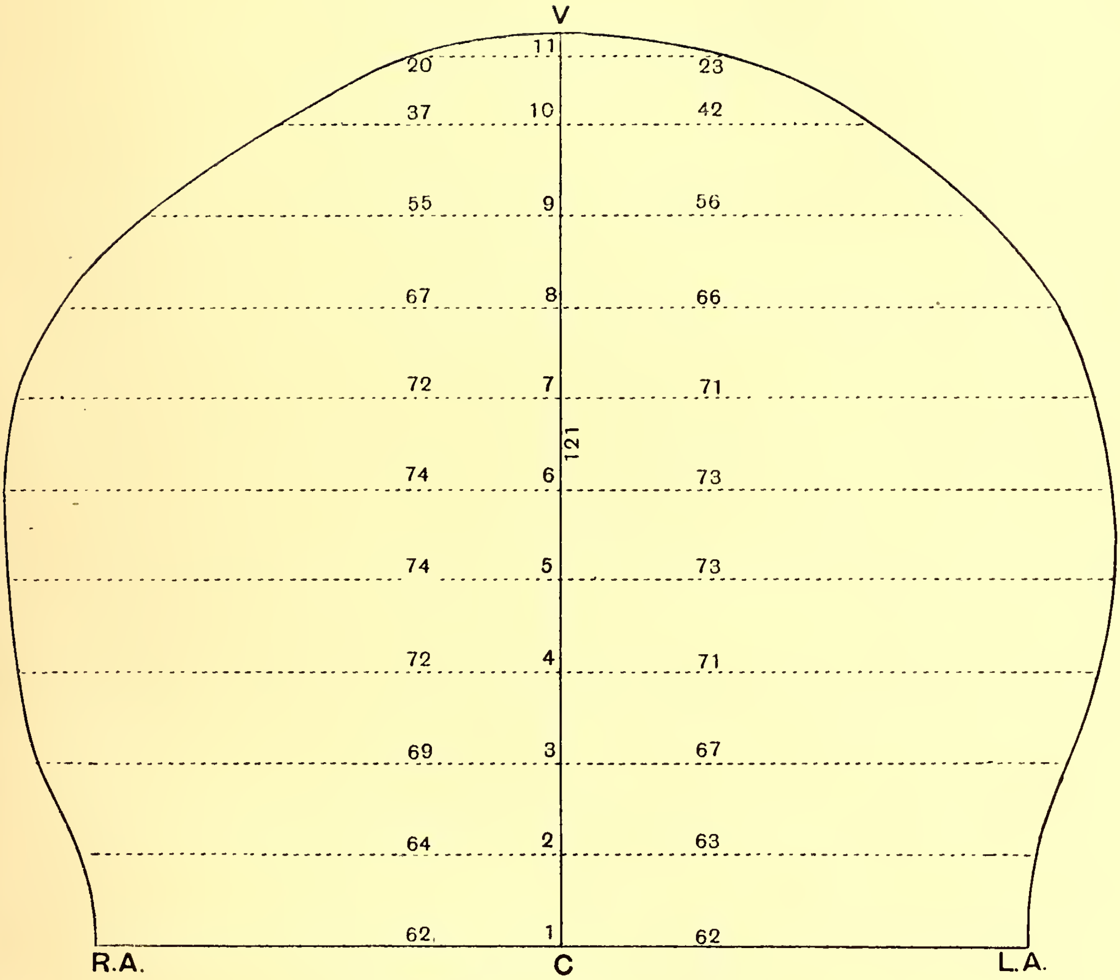


Fig. VII. Cro-Magnon. Transverse Contour.

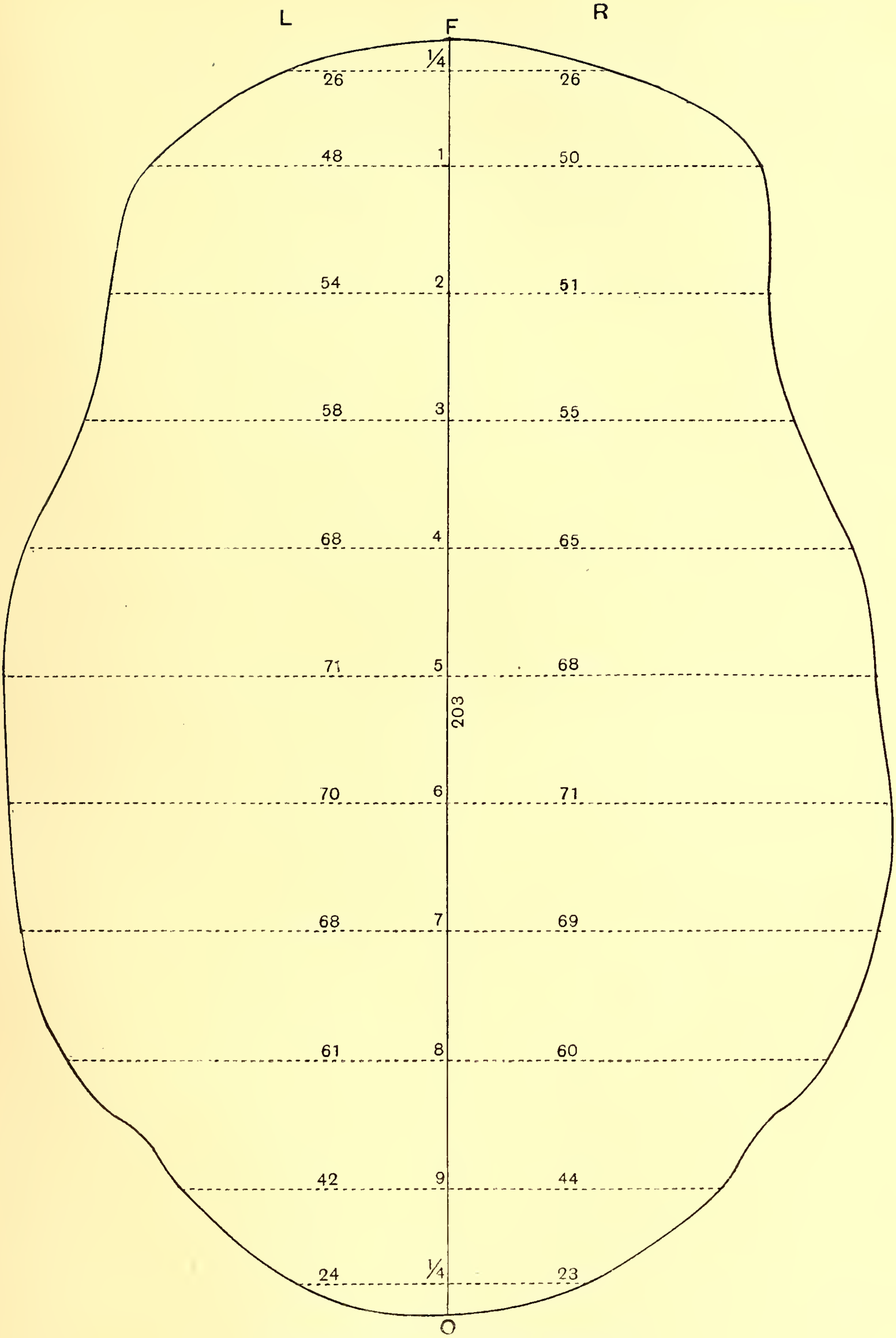


FIG. VIII. Cro-Magnon. Horizontal Contour.

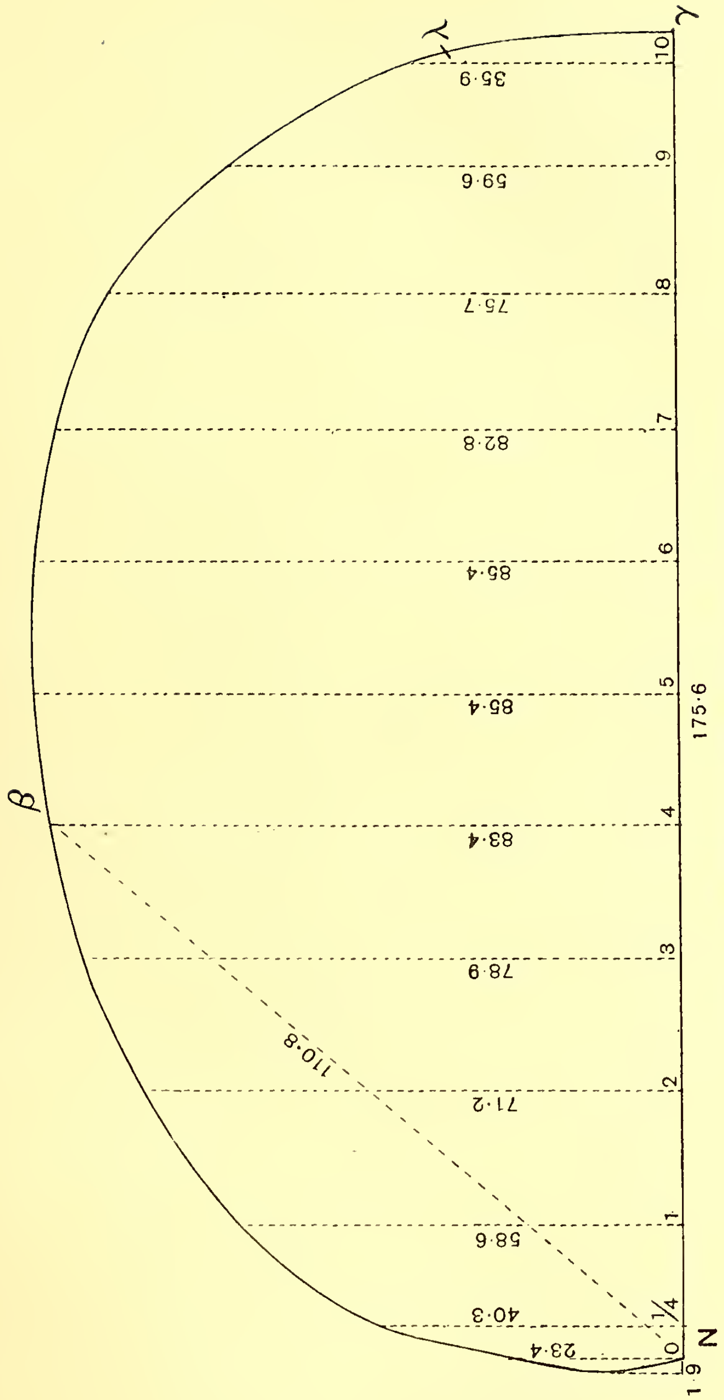


Fig. IX. Congo Crania. Fernand Vaz, 1864. Sagittal Contour.

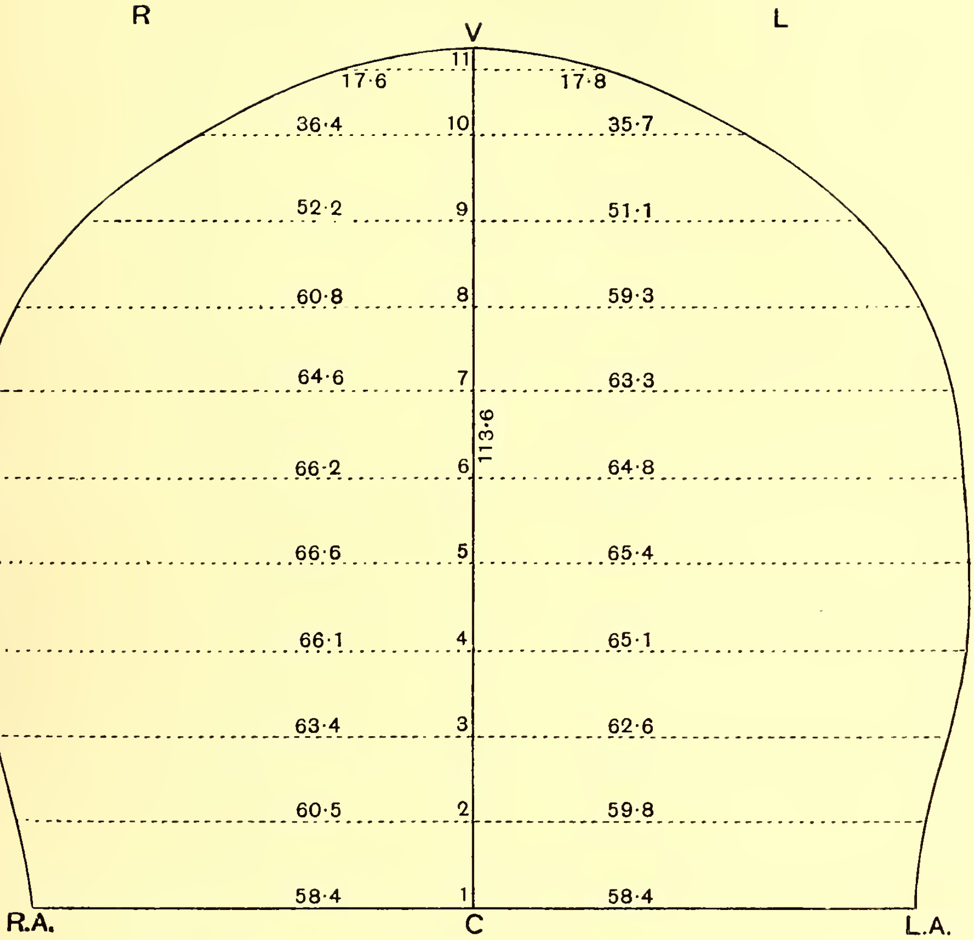


Fig. X. Congo Crania. Fernand Vaz, 1864. Transverse Contour.

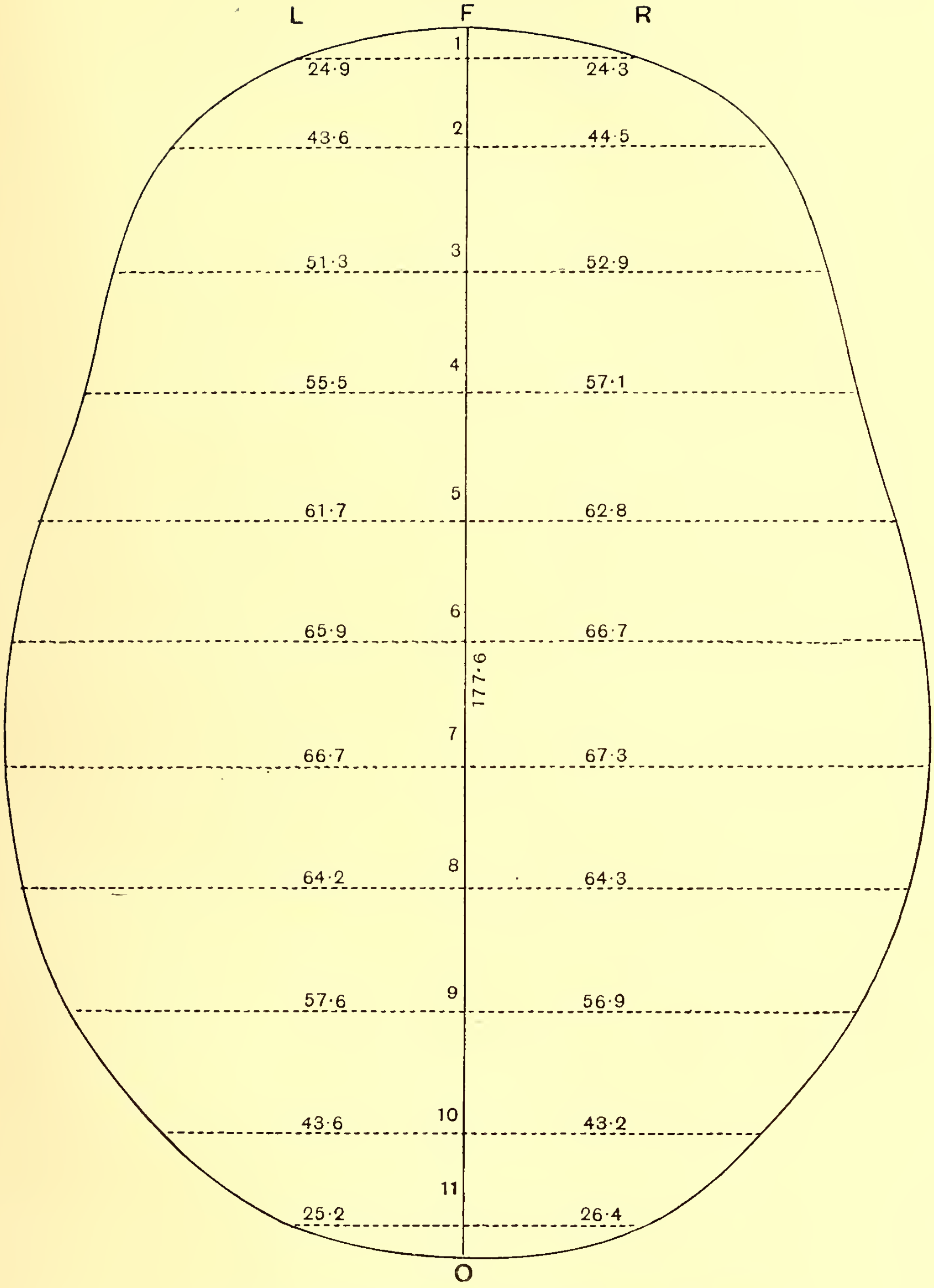


Fig. XI. Congo Crania. Fernand Vaz, 1864. Horizontal Contour.

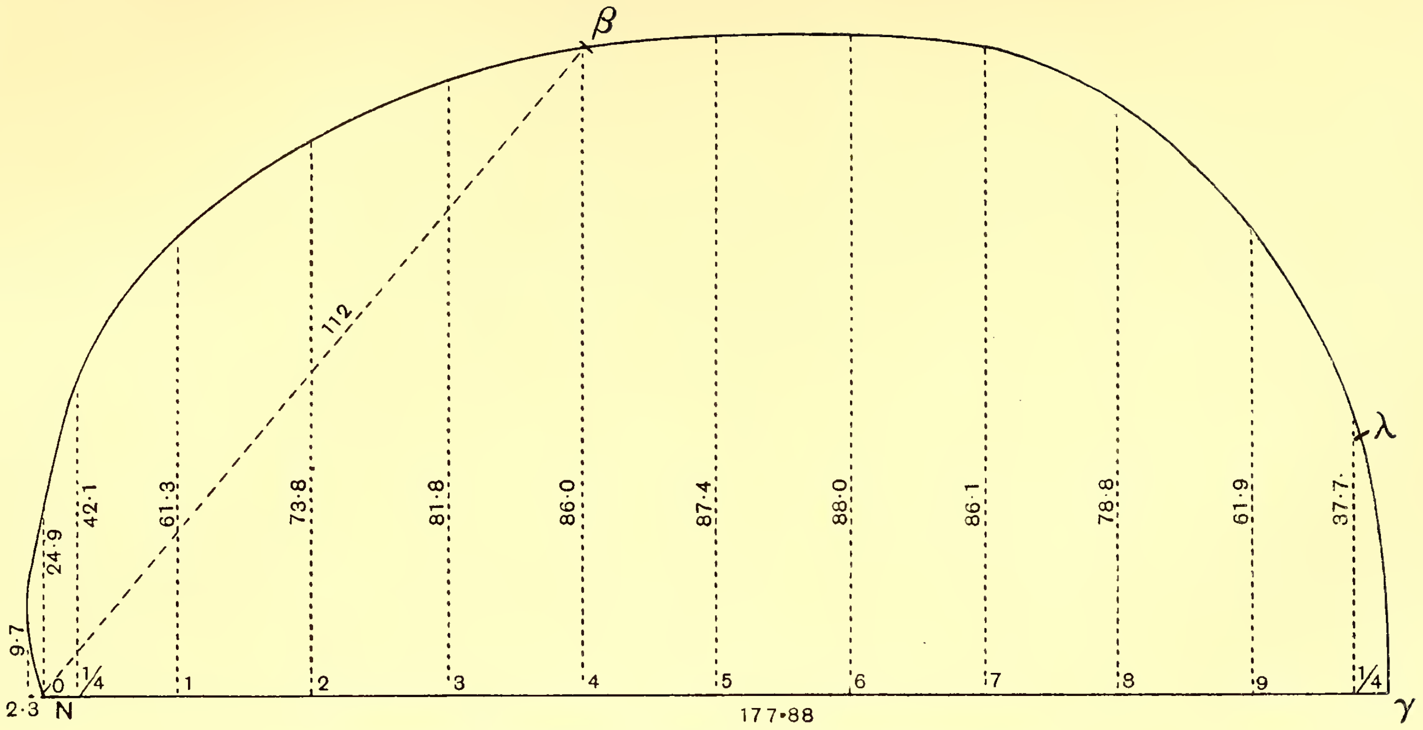


FIG. XII. Congo Crania. Fernand Vaz, 1880. Sagittal Contour.

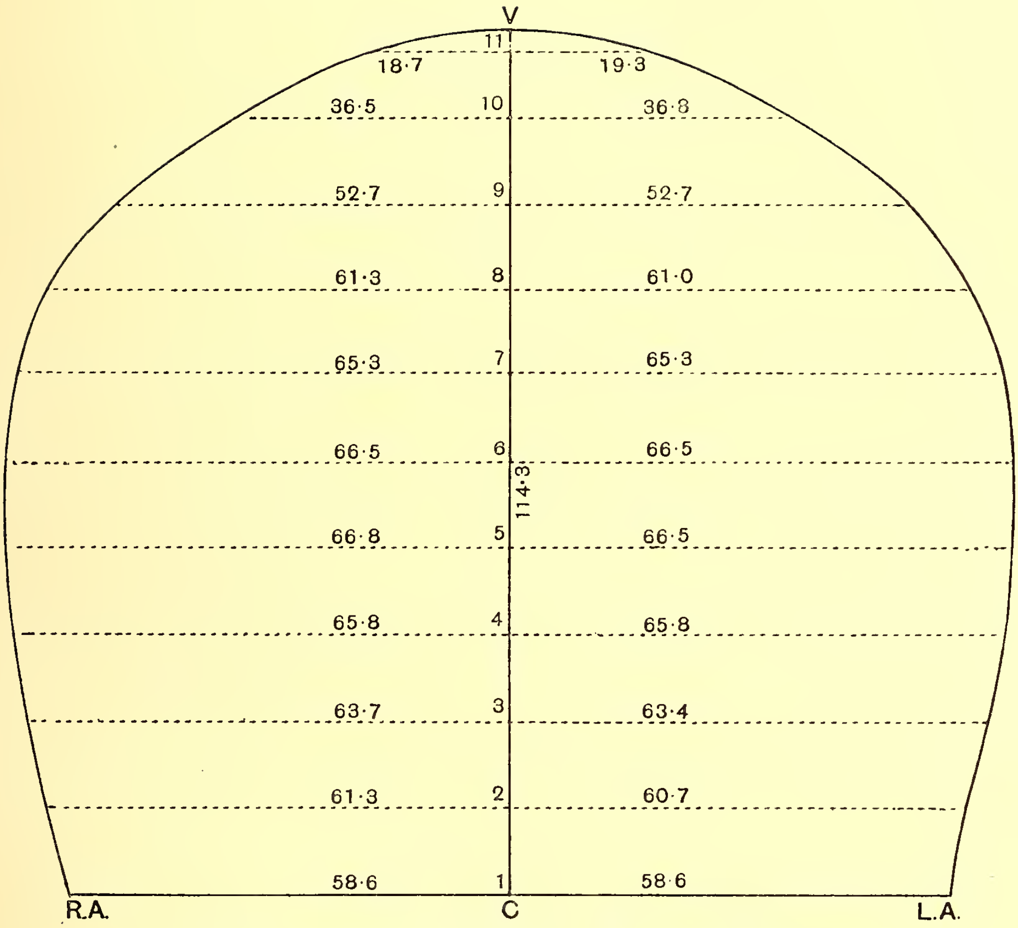


Fig. XIII. Congo Crania. Fernand Vaz, 1880. Transverse Contour.

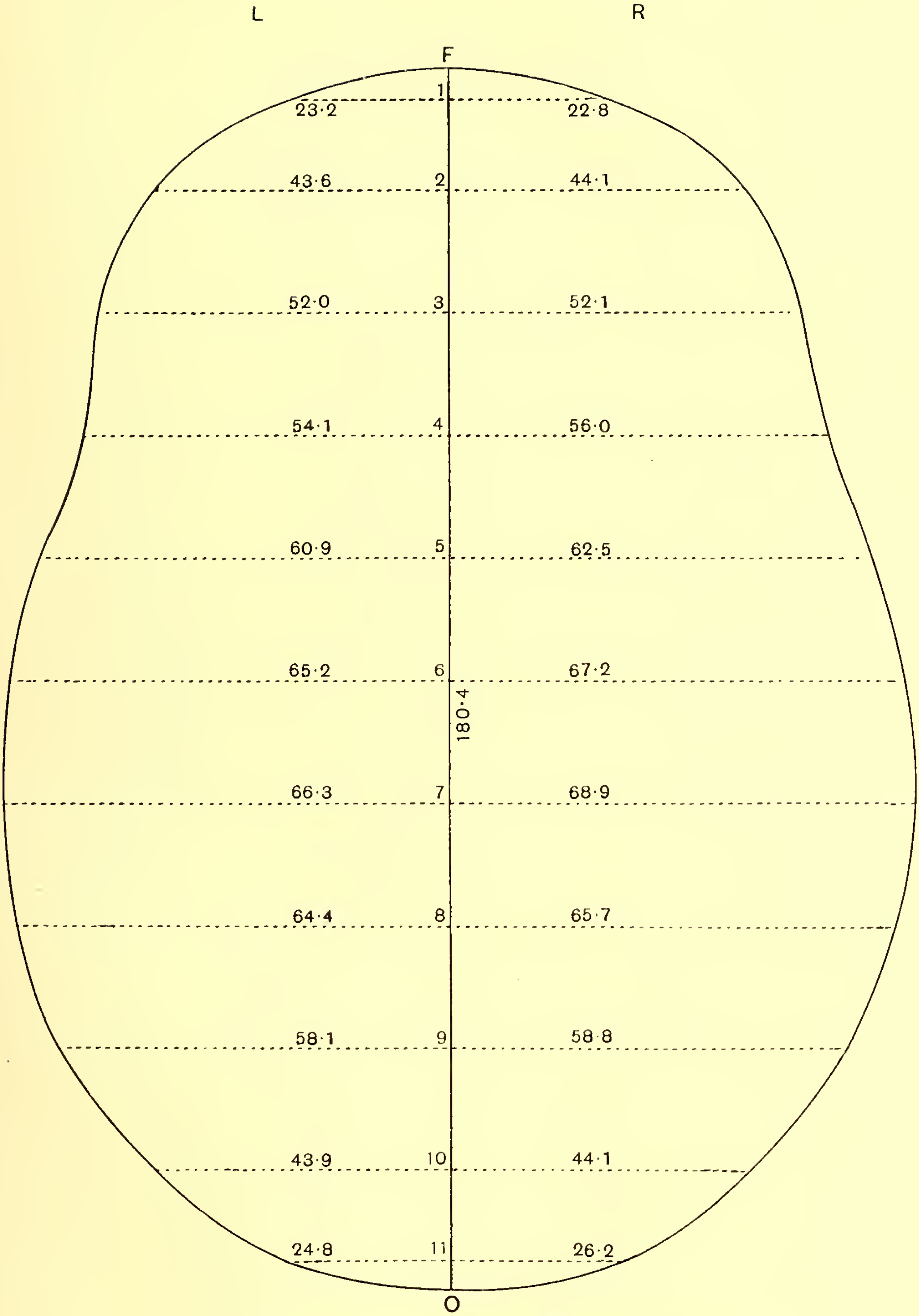


Fig. XIV. Congo Crania. Fernand Vaz, 1880. Horizontal Contour.

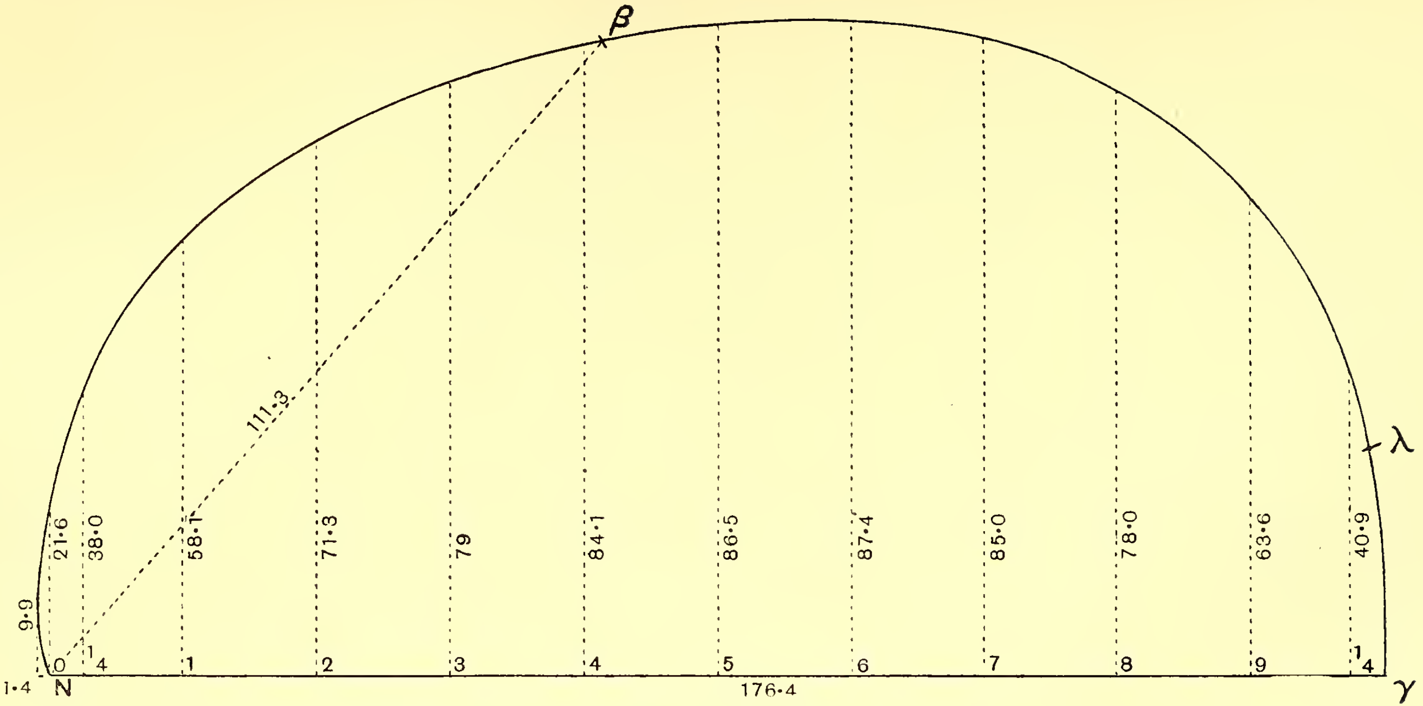


Fig. XV. Congo Crania. Batetelu Race (R.C. of S.). Sagittal Contour.

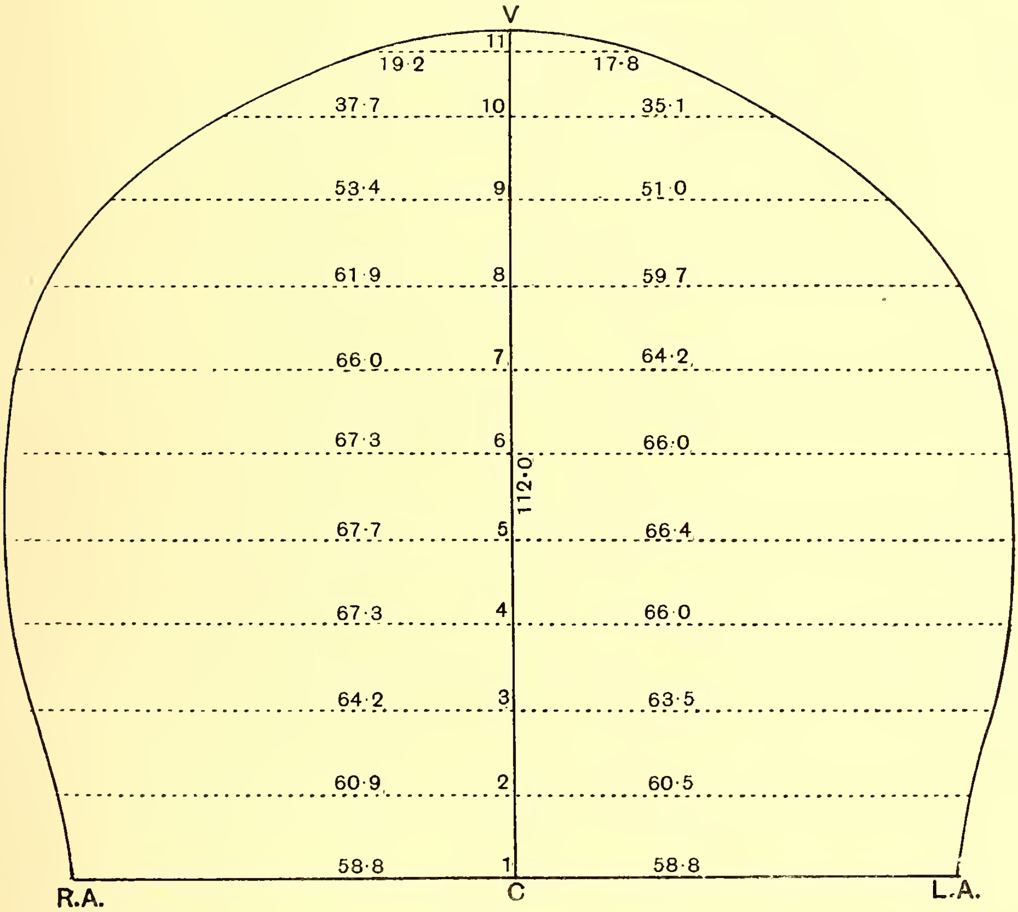


Fig. XVI. Congo Crania. Batetelu Race (R. C. of S.). Transverse Contour.

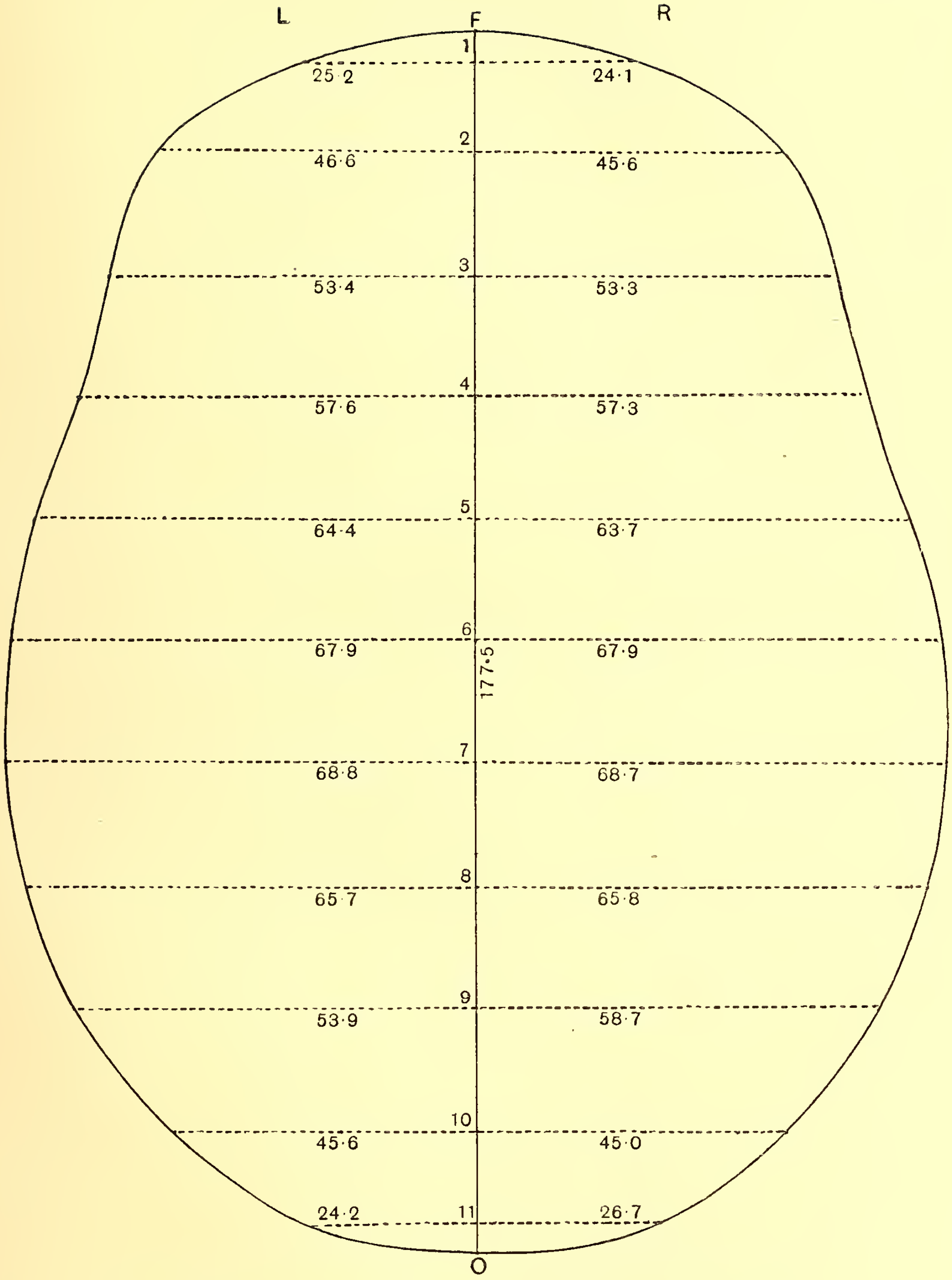


Fig. XVII. Congo Crania. Batetelu Race (R. C. of S.). Horizontal Contour.

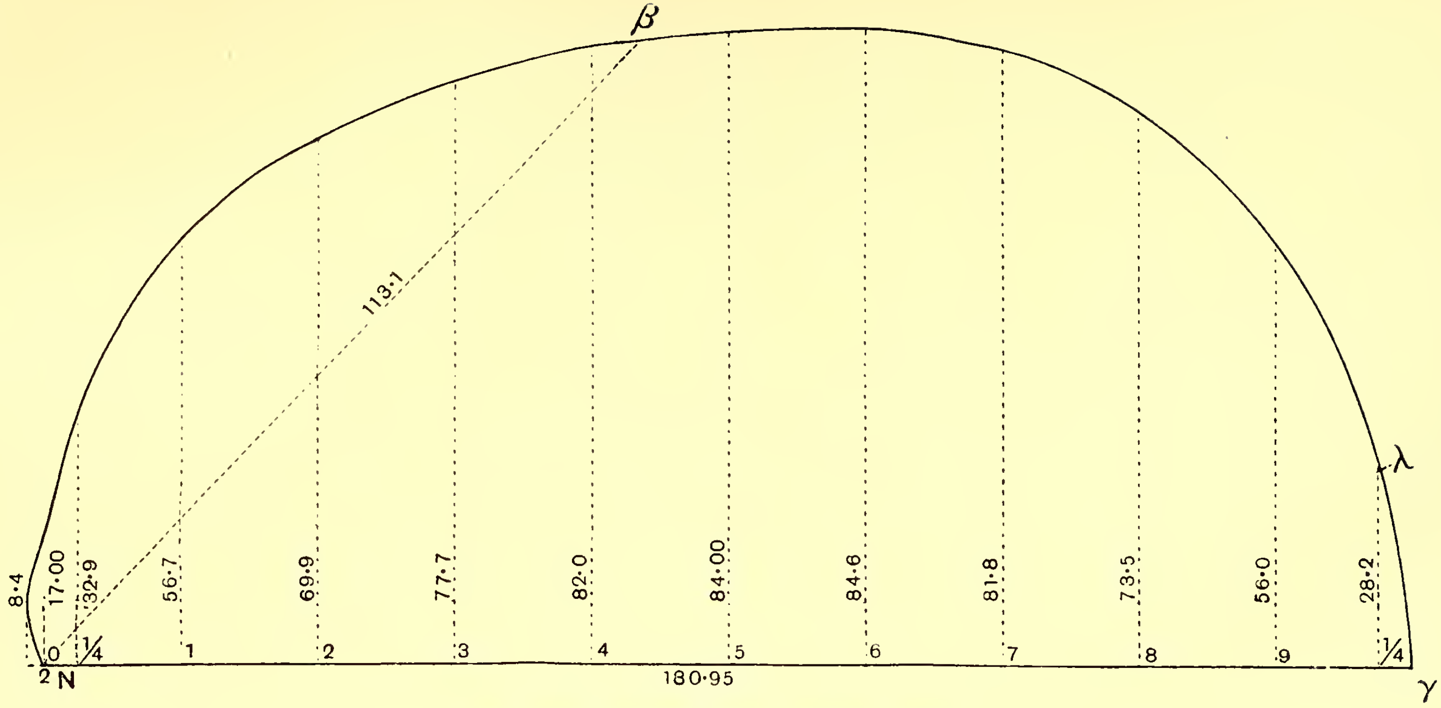


Fig. XVIII. Egyptian Crania. Sagittal Contour.

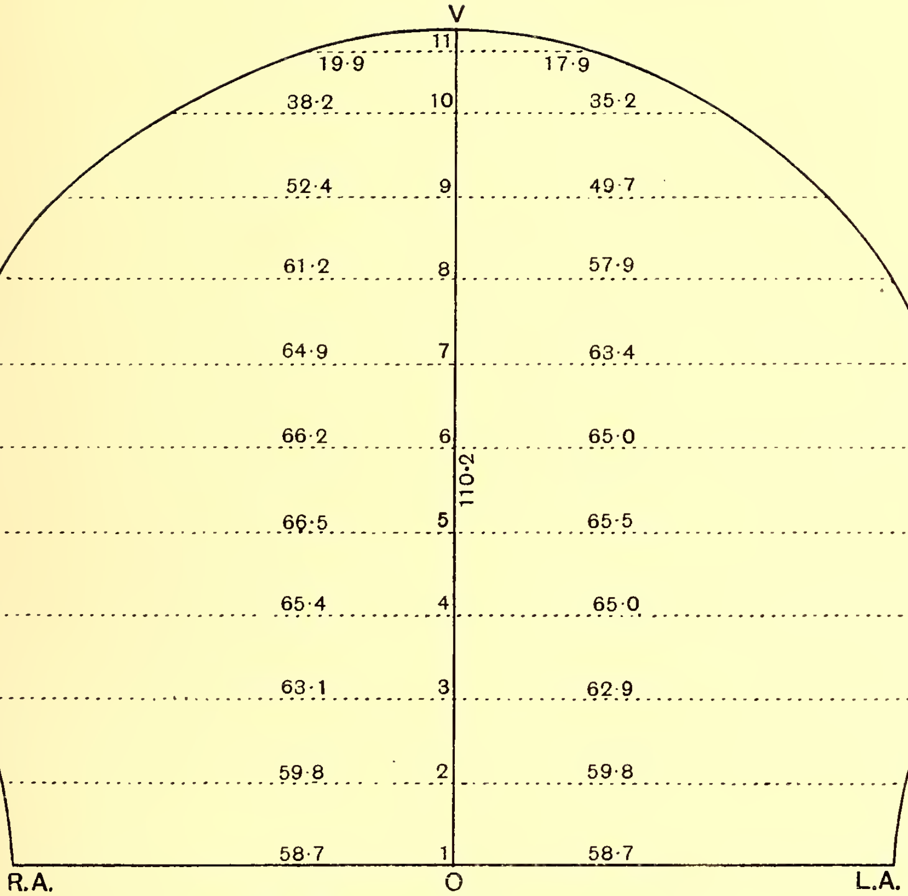


Fig. XIX. Egyptian Crania. Transverse Contour.

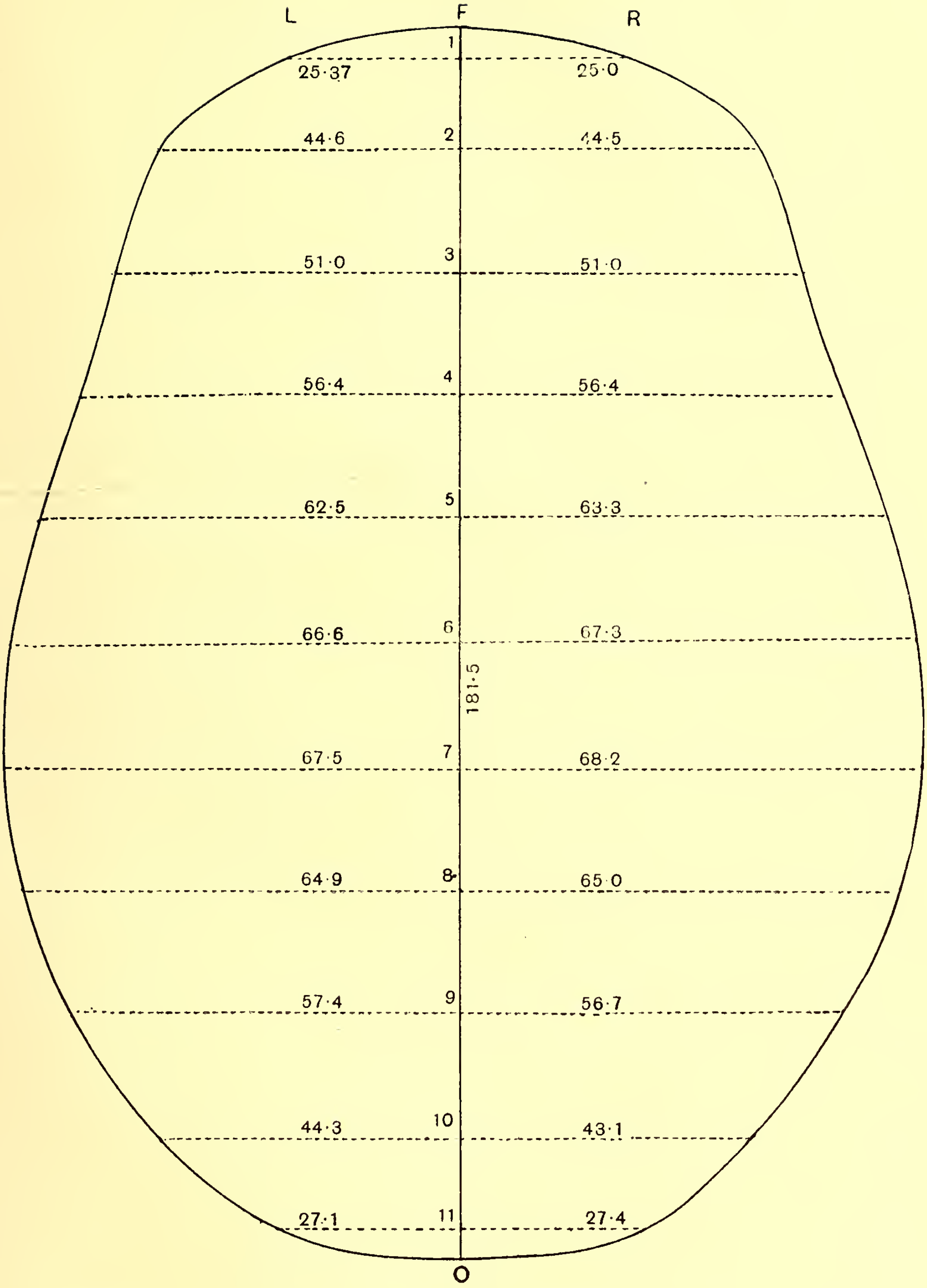


FIG. XX. Egyptian Crania. Horizontal Contour.

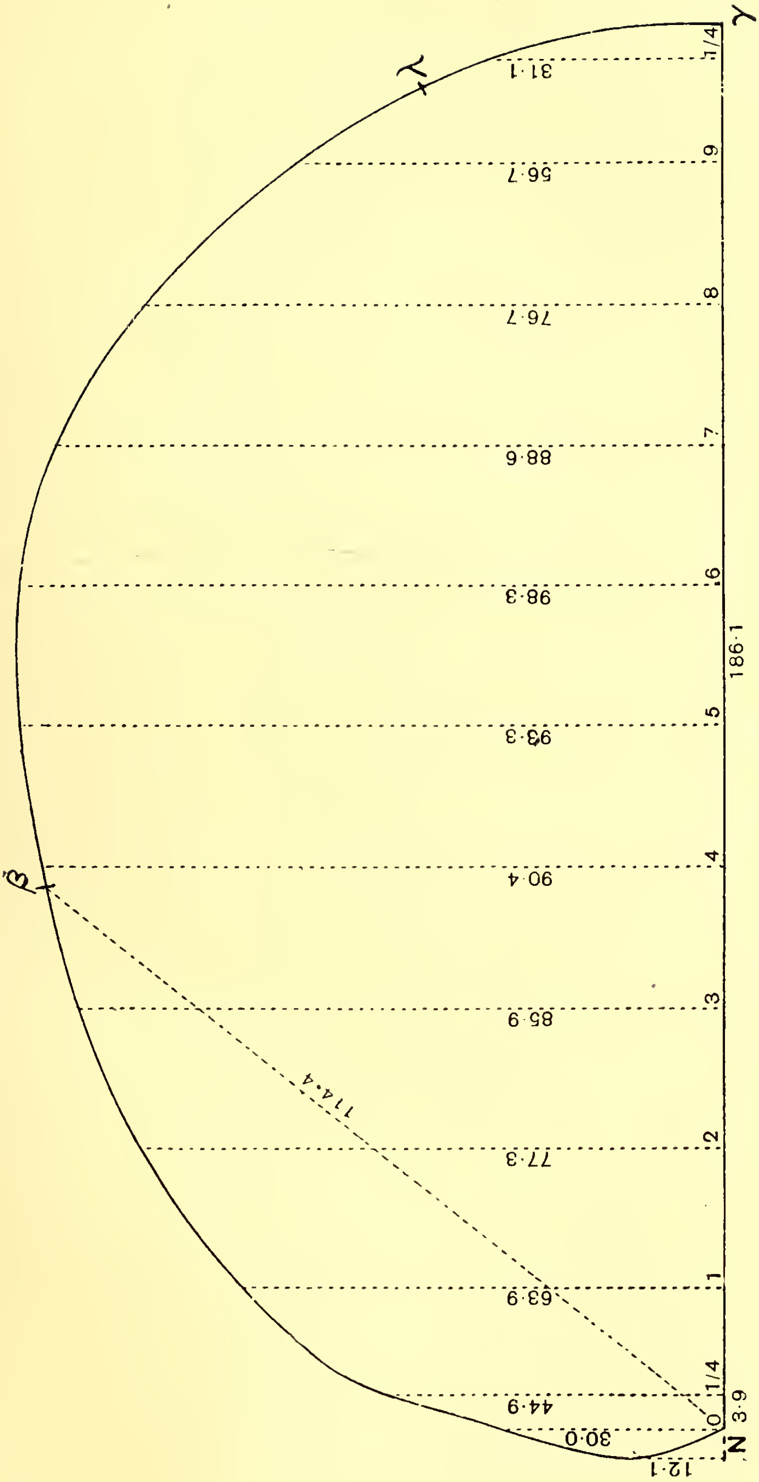


Fig. XXI. Eskimo Crania. Sagittal Contour.

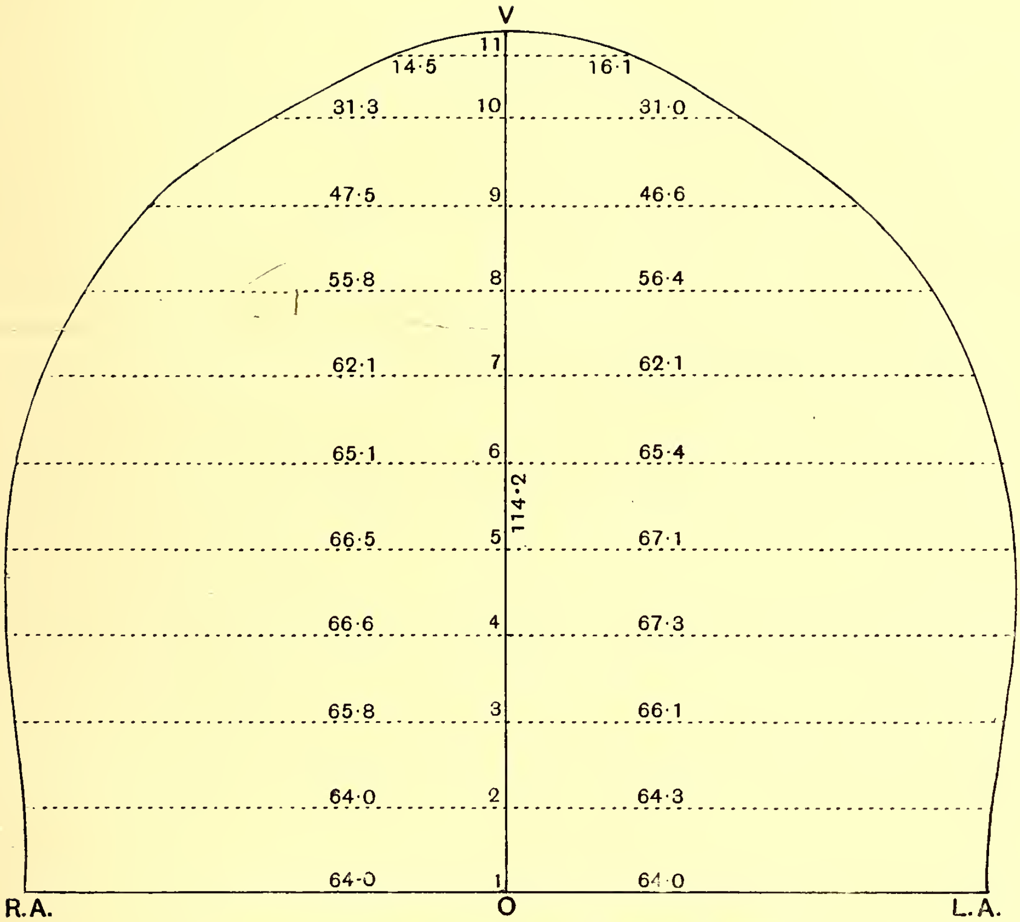


Fig. XXII. Eskimo Crania. Transverse Contour.

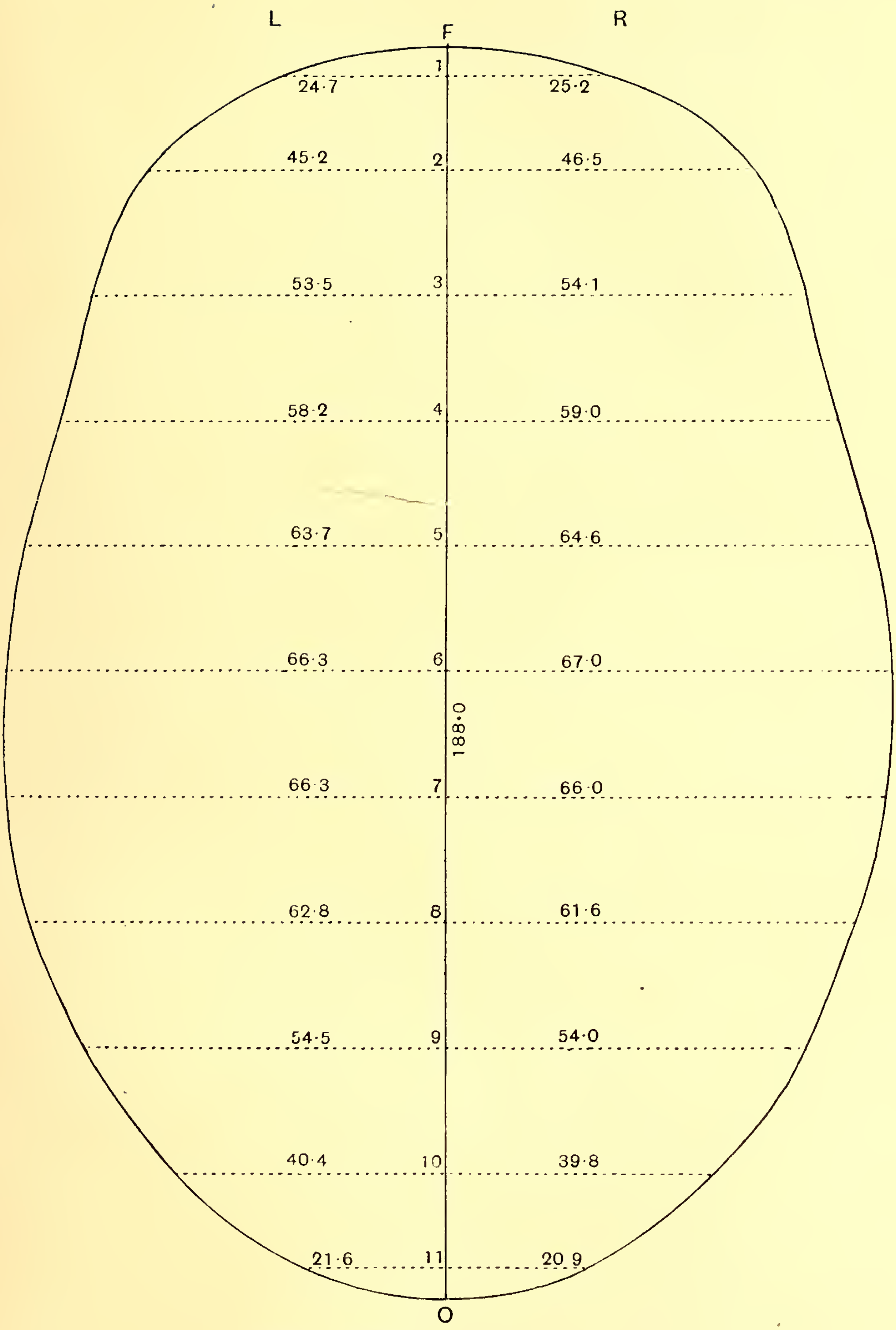


Fig. XXIII. Eskimo Crania. Horizontal Section.

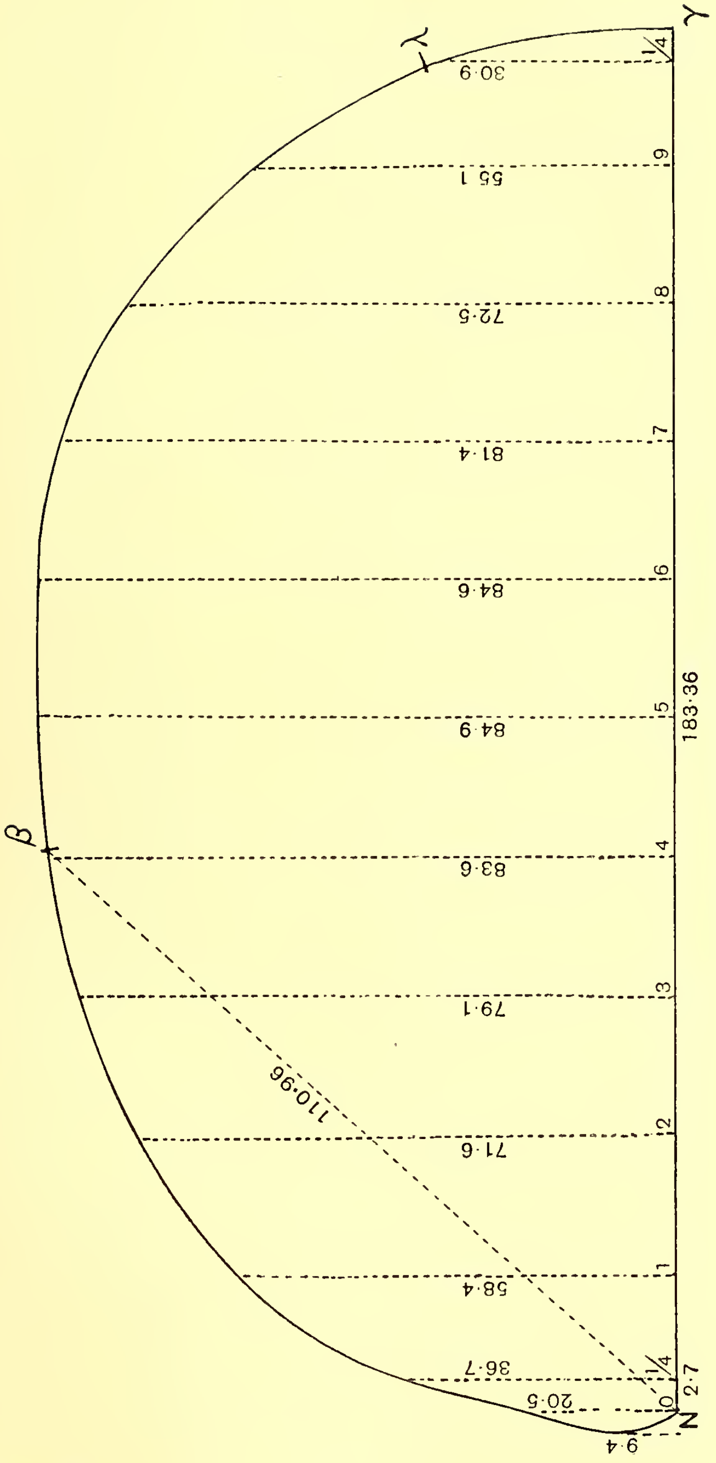


Fig. XXIV. Guanche Crania. Sagittal Contour.

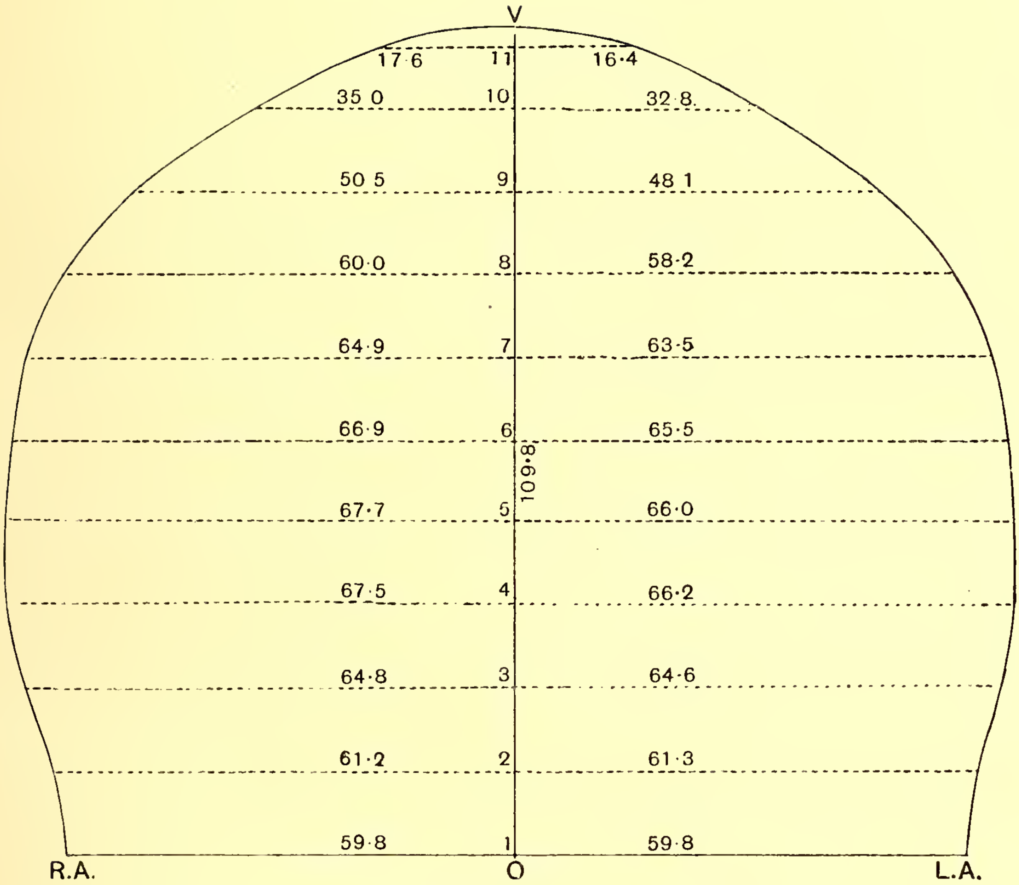


Fig. XXV. Guanche Crania. Transverse Contour.

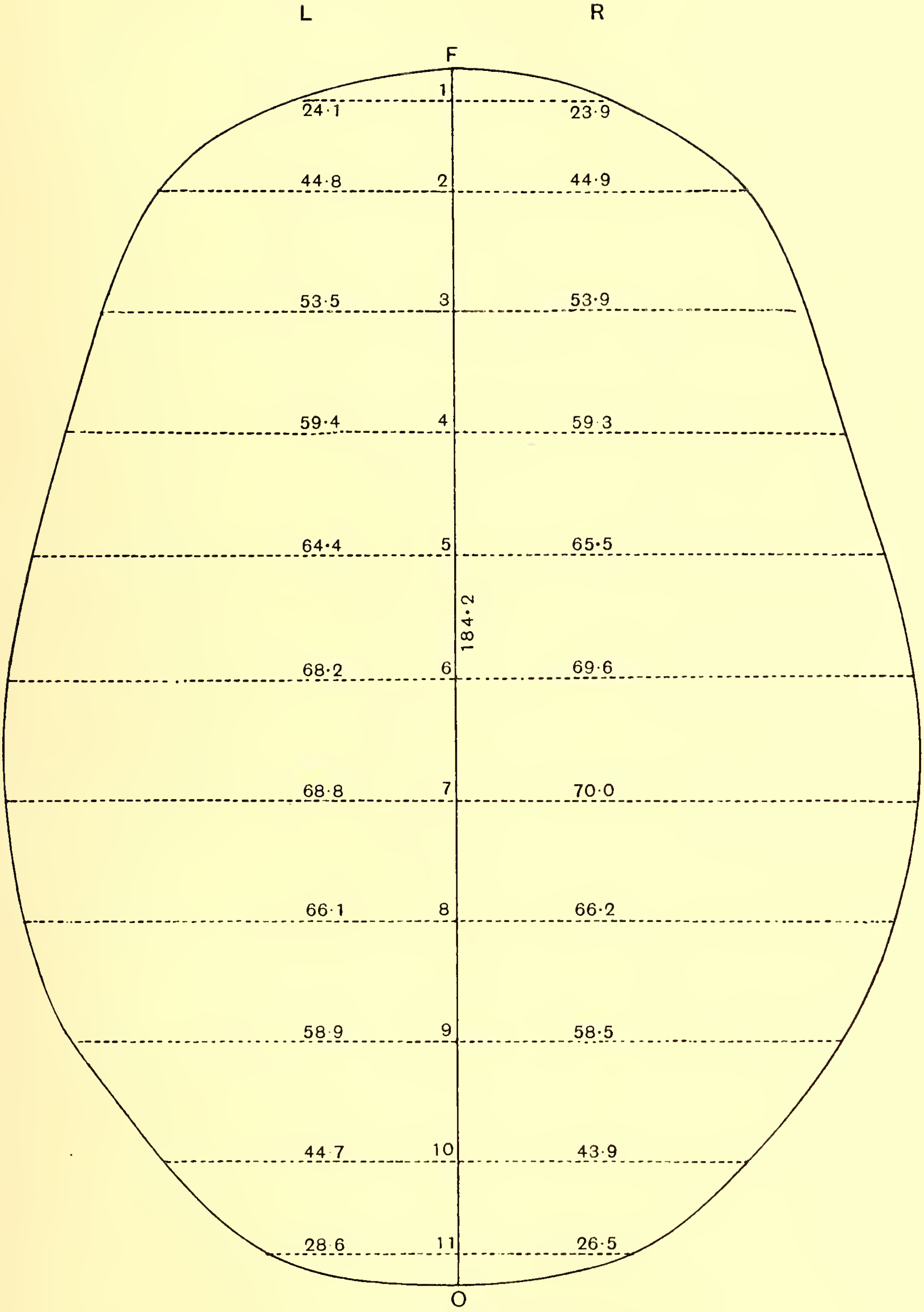


Fig. XXVI. Guanche Crania. Horizontal Contour.

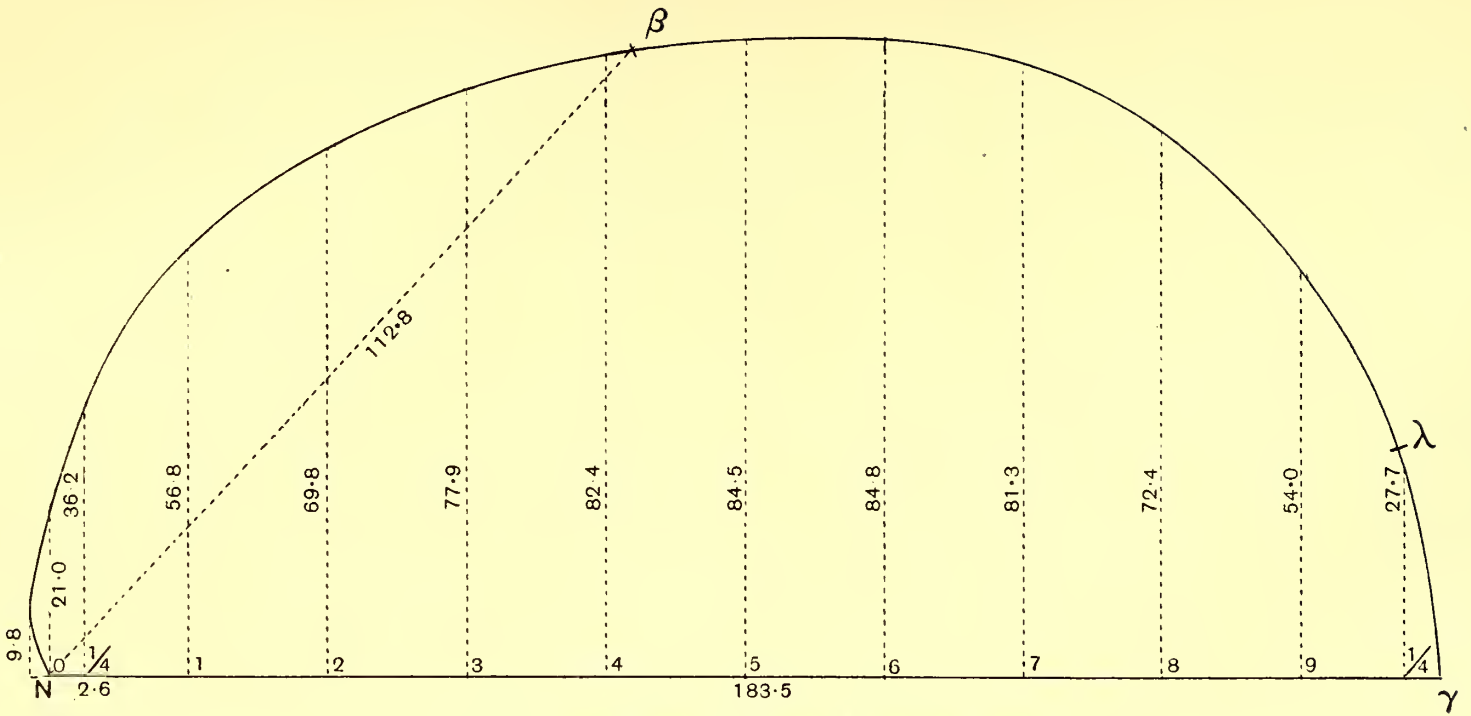


Fig. XXVII. English Crania, 17th Century. Sagittal Contour.

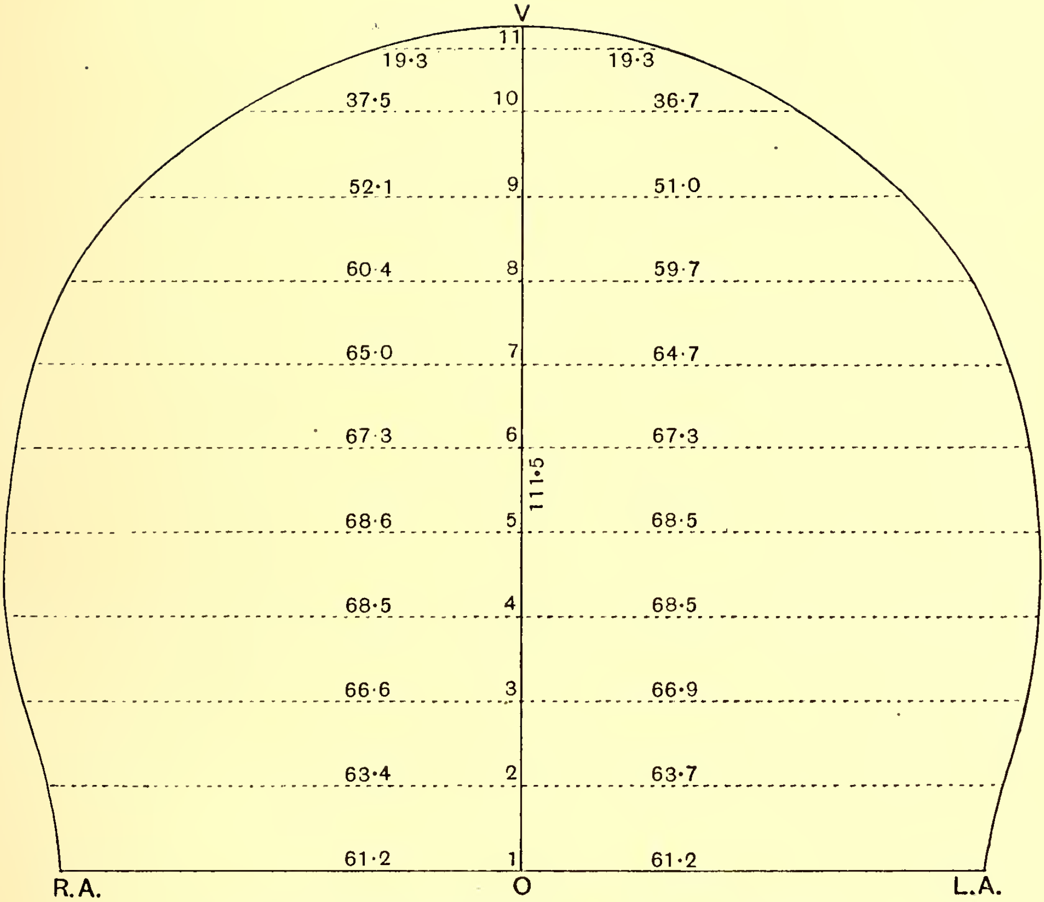


Fig. XXVIII. English Crania, 17th Century. Transverse Contour.

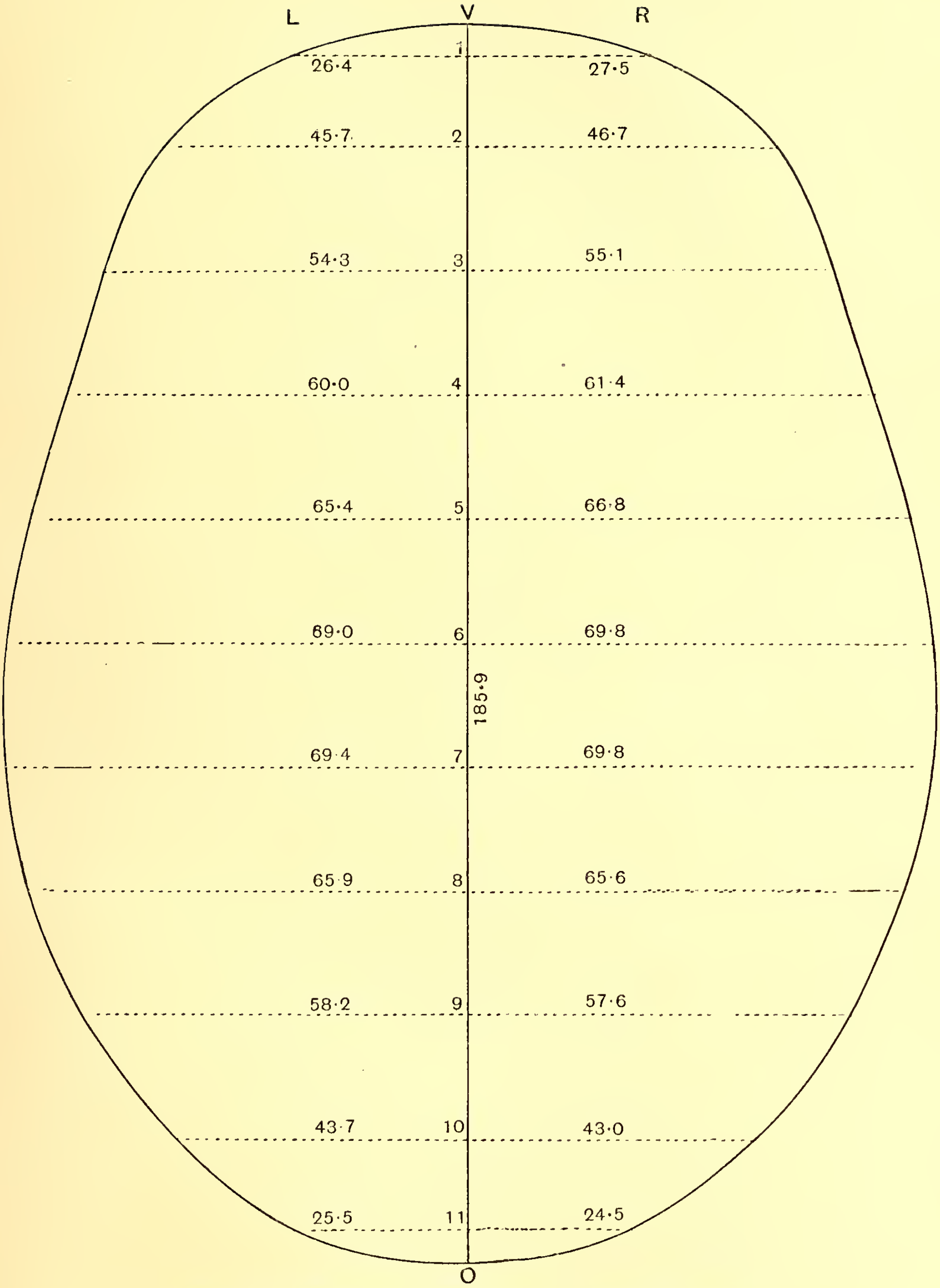


Fig. XXIX. English Crania, 17th Century. Horizontal Contour.

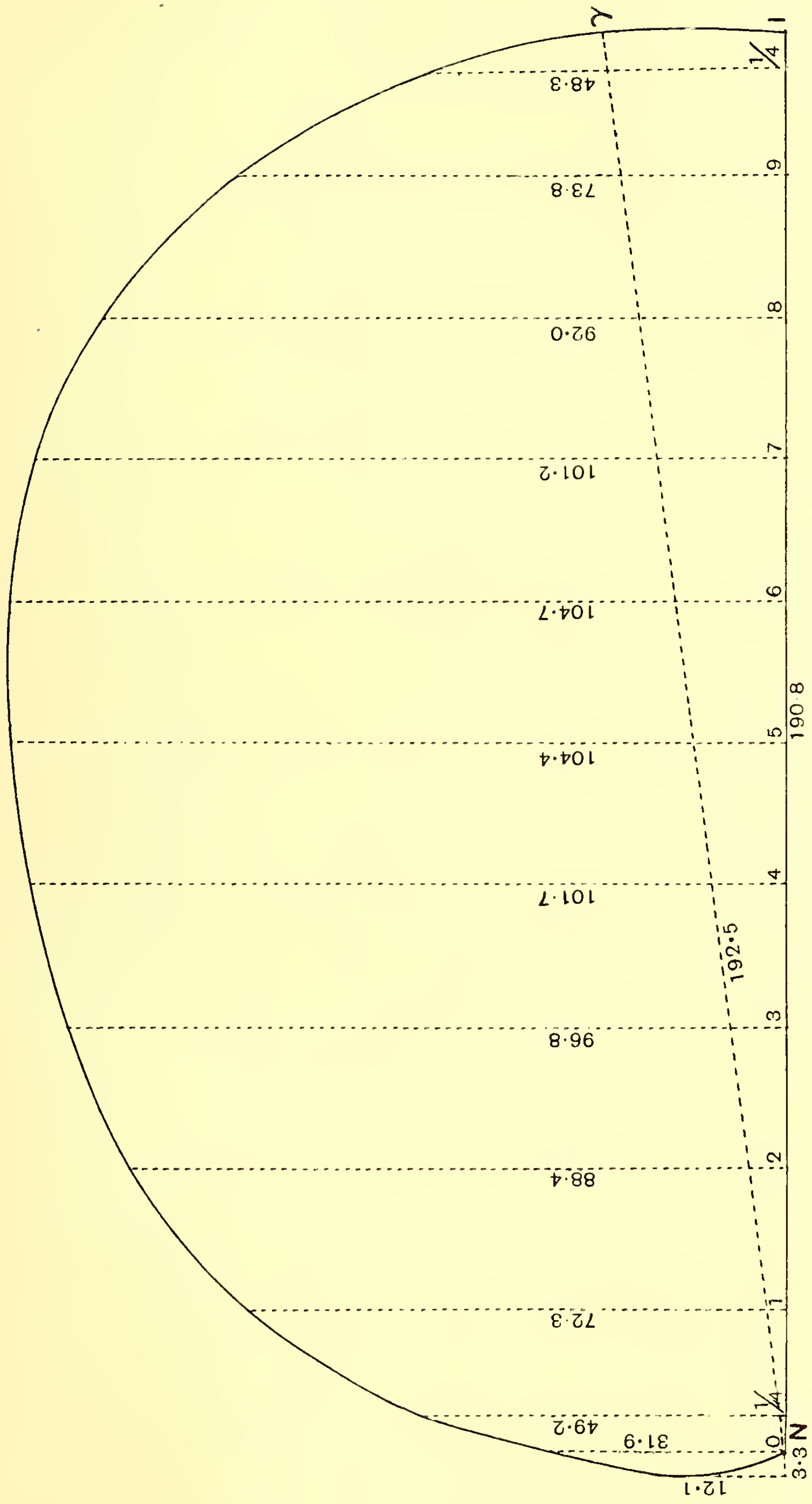


Fig. XXX. Living English Head. Sagittal Contour.

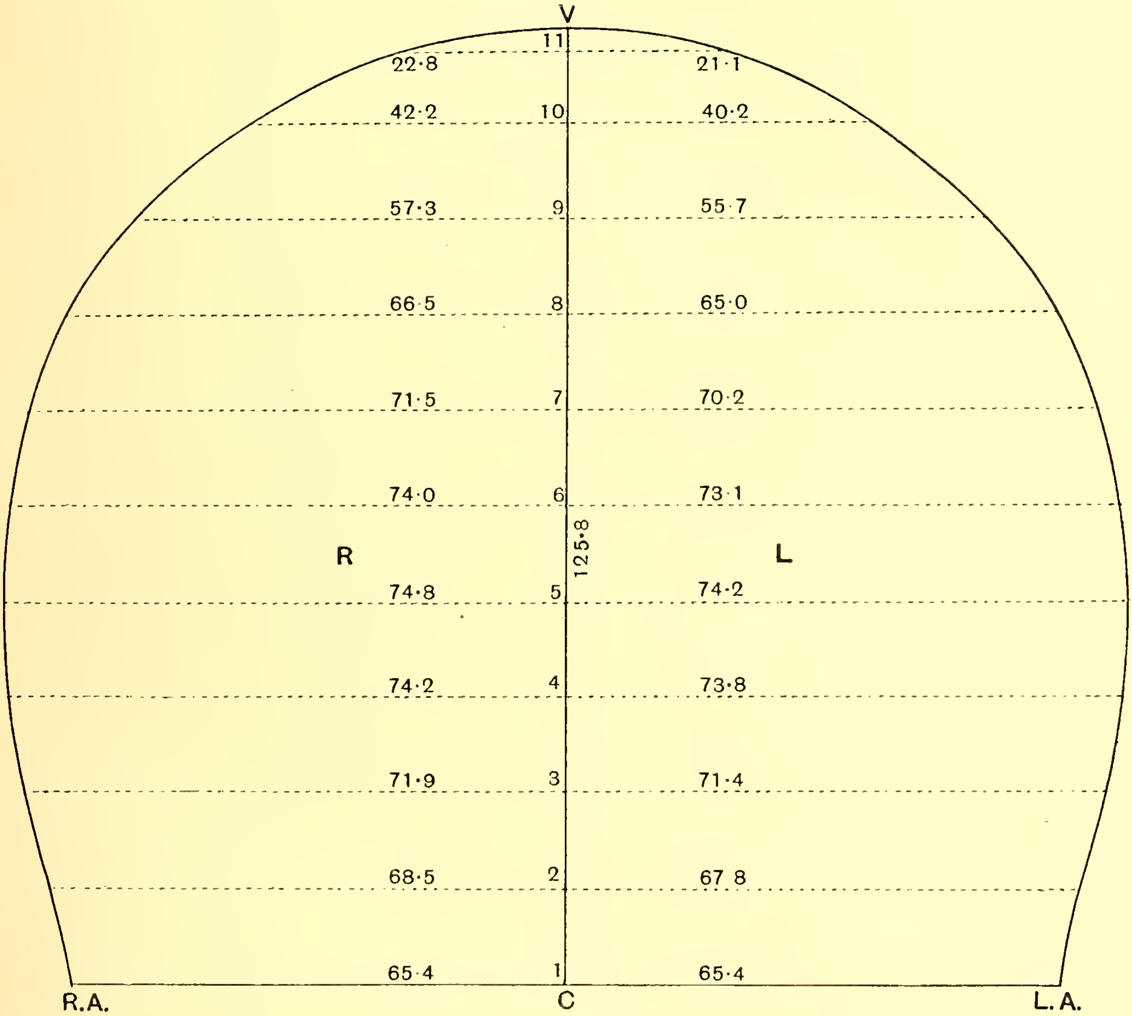


FIG. XXXI. Living English Head. Transverse Contour.

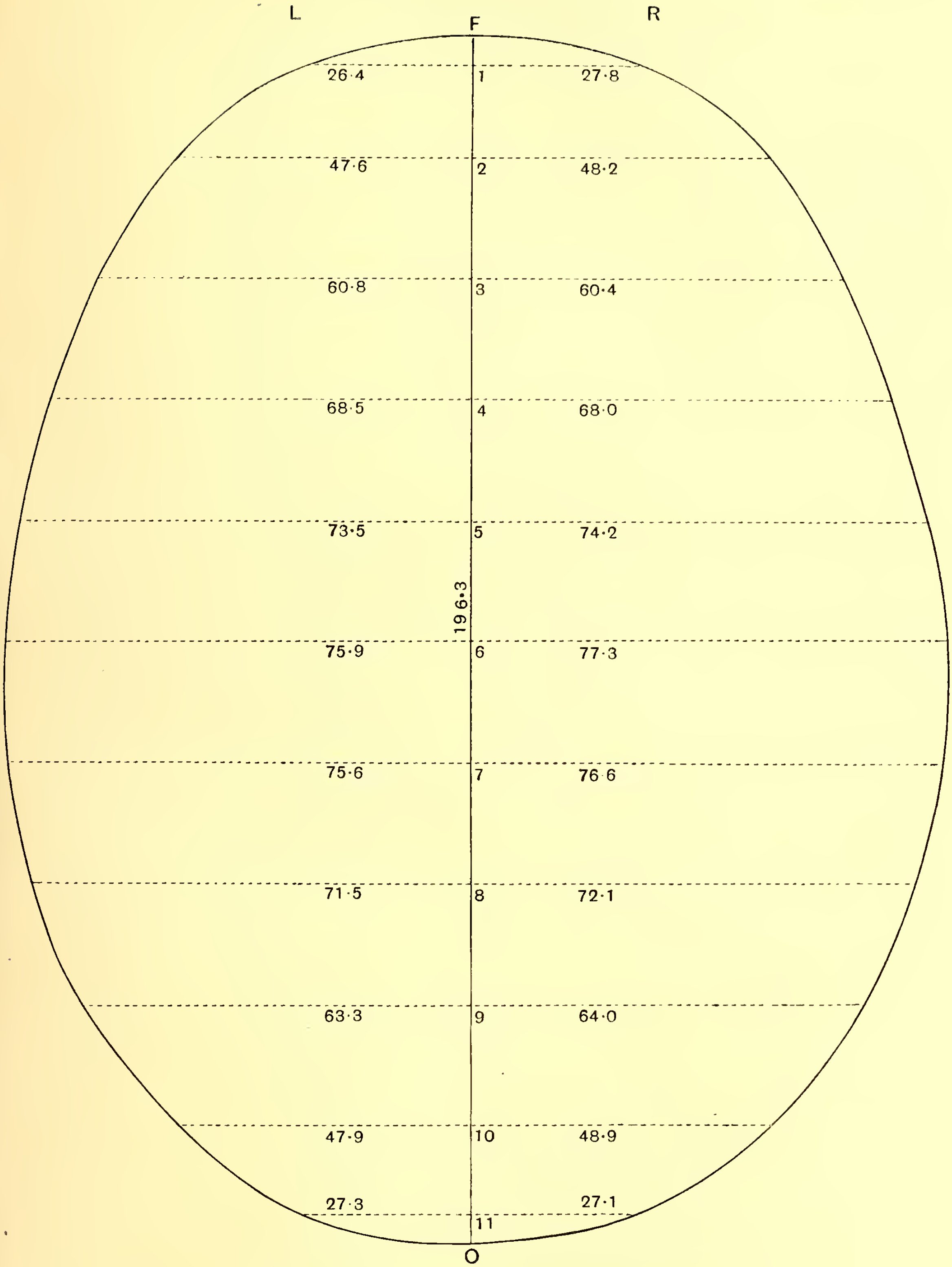


Fig. XXXII. Living English Head. Horizontal Contour.

THE OPSONIC INDEX—"MATHEMATICAL ERROR AND FUNCTIONAL ERROR."

By KARL PEARSON, F.R.S.

(1) In a recent paper on *Vaccine Therapy: its Administration, Value and Limitations**, Sir Almroth E. Wright has raised the interesting point of the relative influence of what he terms "mathematical error" and "functional error" in producing variation in the opsonic index. He very properly points out that the variation in the value of this index may depend upon two sources: (i) the functional error of the individual observer, and (ii) the error of method. By the last term I understand him to mean what we, as statisticians, should term the error of random sampling, *i.e.* the variation produced when random samples are taken from a population, the samples consisting of relatively small numbers selected without bias of any kind whatever. Sir Almroth admits that this last variation must be a subject for mathematical evaluation; at least, I so understand him. On the other hand the first variation, "functional error" as he terms it,

> is an error which attaches only to methods which involve a certain amount of skilful functioning. It attaches to the *operator*. It has a different value for every operator. It may in the case of one and the same operator vary from hour to hour with his physiological efficiency. Its value can be diminished by practice and attention. It cannot be evaluated by a mathematician. It can be pretty accurately gauged by the operator himself (p. 28).

Now it is difficult to see wherein the functional error of the vaccine therapy observer differs from that of the astronomical observer. His personal equation† also varies from hour to hour with his physiological efficiency, and it has a different value for every operator. What sort of reply would Gauss and Bessel have given to the astronomer who said that his functional error could not "be evaluated by a mathematician," but could be "pretty accurately gauged" by

---

* *Proceedings of the Royal Society of Medicine*, Vol. III. 1910. My references are to the pages of the offprint.

† That the "personal equation" is always in flux and is correlated with environment or "atmosphere" is well known: see my memoir *Phil. Trans.* Vol. 198, A, pp. 235—299.

26—2

himself? Have we not here an instance of how disastrous is the specialisation of modern science, which so completely prevents a distinguished specialist from knowing the history of branches of science outside his own little field? The modern theory of errors did not arise from "such an exercise as counting the number of pips on a series of cards," where "no skilful functioning is required," but from the needs of a science like vaccine therapy, where skilful functioning was essential.

The insistence of Sir Almroth Wright on the distinction between variations due to random samples and variations due to observational skill is perfectly legitimate and of vital importance, but his argument that the one and not the other is capable of exact treatment shows not only limitation of his knowledge of statistical methods, but has led him into the following statement which will form the text of the present paper:

> I have satisfied myself, and all my fellow-workers have satisfied themselves, and I am glad to say a very large and increasing number of bacteriological workers all over the world have satisfied themselves, that when the "functional error" has been reduced, as it can be by practice and patience, to small dimensions, and when, in connection with tubercle, the customary counts of 100 or more leucocytes are made, the "mathematical limit of error" of the opsonic index is such as need not seriously be taken into account. In view of this, I suggest that those critics who have put forward figures showing enormous working errors in opsonic estimations may have supplied the world data with regard to their own functional errors, instead of—as self-esteem assured them—data with regard to errors inherent in the opsonic method (p. 29).

"I have satisfied myself," writes Sir Almroth,—but the day of authority in any branch of science has gone by, and we ask legitimately and instinctively for the data from which this satisfaction was extracted.

It is surely a question of statistics and not of individual satisfaction. Possibly Sir Almroth Wright may be correct, possibly he may not be. *A priori* no statement can be made. What we need is a sufficiently long series to establish a definite result by one in whom "the 'functional error' has been reduced, as it can be by practice and patience, to small dimensions." A comparison of that series with those obtained by other observers—i.e. those critics whom Sir Almroth Wright suggests have enormous functional errors—would at once enable us to see the relative value of functional and sampling errors. No doubt Sir Almroth Wright has undertaken an investigation of this kind; but, until he has published it and submitted it to the test of trained statisticians, can he be surprised if they are not wholly convinced by the mere expression of his satisfaction that the difference of functional error is the source of those "enormous working errors" which so far appear in *all* the scientific *statistical* data available for the estimation of opsonic index variation?

(2) Sir Almroth Wright has taken up the ground that "the enormous working errors in the opsonic estimations" are due to functional error*. I take

---

* By the very use of the word "error" he introduces bias. The observations show large *variations*, the question before us is whether these are due to personal equation or to random sampling.

it, especially having regard to the other statements in his paper, that this functional error may be due primarily to two sources:

(*a*) the technique of preparing the slide for counting, (*b*) the actual method adopted in counting. We might hope to consider these facts in the following manner:

(i) The slide might be prepared by one of Sir Almroth Wright's own workers.

(α) The actual counting might be undertaken by one of his staff.

(β) The actual counting might be undertaken by somebody else.

(ii) The slide might be prepared by one of the critics suggested by Sir Almroth as having "enormous functional errors."

(α) The actual counting might be undertaken by the same critic.

(β) The actual counting might be undertaken by somebody else, *e.g.* an assistant of Sir Almroth.

It is clear that if we could have the results obtained in these four different ways, we should—with the assistance of a mathematical statistician—be able to throw much light on the problem of whether the variations of sampling or the variations of functioning are the more important factor in opsonic index variation.

Now unfortunately I am not in possession of data covering all the four cases cited above. But it is possible, from published work, to compare the results obtained when:

(i) (α) A slide was prepared and the bacilli counted by Dr Fleming of Sir Almroth Wright's own laboratory*.

(β) A slide was prepared in Sir Almroth Wright's own Laboratory, but counted by Dr Strangeways†.

(ii) A slide was prepared elsewhere and the count made by the operator, Dr White‡.

The whole material dealt with in these cases was for the tubercle bacillus.

(3) The first point I asked myself was this: Does the actual population on the slide differ substantially according to whether the slide has been prepared in Sir Almroth's Laboratory or not? I took therefore the data for Fleming's

---

* *Biometrika*, VI. p. 384, Slide *T. A.* I. Our results here and later differ slightly from Dr Greenwood's (*loc. cit.* pp. 385—6, and Graph XIV.) because, (i) we have taken the *first* thousand (not the whole 1100) of Fleming's returns and (ii) Dr Greenwood used Dr Fleming's means, which we have recalculated as we found them not without error.

† This is the only interpretation I can put on the footnotes to Strangeways' original paper (*Bulletin of the Committee for the Study of Special Diseases*, Vol. I. p. 133) and to Greenwood's paper, *A Statistical View of the Opsonic Index* (*Proceedings of the Royal Society of Medicine*, March 1909, p. 7 of *Offprint*). I am subject to correction, but I do not understand why there should be any secretiveness as to the origin of these slides.

‡ *Biometrika*, Vol. VII. p. 505.

Slide *T. A.* No. I. which gave rise to a count of 1000 and compared it with Greenwood and White's Slide B—selected purely at random because a 1000 count had been made from it. The results were as follows:

Number of Bacilli.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	Totals
Fleming ...	54	99	184	189	156	121	84	48	29	13	9	4	4	1	2	1	2	1000
Greenwood and White	71	131	197	162	137	111	58	47	32	22	13	11	2	2	2	1	1	1000

The actual constants of the distributions were as follows:

	Fleming	Greenwood and White
Mean	3·72	3·61
Mode	2·67	1·94
Standard Deviation	2·40	2·59
μ_2 ...	5·747	6·699
μ_3 ...	15·474	18·945
μ_4 ...	180·740	202·477
Theoretical Range	Infinite	46·3
β_1 ...	1·262	1·194
β_2 ...	5·472	4·512

$$\log y = 27{\cdot}785{,}8387 + 21{\cdot}6550 \log (x - 5{\cdot}2705) - 40{\cdot}2546 \log x$$

Origin at $-8{\cdot}7417$ bacilli.

$$y = 179{\cdot}64 \left(1 + \frac{x}{2{\cdot}4541}\right)^{1{\cdot}2155} \left(1 - \frac{x}{43{\cdot}8558}\right)^{21{\cdot}7136}.$$

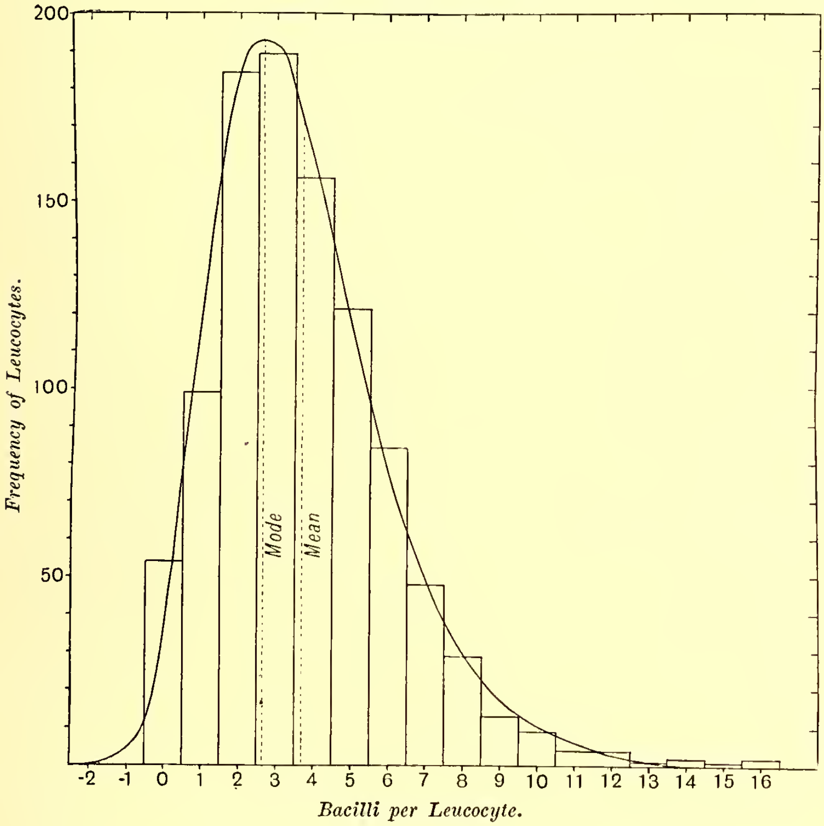
Origin at mode.

The graphs of these results are given in Diagrams I and II. It seems wholly unreasonable, either from mere inspection of the graphs or from any consideration of the constants of the distributions as given above, to suppose one of these frequencies to be due to "functional errors" and the other to be something quite different. Either both show "enormous functional errors," or else both exhibit the variations which occur in a population of this kind whether it be prepared in or outside Sir Almroth's Laboratory. The standard deviation measure of variability is slightly greater for Greenwood and White than for Fleming, 2·59 as against 2·40; but this difference is not beyond what we find in two slides prepared by the same individual; and if it were needful to account for it—which it is not—all that has to be noted would be that our observers are dealing with the serums of different individuals.

On the basis of these samples it seems to me that a population prepared and counted in Sir Almroth Wright's own laboratory gives sensibly the same range of variation as a population prepared and counted elsewhere, and that until this is demonstrated to be erroneous, it is wide of the mark to talk about critics showing enormous working errors in opsonic estimations and to assert that such are really only measures of the critics' own "functional errors."

If a population of a 1000 leucocytes, prepared in two different places and counted by those who prepared them, show practically the same range of variation, then it must be clear that if random samples of 50 or 100 were taken out of these 1000, we should reach the same distribution of possible means of bacilli for both of these populations and that accordingly the opsonic index as determined by the individual as against himself would show just as wide variations as those demonstrated in recent papers in *Biometrika** for Greenwood and White's material. There is only one flaw in this argument—possibly neither Fleming nor Greenwood and White would in an actual count of 50 or 100 really take random samples from their 1000 counts—this is a point to which I shall return later on in this paper.

DIAGRAM I. (Fleming.)



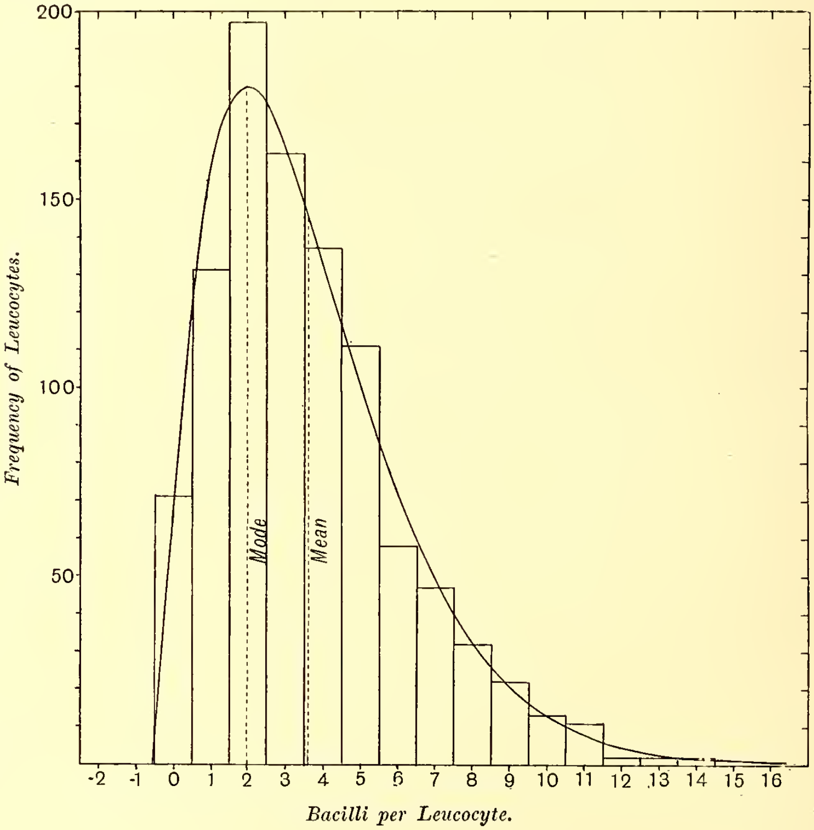
(4) In order to confirm the result of the previous section, namely that whether the slide was or was not prepared in Sir Almroth Wright's Laboratory, it did not affect the variation in the resulting population, I took a slide prepared

---

* See Vol. VII. pp. 505—531, and pp. 531—541.

in that laboratory and counted by Strangeways. In the case of Slide IV he had counted 2000 single cells; this gave me 40 samples of 50*. He had also counted 3000 clumped and single cells from the same slide, thus giving me 60 samples of 50 by a different method of counting. In the one case I had 1600, in the other 3600 possible opsonic indices of the individual patient tested against himself. I was thus able to appreciate the sort of effect that differences of functioning in counting produced in the variation of the opsonic index. Against these two sets of Strangeways' results, I was able to place Greenwood and White's 400 samples of 50 counts. The latter has already been considered by me in a previous paper†. Strangeways' two series were reduced in precisely the same manner as that indicated in the paper just referred to. The following are the constants for the three distributions, and the resulting three curves are plotted, reduced to the scale of 1000 indices, in Diagram III.

Diagram II. (Greenwood and White.)

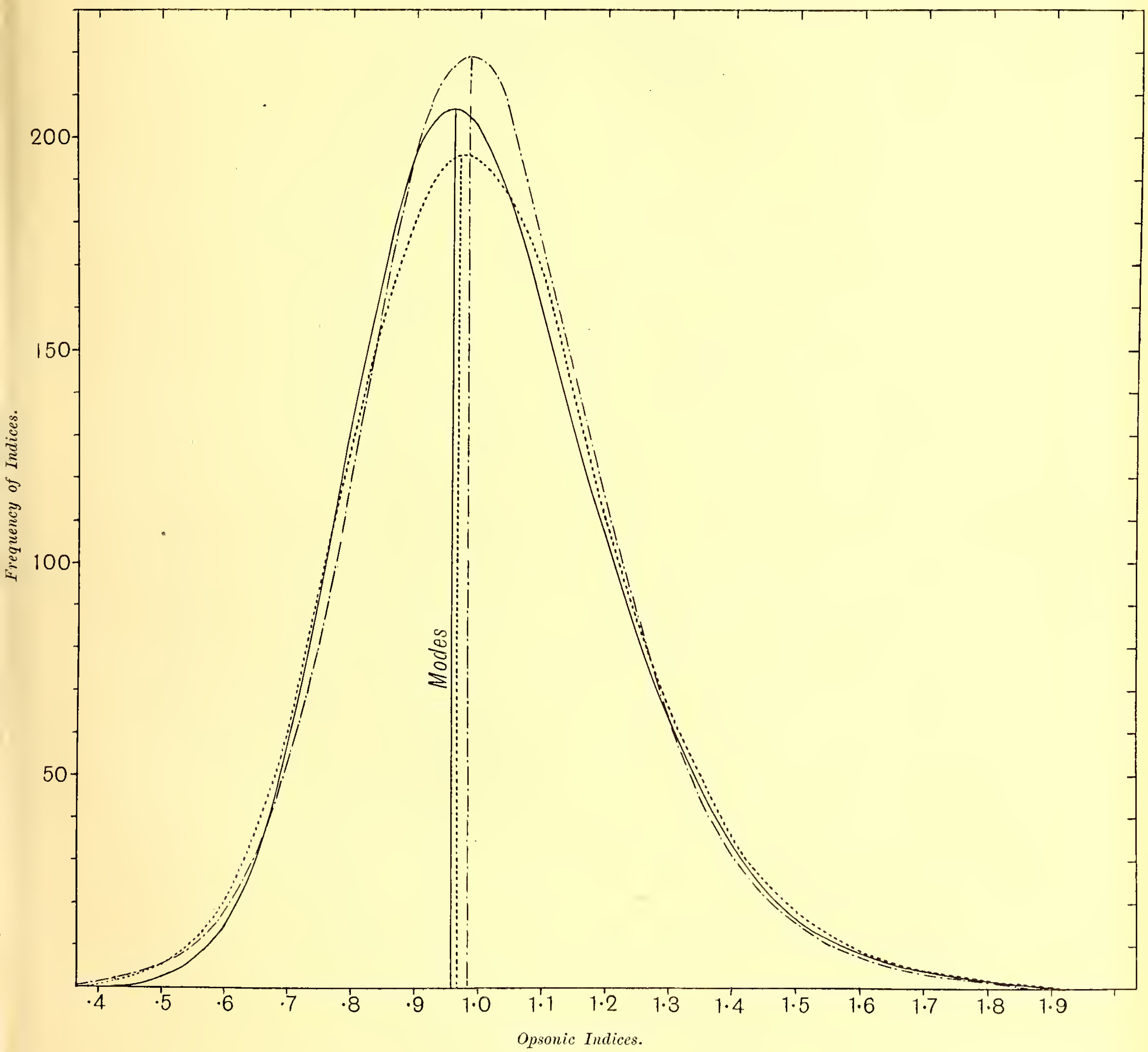


---

* Each sample of 50 was taken in the precise order of counting.

† See *Biometrika*, Vol. vii. pp. 536, 537.

DIAGRAM III.



———	Greenwood and White.
············	Strangeways' Clumped and Single Cells.
—·—·—	Strangeways' Single Cells.

Distribution of Opsonic Indices. Samples of 50.

	I. Strangeways' 2000 single cells	II. Strangeways' 3000 clumped and single cells	III. Greenwood and White's 20,000 cells
Mean Index ...	1·0202	1·0221	1·0205
Mode	·9852	·9734	·9611
Standard Deviation	·2083	·2181	·2072
μ_3 ...	·00510	·00567	·00619
μ_4 ...	·00906	·00892	·00757
β_1 ...	·3184	·2985	·4837
β_2 ...	4·8122	3·9401	4·1022
Origin at Index ...	·8481	·6002	·3409

The equations to the curves, reduced to a total of 1000, are

I. $$y = 156{\cdot}14 \left(1 + \frac{x^2}{{\cdot}2678}\right)^{-4{\cdot}9268} e^{2{\cdot}6106 \tan^{-1}(x/{\cdot}5175)}.$$

II. $$y = 20{\cdot}88 \left(1 + \frac{x^2}{{\cdot}5418}\right)^{-9{\cdot}0480} e^{9{\cdot}1837 \tan^{-1}(x/{\cdot}7360)}.$$

III. $$y = {\cdot}1061 \left(1 + \frac{x^2}{{\cdot}3911}\right)^{-11{\cdot}4301} e^{22{\cdot}6704 \tan^{-1}(x/{\cdot}6254)}.$$

Now it will be clear from an examination of this table of constants that there is no significant difference in the variation of these three series; no one of them can be described as having "enormous working errors" compared with another; they show, for arguments about the range of variation of the opsonic index, practically equivalent results; one of Strangeways' series lying on the one side, and the other upon the other side of Greenwood and White's series. The only conclusion that anyone examining Diagram III, or the above analytical constants of the curves, can reach, is that: If the countings are done by different microscopists, not members of Sir Almroth Wright's Laboratory, the variation in the opsonic index of an individual reckoned against himself shows the same magnitude of range, and this whether the slide was prepared or not in Sir Almroth's Laboratory.

We have now, as far as our data extend, reached two or three interesting results:

(i) The preparation of a slide inside or outside Sir Almroth Wright's laboratory makes very little difference in the population presented by it.

(ii) In counting a large population, say 1000 to 2000 leucocytes, no substantial difference arises according to whether the counter be a member or not of Sir Almroth Wright's staff*.

(iii) The counting or not of clumped cells, when a large population, 2000 to 3000 cells, is being dealt with, has little effect on the range of opsonic index which results when an individual is tested against himself. In other words a

---

* The assumption of course made is that the counter is really a trained microscopist and does his work conscientiously.

point, fertile of differences in technique, produces practically little differentiation in the resulting variation of the index.

It seems to me—as a quite unprejudiced outsider—that a claim to special monopoly of technique in placing the film on the slide or in preparing it in any way needs further justification. In taking a large population, the "enormous working errors in opsonic estimation" appear equally in slides prepared and counted in Sir Almroth Wright's Laboratory, in slides prepared there and counted elsewhere, and in slides both prepared and counted elsewhere. Hence these "enormous working errors" are either variations of random sampling, or are peculiar to the method of counting of both internal and external microscopists[*].

(5) To the purely theoretical statistician the subject might now, for the time being, appear exhausted. If the slide population of leucocytes presents in the above three cases the same range of bacilli-frequency, then if we are seeking the distribution of the opsonic index of the individual as against himself on the basis of a count of 50 or 100 leucocytes, all we have to do is to follow a purely mathematical process, *i.e.* to determine every mean of possible groups of the given number and divide by every other mean. We shall thus reach the final distribution of possible opsonic indices of the individual tested against himself, each index appearing with the frequency of a random distribution.

Now given any frequency of a variate, the frequency of a random sample of the means of 50 or 100 of these variate values is perfectly well known. If β_1 and β_2 be the fundamental frequency-constants of the primitive frequency then

$$B_1 = \beta_1/n \text{ and } B_2 - 3 = (\beta_2 - 3)/n$$

are the fundamental frequency constants[†] of the derived frequency of the means of n variates at a time, such means being selected out of this primitive population by a purely random process. The population of means should thus tend to become rapidly Gaussian. Now Dr Greenwood has shown that the means of 25, 50, or 100 leucocyte results—*as obtained in a particular manner from the slide population*—do not follow this rule, but give far more skew distributions[‡]. This point is one of extraordinary importance, for it suggests, if it may not yet be said to demonstrate, that the 50 or 100 leucocytes counted from the population on the slide is not a random sample of the contents of the slide as more nearly measured when 1000 to 3000 leucocytes are counted. "Student" in a most interesting note[§] on Greenwood and White's paper has suggested that the result is due to "homotyposis," *i.e.* to a certain degree of likeness in consecutive counts.

---

[*] In any further discussion of this subject, it is essentially important that the slides and the counts used should be those made by both parties to the discussion, before the present issues as to "functional errors" and "errors of method" were raised.

[†] The mean of the means is the mean of the primitive population and the standard deviation, the only other constant needed, $=\sigma/\sqrt{n}$.

[‡] *Biometrika*, Vol. VI. p. 400 and Graph 16.

[§] *Biometrika*, Vol. VII. p. 210.

The current theory of random sampling supposes, however, that in actual counting we draw from the leucocytes on the slide our sample of 50 or 100 precisely as we should draw balls from a bag. Now this is not exactly the method which can be followed in a count of thousands. A mechanical stage has to be used in order to prevent any danger of counting the same leucocyte several times. Both Strangeways and White used such a stage. I am not aware whether or no Dr Fleming did. But in using such a stage we count along a line, drawn as it were across the slide, and are liable in doing this to traverse somewhat heterogeneous areas. On the other hand, if the 50 or 100 leucocytes for the mean count are taken at random from the slide, we may tend to even out these heterogeneous areas and so obtain less variation in the distribution of means, while the frequency of bacilli in the leucocyte population remains absolutely the same.

To test possibilities in this direction I took (i) the 1000 count of Dr Fleming, which provided 20 means of 50 and therefore 19 pairs of successive means; (ii) Slide IV of Strangeways' 2000 count, giving me 39 pairs of successive means; and finally (iii) Slide B of Greenwood and White's big 20,000 count giving 19 pairs of successive means. Of course 19 and 39 are very small numbers of pairs to base any really definite conclusions on, but the results may show whether there is very high correlation between successive pairs*. The following are the numerical values obtained:

	Fleming	Strangeways		Greenwood and White
		Without clumps (40)	With clumps (60)	
Mean Number of bacilli	3·714	1·696	1·723	3·605
Standard Deviation ...	·2779	·2560	·2635	·3086
Correlation of successive counts of 50	− ·064 ± ·154	+ ·097 ± ·107	+ ·013 ± ·088	− ·319 ± ·139

Clearly the correlation between successive means of both Fleming and Strangeways is insignificant. Greenwood and White obtain a larger result, but not of any great importance considering its probable error. It is further *negative*, or the successive means tend in opposite directions on this slide. It seemed worth while testing whether the negative correlation obtained from one slide (B) of Greenwood and White's big count would be maintained if the whole series of these authors' data was considered. We were not able to obtain 400 successive means, as this would have involved passing from one slide to a second. A first sample of 193 and then all available 386 successive means were taken; there resulted:

	First sample of 193	Whole material 386
Mean Number of Bacilli	3·679	3·684
Standard Deviation	·2644	·2575
Correlation of successive counts of 50	+ ·259 ± ·045	+ ·245 ± ·032

---

* Dr Strangeways writes: "The cells were counted in successive rows and so far as it was humanly possible every cell was counted and no cell was counted twice...every set of 25 came from the same part of the slide and from a very small area of it as the leucocytes were very plentiful." Letter to Dr Greenwood. 13/2/11.

Now it will be obvious at once from these results that the negative correlation between successive means for the one slide is not of essential significance; there is, as we had anticipated, a slight positive correlation, which is not again, however, of very real importance. Whether Dr Fleming would also have reached this, had he counted 400 means of 50, instead of 20 such means, it is impossible to say. But what is quite obvious is that while Greenwood and White's count extended to more than 20 slides and Fleming's to only one, yet the variability of their 400 means is not greater but rather less than that of Fleming's 20 means. But it is precisely on this variation of means that the variability of the opsonic index as found for one individual tested against himself depends. It is therefore clear that if these "enormous working errors" are due to the "functional error" of observers, and not to the variations of random sampling, they occur equally easily inside and outside Sir Almroth Wright's Laboratory. But if three separate sets of observers working on different material by different methods[*] reach practically like variations in the values of the means, have we not reasonable ground for assuming that we are not dealing with "enormous functional errors," but that the large working errors, to which attention has been drawn by those trained in statistical theory, are for the most part variations of random sampling, *i.e.* what Sir Almroth Wright terms "errors of method," errors which he lightly brushes on one side,—without providing the least quantitative evidence—merely saying that he has satisfied himself that "the 'mathematical limit of error' of the opsonic index is such as need not seriously be taken into account." He has yet to satisfy the biometrician that his "fellow-workers," the "increasing number of bacteriological workers all over the world" and he himself have the knowledge requisite to test critically this insignificance of the "mathematical limit of error[†]."

(6) Taking the actual means of sets of 50 as given in the footnote for Fleming and for Greenwood and White, we have formed the 380 actual opsonic indices which would arise in each series from these means tested against themselves: see Table III. It will be seen from an examination of Diagrams IV and V that while Greenwood and White's Slide B distribution is somewhat more variable than Fleming's[‡], there is absolutely nothing which would enable one to make a substantial differentiation on the ground of the presence and absence of "enormous working errors."

---

[*] In one, if not in two of the series attempt was made to count the bacilli for *every* leucocyte.

[†] As we have seen the variability on Greenwood and White's Slide *B* is in excess of their average variability for the whole series, i.e. ·31 against ·25; Fleming's variability is ·28. It is interesting to look at the whole series of 20 means of 50 due to the two investigations:

Fleming	—	3·16,	3·24,	3·28,	—	3·46,	3·52,	3·56,	3·60
Greenwood and White	3·02,	3·16,	3·22,	3·26,	3·32,	3·40,	3·40,	3·46,	3·54

Fleming	3·62,	3·78,	3·82,	3·84,	3·84,	3·84,	3·88,	3·90,	3·90,	3·98,	4·02,	4·06,	4·14
Greenwood and White	3·60,	—	3·62,	3·64,	3·70,	3·82,	3·84,	—	3·92,	4·00,	4·02,	4·02,	4·14

Thus Greenwood and White have only *one* mean outside Fleming's range and the parallelism is striking!

[‡] We have already seen that Slide *B* of Greenwood and White was 20 % more variable than their average data; accordingly others of their slides *reverse* their relative variability as compared with Fleming's Slide *T. A.* I.

Diagram IV. (Fleming, Independent Means, 380 Indices.)

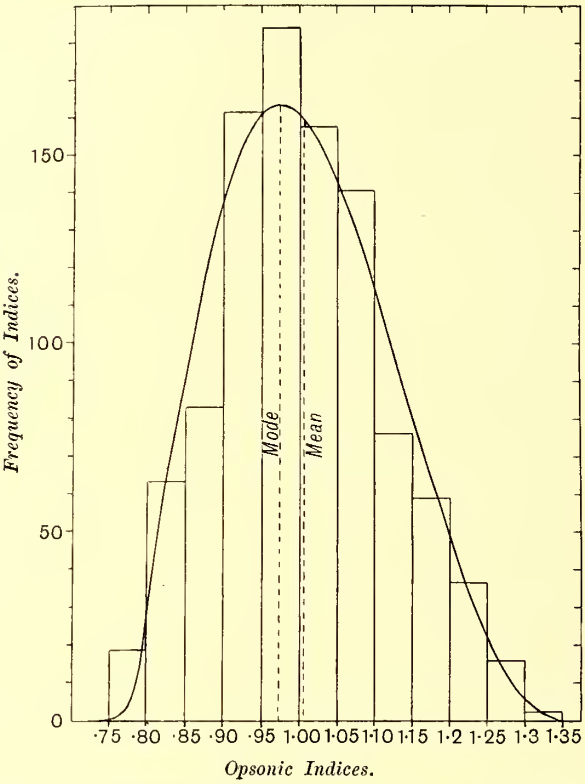
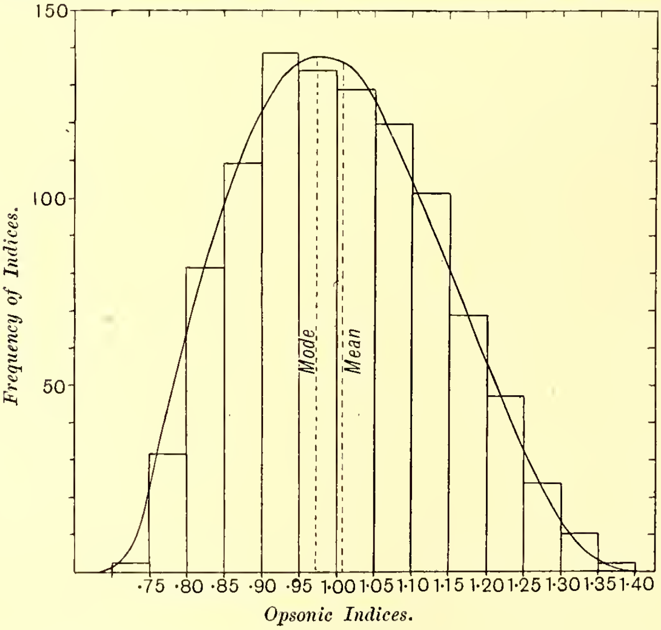


Diagram V. (Greenwood and White, Independent Means, 380 Indices.)



The analytical constants are:

	Fleming	Greenwood and White
Mean	1·0056	1·0089
Mode	·9744	·9743
Standard Deviation	·1104	·1291
μ_3 ...	·000377	·000570
μ_4 ...	·000361	·000673
β_1 ...	·0787	·0700
β_2 ...	2·4338	2·4200

Equation to the curve:

$$y = 163{\cdot}686\left(1+\frac{x}{4{\cdot}0909}\right)^{1{\cdot}3562}\left(1-\frac{x}{7{\cdot}7975}\right)^{2{\cdot}5850}, \qquad y = 137{\cdot}360\left(1+\frac{x}{5{\cdot}3420}\right)^{1{\cdot}5092}\left(1-\frac{x}{8{\cdot}5060}\right)^{2{\cdot}4031}.$$

Unit of x, ·05 of opsonic index and a total frequency of 380.	Unit of x, ·05 of opsonic index and a total frequency of 380.

In ascertaining the indices here from the 20 means of Table V (see p. 223), no mean has been used *with itself*, as the indices are supposed found from independent sets of 50.

(7) I have now reached what I consider the most difficult point in the matter. I have shown:

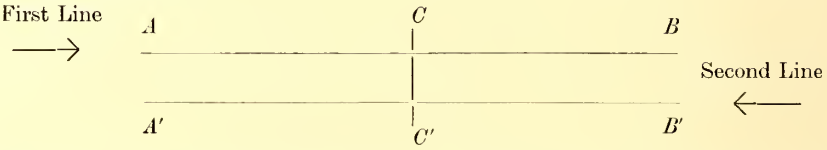
(i) that the frequency distribution of the bacilli per leucocyte is the same for Fleming and for Greenwood and White, if we take populations of 1000 or more.

(ii) that the 20 *independent* means of Fleming and the 20 of Greenwood and White, both from single slides, show also the like degree of variation.

(iii) that the possible opsonic indices deducible from these means for the two investigations do not differ more than is compatible with a difference of material, or (as shown by Greenwood and White's long series) by the difference of two slides from the *same* material.

But notwithstanding these arguments, which would lead to actually the same ranges of opsonic indices, if we take mathematically at random 50 leucocytes from those counted by either of these investigators, there is a very curious and marked difference between their method of counting or recording the bacilli per leucocyte. But the remarkable point about the matter is that Strangeways is in complete practical agreement with White in the results of his counting: see my Diagram III. Now Strangeways and White counted, using a mechanical stage, every available leucocyte as they worked across the slide and recorded them in succession. It is difficult therefore, since Strangeways worked on a slide prepared in Wright's Laboratory, to believe that the difference arises from technique in the preparation of the slide. It must lie in the method of counting.

The difference between the records of Fleming and White is of the following character: While the small numbers and large numbers of bacilli per leucocyte both occur with much the same frequency in the case of the two investigators (see Diagrams I and II), and the independent means have much the same range (see footnote †, p. 213), yet for these records the small and large numbers of bacilli much more closely alternate in Fleming's counts than in Strangeways' or White's returns. If any slide does not really contain a "random distribution" of leucocytes, but is "patchy," then it is conceivable that working with a mechanical stage and crossing the slide along a line, we might in counting our 50 leucocytes cover patches of various degrees of density, and so reach a mean, subject to much the same variation of bacilli, as if we drew from a bag the leucocytes at random; but if we worked back for the next 50 across the slide on a parallel line, thus:



the means of 50 leucocytes along AB or $B'A'$ may not show the same variation as if we worked from CB for 25 and then from $B'C'$ for the second 25 of the 50. For in the latter case the leucocytes are chosen from a more concentrated portion of the slide. This illustration is of course *purely arbitrary*, but it may indicate how heterogeneity on the slide may produce in the frequency of means obtained by mechanical stage counting a differentiation from what is to be expected from pure random sampling.

To test the methods of counting I took 400 means from Fleming's record, by starting with one leucocyte less at the beginning and one leucocyte more at the end of the 50 sample each time. It soon became obvious that the non-independent means thus found gave very different ranges of variation in the Fleming and the White series. Both are fairly irregular (see Diagrams VI and VII), but the Greenwood-White series is 30 °/₀ more variable than the Fleming. The analytical constants of the two distributions are as follows:

	Fleming	Greenwood and White
Mean	3·723	3·609
Mode	3·758	3·718
Standard Deviation ...	·2609	·3394
β_1	·0461	·1957
β_2	2·763	2·836

$$y = 58·93\left(1 + \frac{x}{14·3057}\right)^{9·3528}\left(1 - \frac{x}{8·3288}\right)^{5·4454}, \qquad y = 45·48\left(1 + \frac{x}{18·4572}\right)^{6·4159}\left(1 - \frac{x}{6·6997}\right)^{2·3288}.$$

The test for goodness of fit gave $P = ·09$ for Fleming and $P = ·14$ for Greenwood and White, no great difference*, but what there is lies to the advantage of Greenwood and White.

---

[*] For the bacilli per leucocyte (see Diagrams I and II) $P = ·36$ for Fleming, and ·33 for Greenwood and White, i.e. both have quite reasonable and practically equal goodness of fit.

This difference of distribution between the non-independent and independent means is a noteworthy one, and I am not able fully to account for it. The determination of the opsonic index by wholly different series of leucocytes shows much the same results whether we use a slide and count both due to Sir Almroth Wright's Laboratory, or a slide and count both prepared outside it. But the actual record of the counts differs in this respect—that the large and small numbers of leucocytes alternate more completely in Fleming's than in Greenwood

DIAGRAM VI. (Fleming, 400 dependent Samples of 50.)

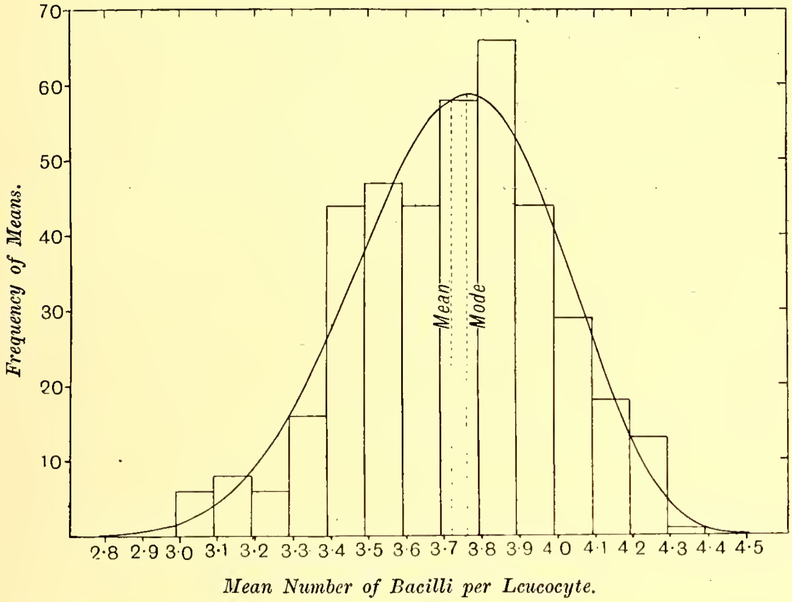
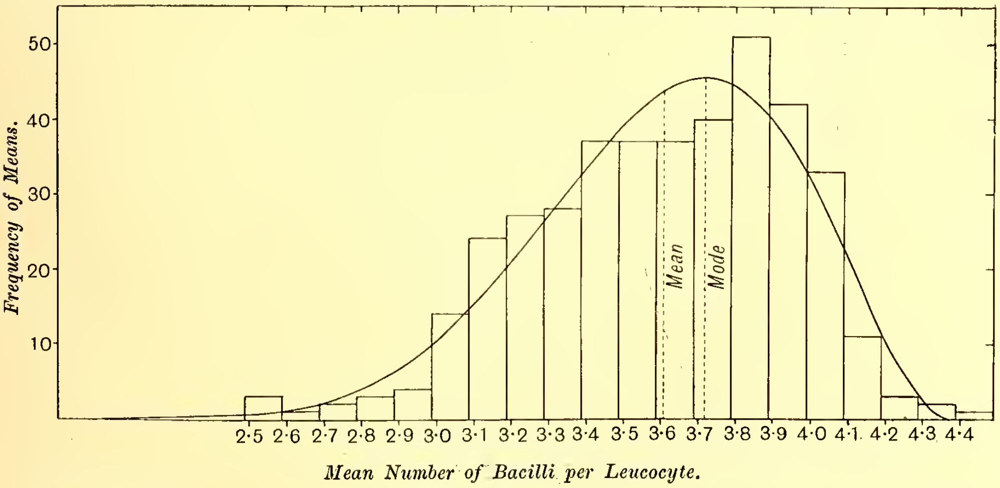
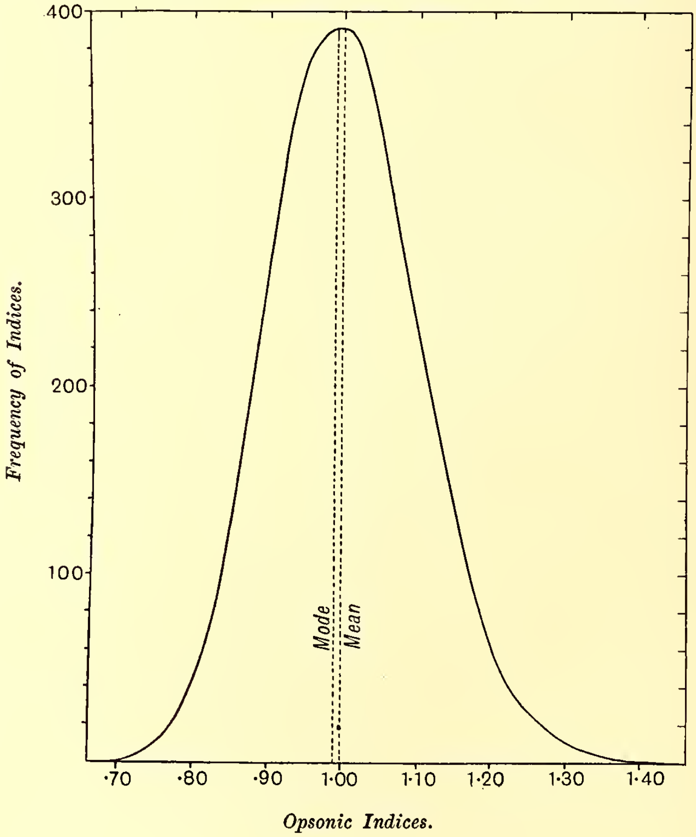


DIAGRAM VII. (Greenwood and White, 400 dependent Samples of 50.)



and White's record. How this result is reached I am unable to say. It cannot apparently be due to a more thorough mixture on the slide, because Strangeways' working with a Wright-Laboratory slide and using practically the same method of counting get variations close to Greenwood and White. It would appear to me that Fleming has, even when counting 1000 leucocytes, a method of more evenly distributing his large and small number of bacilli per leucocyte within his 50. How he manages this I am unable to say; it might possibly be achieved by picking each 50 leucocytes at random from the slide, but in this case it is difficult to understand how, if he does not use a mechanical stage, recounting the same leucocyte is avoided. But, whatever be the nature of the recording, this end is reached, that the mean number of bacilli in 50 leucocytes *starting anywhere in the record* is less variable than Greenwood and White's result. An appreciation of the extent of this lessened variation may be reached by comparing the curve for opsonic indices deduced from 400 non-independent means from Fleming's counts on Slide *T. A.* I. with that found for 400 on Greenwood and White's Slide B: see Diagrams VIII and IX.

DIAGRAM VIII. Opsonic Indices from dependent means. (Fleming.)



The analytical constants are:

	Fleming	Greenwood and White
Mean	1·0054	1·0096
Mode	·9901	·9830
Standard Deviation	·1020	·1401
μ_3 ...	·00032	·00130
μ_4 ...	·00034	·00137
β_1 ...	·0910	·2238
β_2 ...	3·1481	3·5590

Equation to curve*:

$$\log y = 529·5950139 + 58·4127 \log (x - 6·7004) - 594·7245 \log x$$

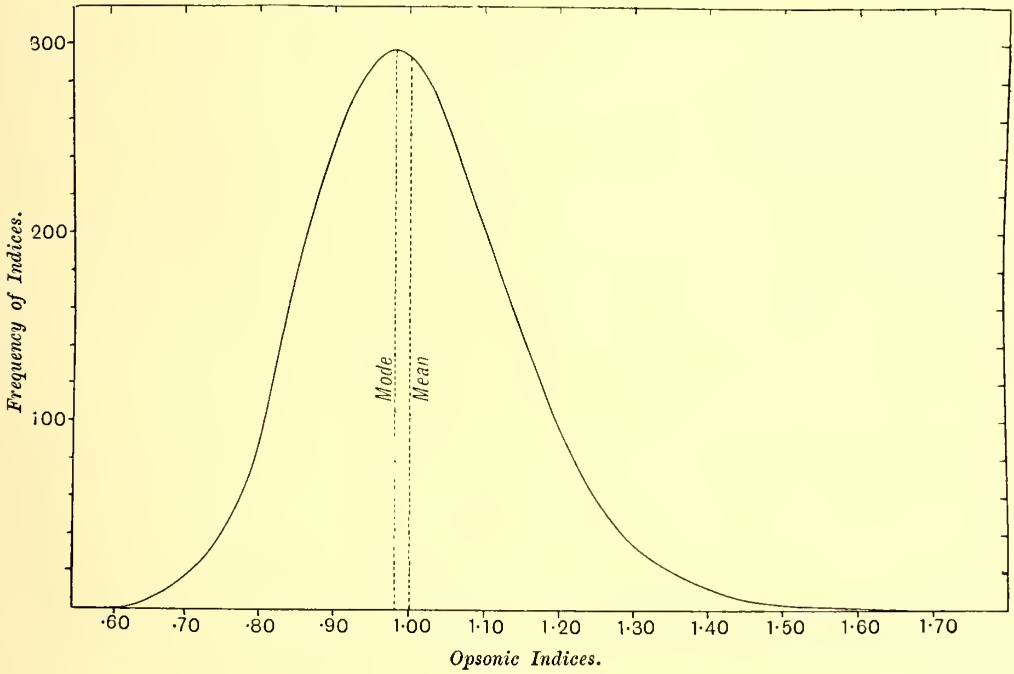
Mode at $x = 7·4302$, Mean at 7·446
and unit of x = unit of opsonic index.

$$y = ·15614 \left(1 + \frac{x^2}{·3592}\right)^{-16·6830} e^{25·4589 \tan^{-1}(x/·5993)}.$$

Origin at Index ·5232
and unit of x = unit of opsonic index.

It will be seen that Greenwood and White's distribution, while sensibly less variable than their "independent means," is more variable than Fleming's†. It will thus I think be clear that while Fleming reaches precisely as great a scatter

DIAGRAM IX. Opsonic Indices from dependent means. (Greenwood.)



---

* Reduced to 1000 total.

† He would be a courageous critic, who asserted even here that Diagram IX differs from Diagram VIII by the existence of "enormous working errors," to which the variation in the former is due! And such a statement would be wide of the mark, for it is the determination of the opsonic index from *independent* means, Diagrams IV and V, which must actually be compared.

in his frequency of bacilli per leucocyte, and a rather greater scatter in his 20 independent means than Greenwood and White in their 400 independent means, he yet records his results in some manner which leads to less variation in the dependent means, i.e. when we take 400 means of samples of 50, starting with the first, second, third, etc. entry in his record. I think the explanation will be found in the fact that Fleming's record of his count was not obtained by working backwards and forwards across the slide with a mechanical stage.

This is the only point where the variation in result of workers inside and outside Sir Almroth Wright's Laboratory appears to me on the present data appreciably different, and it concerns the nature of the record, rather than the actual contents of a slide or the results of counting the bacilli per leucocyte.

(8) *Conclusions.*

Counts of at least 1000 leucocytes have been considered in this paper when :

(*a*) the slide was prepared and recorded by a member of Sir Almroth Wright's staff.

(*b*) the slide was prepared by an "internal," but recorded by an "external," investigator.

(*c*) the slide was both prepared and counted by an "external" investigator.

The three slides all refer to sera mixed with tubercle bacilli, and have not been specially selected by me, beyond imposing the condition that a 1000 leucocytes at least should have been dealt with. Before the reduction of the numbers I had no idea what would arise from them.

I find that:

(i) The same degree of variation in the opsonic index of an individual tested against himself arises when external investigators count slides prepared inside or outside Sir Almroth Wright's Laboratory. It seems improbable therefore that "enormous working errors" are functional errors due to the defective preparation of the slide.

(ii) The same degree of variation in the bacilli per leucocyte arises when the slides are prepared and counted (*a*) inside or (*b*) outside Sir Almroth's Laboratory. The population of a slide therefore shows the same high degree of variation under both conditions.

(iii) The variation of twenty independent means of 50 from the same individual is of the same order in the case of internal and external workers. The variation of the opsonic index of the same patient tested against himself is as great for the slides tested whether they be of internal or external origin.

(iv) There is a difference in the nature of the written record of the counts. The "external" counters worked backwards and forwards across the slide, recording continuously by aid of a mechanical stage. I have no knowledge of how Dr Fleming's record was entered but, while it exhibits the same order of variation in the bacilli per leucocyte, the large and small numbers of bacilli appear to be distributed more at random within the groups of 50 in the record.

(v) The results reached are consonant with the "enormous working errors," being for the most part the variations of random sampling; they are not consonant with any wide differentiation resulting from defective technique or from serious individual peculiarities in counting and recording.

It seems to me accordingly that Sir Almroth Wright's statement that he, "his fellow-workers" and "a very large and increasing number of bacteriological workers all over the world" have satisfied themselves that "the enormous working errors in opsonic estimations" are merely the functional errors of his critics, who in "self-esteem" have attributed them to errors peculiar to the method, requires substantiation. I have no prejudice for or against the opsonic index method; for me it is a nice problem in statistics, that is all; but I should much like to see a count of 1000 leucocytes made on a Wright Laboratory slide by one of his staff, and then made on the same slide by an independent microscopist not trained in Sir Almroth Wright's Laboratory. I hardly think, as at present advised, that there would be an appreciable difference in the result. Until this be done, it is scarcely scientific—without publishing evidence of any kind—to appeal vaguely to the "satisfaction" of "a large and increasing number of bacteriological workers all over the world." Statistics on the table, please! I may be quite in error, but at any rate the evidence on which *my* conclusions are based is here provided for criticism or correction.

TABLES OF DATA USED IN THE PAPER.

I. *Bacilli per leucocyte.*

Number of Bacilli	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	Totals
(*a*) Fleming, Slide *T. A.* I. ...	54	99	184	189	156	121	84	48	29	13	9	4	4	1	2	1	2	1000
(*b*) Greenwood and White, Slide B	71	131	197	162	137	111	58	47	32	22	13	11	2	2	2	1	1	1000

II. *Distribution of Means.*

	1·24	*1·34*	*1·44*	*1·54*	*1·64*	*1·74*	*1·84*	*1·94*	*2·04*	*2·14*	*2·24*	*2·34*	*2·44*	*2·54*	Totals
(*a*) Strangeways' 2000 simple cells ...	1	2	5	9	3	8	4	3	4	—	—	—	—	1	40
(*b*) Strangeways' 3000 simple and clumped	2	1	11	9	3	10	9	7	4	2	—	1	—	1	60

(*c*) Fleming's 1000 cells, Slide *T. A.* I. } see footnote†, p. 213.
(*d*) Greenwood and White, Slide B }

III. *Frequency of Opsonic Indices (Independent Means).*

	·725	*·775*	*·825*	*·875*	*·925*	*·975*	*1·025*	*1·075*	*1·125*	*1·175*	*1·225*	*1·275*	*1·325*	*1·375*	Totals
Fleming, Slide *T. A.* I. ...	—	7	24	31·5	61·5	70	60	53·5	29	22·5	14	6	1	—	380
Greenwood and White, Slide B	1	12	31	41·5	52·5	51	49	45·5	38·5	26	18	9	4	1	380

IV. *Frequency of* 400 "*Dependent*" *Means of* 50.

Number of Bacilli	*2·54*	*2·64*	*2·74*	*2·84*	*2·94*	*3·04*	*3·14*	*3·24*	*3·34*	*3·44*	*3·54*	*3·64*	*3·74*	*3·84*	*3·94*	*4·04*	*4·14*	*4·24*	*4·34*	*4·44*	Totals
(*a*) Fleming	—	—	—	—	—	6	8	6	16	44	47	44	58	66	44	29	18	13	1	—	400
(*b*) Greenwood and White	3	1	2	3	4	14	24	27	28	37	37	37	40	57	42	33	11	3	2	1	400

V. *Successive "Independent" Means.*

(*a*) 20 Successive Means: Fleming: 3·90, 3·90, 3·84, 3·16, 3·60, 3·78, 3·56, 3·62, 3·98, 3·24, 3·52, 4·02, 3·84, 3·46, 3·82, 3·88, 3·28, 4·06, 4·14, 3·84.

(*b*) 40 Successive Means: Strangeways: 1·56, 1·98, 1·50, 1·86, 1·80, 1·70, 1·76, 1·34, 1·52, 1·76, 2·06, 1·70, 2·58, 1·70, 1·50, 1·22, 1·32, 1·56, 1·70, 2·00, 1·40, 1·40, 1·66, 1·46, 1·92, 1·86, 1·54, 1·58, 1·66, 1·48, 1·92, 1·66, 1·52, 2·00, 1·88, 1·78, 1·72, 2·04, 1·54, 1·42.

(*c*) 60 Successive Means: Strangeways: 2·00, 1·98, 1·88, 2·58, 1·68, 1·58, 1·86, 1·26, 1·42, 1·46, 1·84, 1·66, 2·04, 1·84, 1·58, 2·08, 1·52, 2·04, 1·78, 1·82, 1·44, 1·52, 1·44, 1·34, 1·48, 1·42, 1·74, 1·78, 1·68, 1·74, 1·72, 1·56, 1·94, 1·42, 1·56, 1·80, 1·70, 1·72, 1·82, 1·52, 1·90, 1·76, 1·88, 1·98, 1·98, 2·12, 1·52, 1·96, 1·88, 1·42, 1·58, 1·46, 2·36, 1·42, 1·42, 1·76, 1·26, 2·12, 1·70, 1·90.

VI. *Correlation between Successive Means of Samples of* 50. *Greenwood and White's* 20,000 *Count.*

Following Mean.

Preceding Mean.	*2·31*	*2·51*	*2·71*	*2·91*	*3·11*	*3·31*	*3·51*	*3·71*	*3·91*	*4·11*	*4·31*	*4·51*	*4·71*	*4·91*	*5·11*	*5·31*	*5·51*	*5·71*	*5·91*	*6·11*	Totals
2·31	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
2·51	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
2·71	1	—	3	—	3	2	1	1	—	3	—	—	1	—	—	—	—	—	—	—	15
2·91	—	—	—	1	4	4	1	4	4	3	—	1	—	—	—	—	—	—	—	—	22
3·11	1	—	4	2	1	3	6	7	1	1	1	—	—	—	—	—	—	—	—	—	27
3·31	—	1	—	2	2	8	10	9	7	4	1	1	1	—	—	—	—	—	—	—	46
3·51	—	—	—	8	6	5	13	10	6	7	5	1	2	—	—	—	—	—	—	—	63
3·71	—	—	2	1	2	9	11	13	11	5	4	2	2	—	—	—	—	—	—	—	62
3·91	—	—	2	3	—	8	6	3	12	10	1	5	—	1	1	—	—	—	—	—	52
4·11	—	—	2	3	1	6	8	5	5	4	3	1	2	1	—	—	1	—	—	—	42
4·31	—	—	—	1	2	2	7	2	3	3	4	—	—	1	—	—	—	—	—	1	26
4·51	—	—	—	—	—	—	—	3	2	4	3	—	—	—	—	—	—	—	—	—	12
4·71	—	—	—	—	2	1	—	1	4	—	1	—	—	—	—	—	—	—	—	—	9
4·91	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	—	—	—	3
5·11	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
5·31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·51	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
5·71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·91	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·11	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	1	14	21	24	48	64	60	55	44	25	12	9	3	2	—	1	—	—	1	386

2·31 = subrange from 2·21 to 2·41, or from and including 111 to 120 bacilli per 50 leucocytes ; the other subranges are to be interpreted in like manner.

MISCELLANEA.

I. A Note on the Age Distribution of Deaths from Diabetes Mellitus.

By G. D. MAYNARD.

In a paper entitled "A Statistical Study in Cancer Death-rates," published in *Biometrika*, Vol. VII. Part 3, brief mention was made of the fact that the Diabetes death-rates at age periods could not be fitted satisfactorily by any single frequency curve.. The form of the curve obtained seemed to indicate that the population dealt with under the term Diabetes Mellitus was not homogeneous.

I have attempted therefore to resolve the complex distribution into its components. After some considerable labour, on the principle of trial and error, three curves have been found that give a very close fit to the original observations. For the sake of convenience I have termed the first sub-curve the Growth Curve, as its range is from birth to about 20 years; this curve is symmetrical having its mean at 10 years, and being platykurtic belongs therefore to Type II. The second or Reproductive Curve approximately corresponds to the period of reproduction, that is from 15 to 50 ; this curve is also symmetrical and platykurtic, except in the case of the U.S.A. data where it was best fitted by the normal curve. The third or Old-age Curve is skew and of Type I.

Finding, as might be expected, that the irregularities due to random sampling were of considerable inconvenience when dealing with only one year's returns, the figures dealt with in the paper above referred to were abandoned in favour of five-yearly returns for the period 1900—1904. Three tables were available, England and Wales Male, England and Wales Female, and U.S.A. Male and Female; separate tables for males and females in the case of U.S.A. were not obtainable for a five-yearly period. These data were then reduced to rates per million at each five-yearly age period. The higher rates observed in the U.S.A. table are largely due to the fact that the returns used are obtained chiefly from urban areas, whereas the English figures are compiled from the whole country.

It will be seen that the various sub-curves agree very closely with each other, both as to range and position of the mean, but the final test as to the suitability of the resolution rests in the result of the application of the test for 'Goodness of Fit.' A total theoretical distribution was then constructed, by adding together the areas of the various components standing on the same five-yearly bases. The value of χ^2 was then found and P, the probability of goodness of fit, obtained from the table in *Biometrika*, Vol. I. In both the English distributions the values of P could hardly be improved. In the U.S.A. figures the result is not quite so satisfactory when all the groups are taken into consideration. The number of cases of diabetes from which the rates were calculated is very small after the 80—85 age period and considerable variation may be expected; in the 95—100 group only 3 cases were recorded in the 5 years. The effect therefore of a single case, or the misstatement of age in one case, will very materially alter the rate. I have therefore calculated χ^2 for the first 19 and also 17 groups. The exclusion of the 20th group increases the value of P from ·6726 to ·9968, and if only the first 17 groups are taken $P = ·9994$, which is I think a very satisfactory value.

Medically this analysis is of some interest. Diabetes Mellitus has presented considerable difficulties and different types are well known. For instance, it has long been recognised that diabetes in the young adult is a much more serious disease than it is when it occurs in a person of advanced life. Then, again, pathologically there is the form associated with pancreatic disease, and a type in which hepatic disturbances are found. It is conceivable that the Growth Type may be associated with some pathological condition of an organ mainly of importance during development. The statistical analysis indicates that a different group of cause factors is concerned in the production of the three types; what these are can only be discovered by the clinician and pathologist. Each type has its peculiar age period and one can say that, except in very rare instances, any case occurring in a child under 10 years of age will belong to the Growth Type, if the disease is present in a young adult between 20 and 30 most probably to the Reproductive Type, and in persons over 55 to the Old-age Type.

Diabetes Mellitus.

Constants of Curves.

	U. S. A. ♂ and ♀	England and Wales ♂, 1900—1904	England and Wales ♀
		Total Observations	
ν_2	10·707757	8·4462	9·2298
ν_3	−39·898144	−35·6000	−37·6492
ν_4	582·482889	363·5801	422·0587
σ	3·272271	2·9062	3·0381
β_1	1·296611	2·1034	1·8027
β_2	5·080271	5·0966	4·9544
κ_1	·270709	− 2·1170	− 1·4993
Mean	71·793190	66·0115	65·6125
Mode	81·108435	88·0993	81·9805
		Old-Age Curve, III	
ν_2	6·523346	4·427590	4·433679
ν_3	−5·344153	− 6·656844	−5·300318
ν_4	117·214998	65·102404	60·322009
σ	2·554084	2·104184	2·105630
β_1	·102884	·510546	·322338
β_2	2·754497	3·320949	3·068650
κ_1	− ·799658	− ·889740	− ·829714
b	19·601235	18·655191	17·836714
y_0	913·3	775·55	561·583
Mean	73·5556	69·370275	68·875840
Mode	76·3920	74·601783	72·989120
		Reproductive Curve, II	
ν_2	2·9319	3·4115	3·0853
ν_4	24·7888	25·1364	22·0005
σ	1·6878	1·8470	1·7565
β_2	2·8778	2·1598	2·3112
a	—	4·1880	4·5502
y_0	51·53	51·81	48·14
Mean	30·9175	30·8450	30·3960
		Growth Curve, I	
ν_2	1·0444	·7400	·7100
ν_4	2·0699	1·3640	1·2210
σ	1·0219	·8602	·8426
β_2	1·8976	2·4909	2·4221
a	1·8962	2·7804	2·4395
y_0	23·80	8·268	11·31
Mean	10·375	9·5000	10·1925

Diagram (i) U.S.A. ♂ and ♀	Total	$y=939{\cdot}6e^{\cdot5368x}\left(1-\frac{x}{3{\cdot}8844}\right)^{2\cdot0850}.$
	III	$y=913{\cdot}3\left(1+\frac{x}{13{\cdot}3157}\right)^{7\cdot0598}\left(1-\frac{x}{6{\cdot}2856}\right)^{3\cdot3326}.$
	II	$y=51{\cdot}53e^{-x^2/5\cdot6972}.$
	I	$y=23{\cdot}80\left(1-\frac{x^2}{3{\cdot}5955}\right)^{0\cdot2213}.$
Diagram (ii) England and Wales ♂	Total	$y=1246{\cdot}3\left(1+\frac{x}{23{\cdot}4829}\right)^{3\cdot8931}\left(-1+\frac{x}{1{\cdot}4718}\right)^{-\cdot2440}.$
	III	$y=775{\cdot}55\left(1+\frac{x}{15{\cdot}7145}\right)^{8\cdot5993}\left(1-\frac{x}{2{\cdot}9407}\right)^{1\cdot6092}.$
	II	$y=51{\cdot}81\left(1-\frac{x^2}{17{\cdot}5390}\right)^{1\cdot0706}.$
	I	$y=8{\cdot}268\left(1-\frac{x^2}{7{\cdot}7305}\right)^{3\cdot3928}.$
Diagram (iii) England and Wales ♀	Total	$y=713{\cdot}1\left(1+\frac{x}{28{\cdot}4401}\right)^{6\cdot5528}\left(1-\frac{x}{0{\cdot}2517}\right)^{\cdot0580}.$
	III	$y=561{\cdot}583\left(1+\frac{x}{14{\cdot}1129}\right)^{8\cdot4094}\left(1-\frac{x}{3{\cdot}7238}\right)^{2\cdot2189}.$
	II	$y=48{\cdot}14\left(1-\frac{x^2}{20{\cdot}7048}\right)^{1\cdot8554}.$
	I	$y=9{\cdot}824\left(1-\frac{x^2}{5{\cdot}2500}\right)^{2\cdot0000}.$

Diabetes Death-Rates. U. S. A., 1900—1904.

Male and Female.

Age	Observations	Calculated Curves			Totals
		I	II	III	
2·5	15	15	—	—	15
7·5	21	23	1	—	24
12·5	32	24	5	—	29
17·5	32	18	15	—	33
22·5	33	—	31	1	32
27·5	46	—	47	2	49
32·5	58	—	50	8	58
37·5	67	—	38	28	66
42·5	94	—	20	70	90
47·5	149	—	8	146	154
52·5	266	—	2	263	265
57·5	418	—	—	417	417
62·5	604	—	—	593	593
67·5	768	—	—	760	760
72·5	894	—	—	877	877
77·5	924	—	—	904	904
82·5	805	—	—	822	822
87·5	663	—	—	633	633
92·5	400	—	—	385	385
97·5	200	—	—	160	160
102·5	—	—	—	29	29

χ^2 to 82·5 = 3·461, $P=\cdot9994$,
χ^2 to 92·5 = 5·765, $P=\cdot9968$,
χ^2 to 97·5 = 15·765, $P=\cdot6726$.

Diabetes Death-Rates. England and Wales, 1900—1904.

	Male.					Female.				
Age	Observations	Calculated Curves			Totals	Observations	Calculated Curves			Totals
		I	II	III			I	II	III	
2·5	3	3	—	—	3	3	3	—	—	3
7·5	8	8	—	—	8	9	10	—	—	10
12·5	18	7	11	—	18	19	10	8	—	18
17·5	32	2	30	—	32	26	3	23	—	26
22·5	44	—	43	1	44	34	—	38	—	38
27·5	55	—	50	3	53	48	—	47	1	48
32·5	61	—	51	9	60	54	—	47	6	53
37·5	68	—	46	24	70	60	—	40	17	57
42·5	90	—	35	55	90	66	—	26	41	67
47·5	129	—	18	110	128	98	—	10	88	98
52·5	199	—	1	198	199	158	—	—	163	163
57·5	323	—	—	325	325	285	—	—	268	268
62·5	479	—	—	484	484	385	—	—	395	395
67·5	651	—	—	648	648	496	—	—	504	504
72·5	761	—	—	761	761	562	—	—	560	560
77·5	747	—	—	742	742	521	—	—	512	512
82·5	517	—	—	511	511	331	—	—	334	334
87·5	90	—	—	98	98	90	—	—	94	94

$\chi^2 = 0·992$. $\chi^2 = 2·781$.

$P = 1$. $P = ·9999945$.

[NOTE. The application of the theory of frequency curves to *rates* as distinguished from total frequencies is possibly open to criticisms, which might touch also the applicability of the χ^2 test for goodness of fit. That test essentially concerns total numbers dealt with, and is not necessarily a test of good graduation formulae. The groups given by Dr Maynard, however, seem of much interest and suggestiveness. EDITOR.]

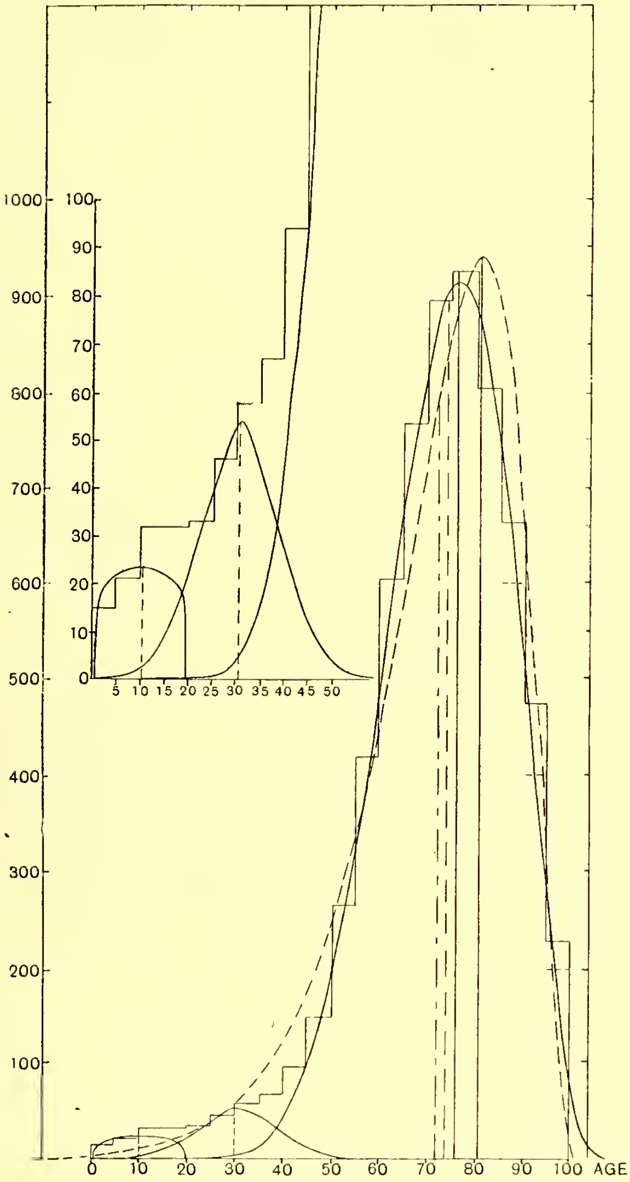


DIAGRAM (i). Age Polygon of Diabetes Death-Rates, ♂ and ♀. U.S.A., 5 years, 1900—1904.

$$\left\{\begin{array}{ll} \text{III} & y = 913{\cdot}3\left(1+\dfrac{x}{13{\cdot}3157}\right)^{7{\cdot}0598}\left(1-\dfrac{x}{6{\cdot}2856}\right)^{3{\cdot}3326}. \\ \text{II} & y = 51{\cdot}53\,e^{-x^2/5{\cdot}6972}. \\ \text{I} & y = 23{\cdot}80\left(1-\dfrac{x^2}{3{\cdot}5955}\right)^{0{\cdot}2213}. \end{array}\right.$$

$$\text{Total}\quad y = 939{\cdot}6\,e^{+0{\cdot}5368x}\left(1-\frac{x}{3{\cdot}8844}\right)^{2{\cdot}0850}.$$

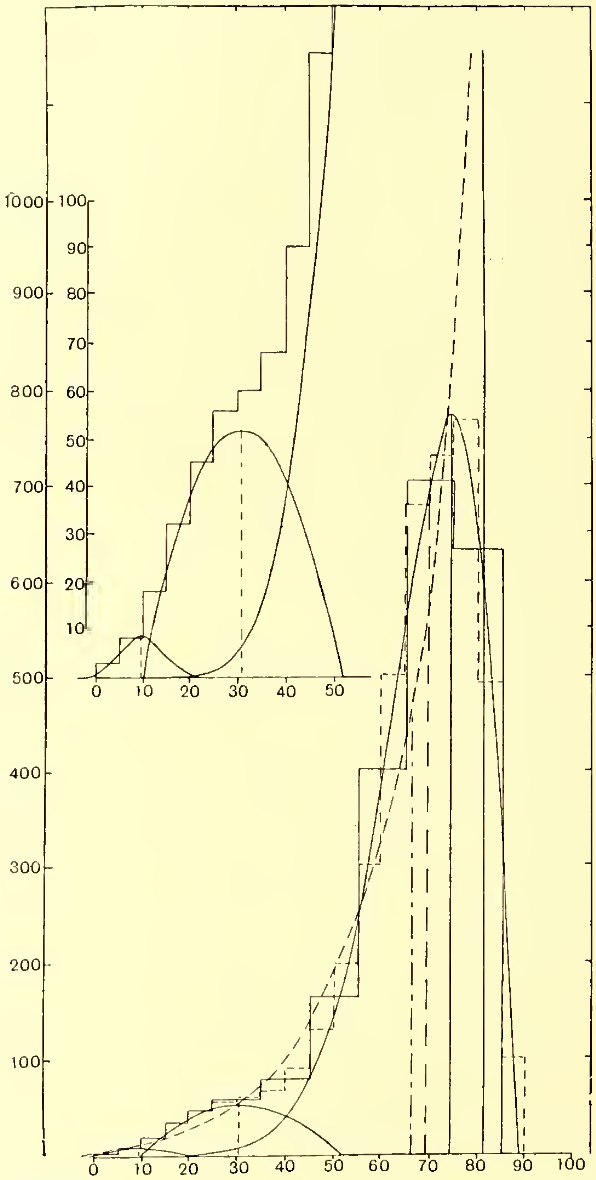


DIAGRAM (ii). Age Polygon of Diabetes Death-Rates, ♂. England and Wales, 1900—1904. Continuous rectangles rates given. Dotted rectangles rates used in calculations.

$$\left\{\begin{array}{ll}\text{III} & y=775\cdot55\left(1+\frac{x}{15\cdot7145}\right)^{8\cdot5993}\left(1-\frac{x}{2\cdot9407}\right)^{1\cdot6092}.\\ \text{II} & y=51\cdot81\left(1-\frac{x^2}{17\cdot5390}\right)^{1\cdot0706}.\\ \text{I} & y=8\cdot268\left(1-\frac{x^2}{7\cdot7305}\right)^{3\cdot3928}.\end{array}\right.$$

$$\text{Total}\quad y=1246\cdot3\left(1+\frac{x}{23\cdot4829}\right)^{3\cdot8931}\left(-1+\frac{x}{1\cdot4718}\right)^{-\cdot2440}.$$

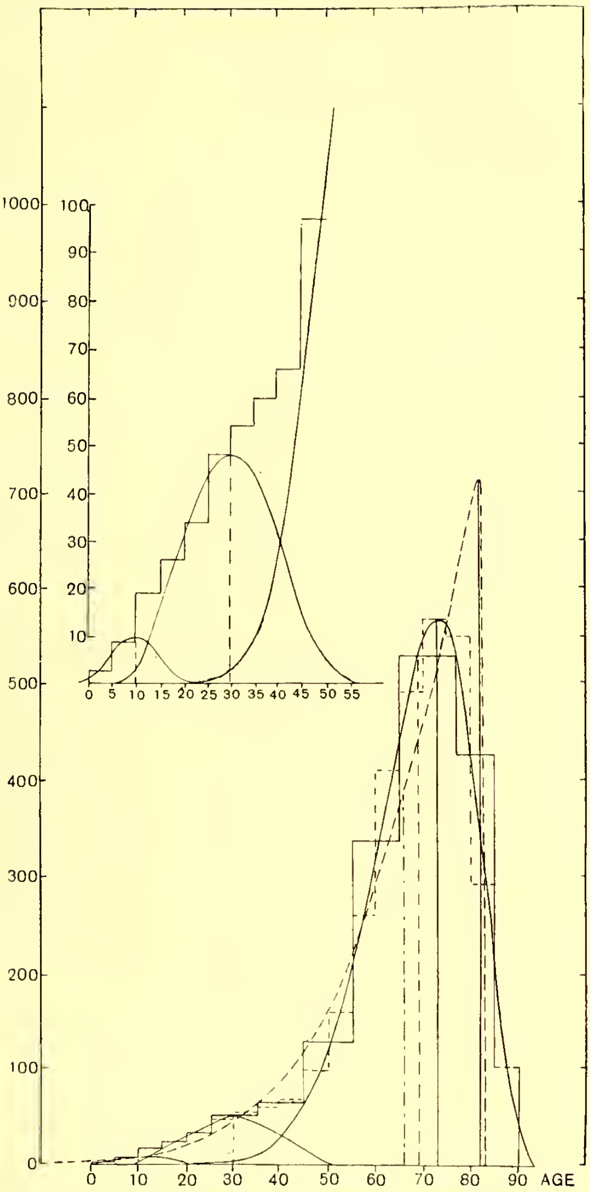


DIAGRAM (iii). Age Polygon of Diabetes Death-Rates, ♀. England and Wales, 1900—1904.

$$\left\{\begin{array}{ll}\text{III} & y=561{\cdot}58\left(1+\dfrac{x}{14{\cdot}1129}\right)^{8{\cdot}4094}\left(1-\dfrac{x}{3{\cdot}7238}\right)^{2{\cdot}2189}.\\ \text{II} & y=48{\cdot}14\left(1-\dfrac{x^2}{20{\cdot}7048}\right)^{1{\cdot}8554}.\\ \text{I} & y=9{\cdot}824\left(1-\dfrac{x^2}{5{\cdot}2500}\right)^{2{\cdot}0000}.\end{array}\right.$$

$$\text{Total}\quad y=713{\cdot}1\left(1+\frac{x}{28{\cdot}4401}\right)^{6{\cdot}5528}\left(1-\frac{x}{0{\cdot}2517}\right)^{0{\cdot}0580}.$$

II. On Pulse and Breathing Rates and their Relation to Stature.

By JULIA BELL, M.A.

In the *Deutsches Archiv für klinische Medizin* (December 1910, S. 267—282), Dr Kornél Körösy published a very interesting medico-statistical paper entitled "Studien über Puls- und Atmungsfrequenz." The author obtained the statistics upon which his paper is based by observing the pulse and breathing of 255 military recruits stationed at Budapest. He took great pains to ensure normal conditions and homogeneous data; thus all his men were aged 20 to 24 years. Any observation taken on a man found to have a not perfectly healthy heart was rejected; the observations were invariably taken in the early morning soon after the men were wakened and whilst they were still in a position of rest. Further, they were all taken at the same time of the year, the end of September and the beginning of October.

The following tables are given. Table I, a correlation table of Pulse classified according to Stature, and Table II, a correlation table of Breathing classified according to Stature. Dr Körösy calculates the mean of the Pulse and Breathing frequencies for each array corresponding to a given stature, and finds that there is no systematic reduction in these means as the stature increases. From this fact he deduces that there is no relation between Pulse and Stature or Breathing and Stature. It seems however worth while to carry the enquiry a little further and measure the degree of relationship more accurately. I have therefore calculated the correlation coefficient for each of his tables and find:

For Pulse and Stature

$$r_{\text{P.S.}} = -\cdot072 \pm \cdot042,$$

For Breathing and Stature

$$r_{\text{B.S.}} = -\cdot042 \pm \cdot042.$$

Thus we find no relationship existing between Breathing and Stature, whilst there appears to be a slight tendency to a slower pulse with increasing stature; we cannot however attach much importance to this slight relationship based upon only 253 observations. It is possible owing to the courtesy of Dr Körösy, who has kindly provided us with the data represented in our Table III, not given in his paper, to obtain the relationship between Pulse and Breathing.

For Pulse and Breathing

$$r = +\cdot108 \pm \cdot042,$$

or, there is a small if possibly significant relationship between breathing and pulse rates*.

Dr Körösy also gives graphs of the two frequency distributions for Pulse and Breathing but they are unfortunately a little misleading from the fact that he does not bring his curve down to the zero base line for the groups which correspond to a zero frequency and thus his distributions appear more unsymmetrical than they really are. He calculates approximately some of the statistical constants characteristic of the two distributions, and again we have carried his work a little further and found the best fitting curves to the two series.

For the Pulse frequency distribution we find the following constants:

$\mu_1 = +\cdot1765$, $\beta_1 = \cdot8587$, Mean Pulse $= 64\cdot206$ per minute,

$\mu_2 = 4\cdot5012$, $\beta_2 = 5\cdot8368$, Standard Deviation $= 8\cdot486$,

$\mu_3 = +8\cdot8494$, $\kappa = +\cdot2606$, Mode is at pulse $61\cdot92$.

$\mu_4 = 118\cdot2576$,

---

* For these 255 cases: mean pulse $= 64\cdot200$, mean breathing $15\cdot839$; standard deviation for pulse $8\cdot588$, for breathing $2\cdot349$.

Thus the best fitting curve to the series is an unsymmetrical curve of Type IV. and calculating the constants of the curve in terms of the moments, its equation is found to be

$$y=19{\cdot}57\left(1+\frac{x^2}{22{\cdot}319}\right)^{-4{\cdot}8529} e^{4{\cdot}5744\tan^{-1}\frac{x}{4{\cdot}7243}}. \quad \text{(See fig. 1.)}$$

The origin of the curve is at a pulse of 52·99.

For the case of the Breathing distribution we find :

$\mu_1 = -\ {\cdot}1608$,	$\beta_1 = \ {\cdot}0034$,	Mean breathing = 15·839 per minute,
$\mu_2 = 5{\cdot}5183$,	$\beta_2 = 3{\cdot}6262$,	Standard Deviation = 2·349.
$\mu_3 = +\ {\cdot}7574$,	$\kappa = +{\cdot}0021$,	
$\mu_4 = 110{\cdot}4248$,		

This time we see that the best fitting curve is symmetrical and of Type II. Calculating the constants we have as the equation to the curve

$$y=46{\cdot}04\left(1+\frac{x^2}{63{\cdot}9107}\right)^{-7{\cdot}2908}. \quad \text{(See fig. 2.)}$$

The series for this case is very irregular and it would not be possible to find any curve which would give a very close fit. Doubtless this is largely due to the small number of observations, but again the nature of the irregularities suggests that there may possibly be some unsuspected source of heterogeneity in the data.

My thanks are due to Miss H. Gertrude Jones who has very kindly drawn the diagrams.

TABLE I.

Stature in cms.

Pulse per minute.	—159	160—164	165—169	170—174	175—179	180—184	Totals
42— 45	—	—	—	2	—	—	2
46— 49	—	1	1	1	1	—	4
50— 53	—	2	6	—	3	—	11
54— 57	1	6	14	10	4	1	36
58— 61	1	9	12	15	10	—	47
62— 65	1	13	24	10	4	1	53
66— 69	—	12	14	13	5	1	45
70— 73	1	7	6	6	2	—	22
74— 77	—	3	6	3	2	1	15
78— 81	1	3	1	5	1	—	11
82— 85	—	1	3	1	—	—	5
86— 89	—	—	—	—	—	—	—
90— 93	—	—	—	—	—	—	—
94— 97	—	—	—	—	—	—	—
98—101	—	—	—	1	—	—	1
102—105	—	—	—	—	—	—	—
106—109	—	—	1	—	—	—	1
Totals	5	57	88	67	32	4	253

Two additional observations of Pulse fall into the groups 58—61 and 66—69 respectively. They do not appear in this table as the corresponding Stature was unknown.

TABLE II.

Stature in cms.

Breathing per minute.	—159	160—164	165—169	170—174	175—179	180—184	Totals
8·9— 9·5	—	1	—	—	—	—	1
9·5—10·5	—	—	2	1	1	—	4
10·5—11·5	—	1	1	—	—	—	2
11·5—12·5	—	2	5	5	3	—	15
12·5—13·5	—	4	8	5	1	—	18
13·5—14·5	—	4	12	8	5	—	29
14·5—15·5	2	4	16	6	4	—	32
15·5—16·5	1	14	18	11	7	3	54
16·5—17·5	—	12	13	13	7	—	45
17·5—18·5	2	7	5	11	1	—	26
18·5—19·5	—	1	5	2	1	—	9
19·5—20·5	—	4	3	3	1	—	11
20·5—21·5	—	2	—	2	1	1	6
21·5—22·5	—	—	—	—	—	—	—
22·5—23·5	—	—	—	—	—	—	—
23·5—24·5	—	—	—	—	—	—	—
24·5—25·5	—	1	—	—	—	—	1
Totals	5	57	88	67	32	4	253

Two additional observations of Breathing where the Stature was unknown fall into the groups 13·5—14·5 and 14·5—15·5 respectively.

TABLE III.

Breathing per minute.

Pulse per minute.	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Totals
41— 43	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
44— 46	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
47— 49	—	—	—	—	—	—	—	—	2	2	—	—	—	—	—	—	—	4
50— 52	—	—	—	1	1	1	—	1	1	1	1	—	—	—	—	—	—	7
53— 55	1	—	—	2	5	3	1	3	3	—	—	2	—	—	—	—	—	20
56— 58	—	—	—	5	2	5	5	10	3	1	1	—	1	—	—	—	—	33
59— 61	—	—	—	1	1	4	7	2	9	7	2	2	—	—	—	—	—	35
62— 64	—	1	1	1	4	7	6	9	5	3	2	1	1	—	—	—	—	41
65— 67	—	—	1	1	—	3	5	9	9	5	—	2	2	—	—	—	—	37
68— 70	—	1	—	1	2	4	4	8	3	2	1	3	—	—	—	—	—	29
71— 73	—	1	—	2	1	—	3	3	2	2	—	—	—	—	—	—	—	14
74— 76	—	—	—	1	—	1	1	3	2	1	1	—	1	—	—	—	—	11
77— 79	—	1	—	—	1	—	—	2	2	—	—	—	1	—	—	—	1	8
80— 82	—	—	—	—	1	1	—	1	3	—	1	1	—	—	—	—	—	8
83— 85	—	—	—	—	—	1	—	1	—	2	—	—	—	—	—	—	—	4
86— 88	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
89— 91	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
92— 94	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
95— 97	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
98—100	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
101—103	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
104—106	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
107—109	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	4	2	15	18	30	33	54	45	26	9	11	6	—	—	—	1	255

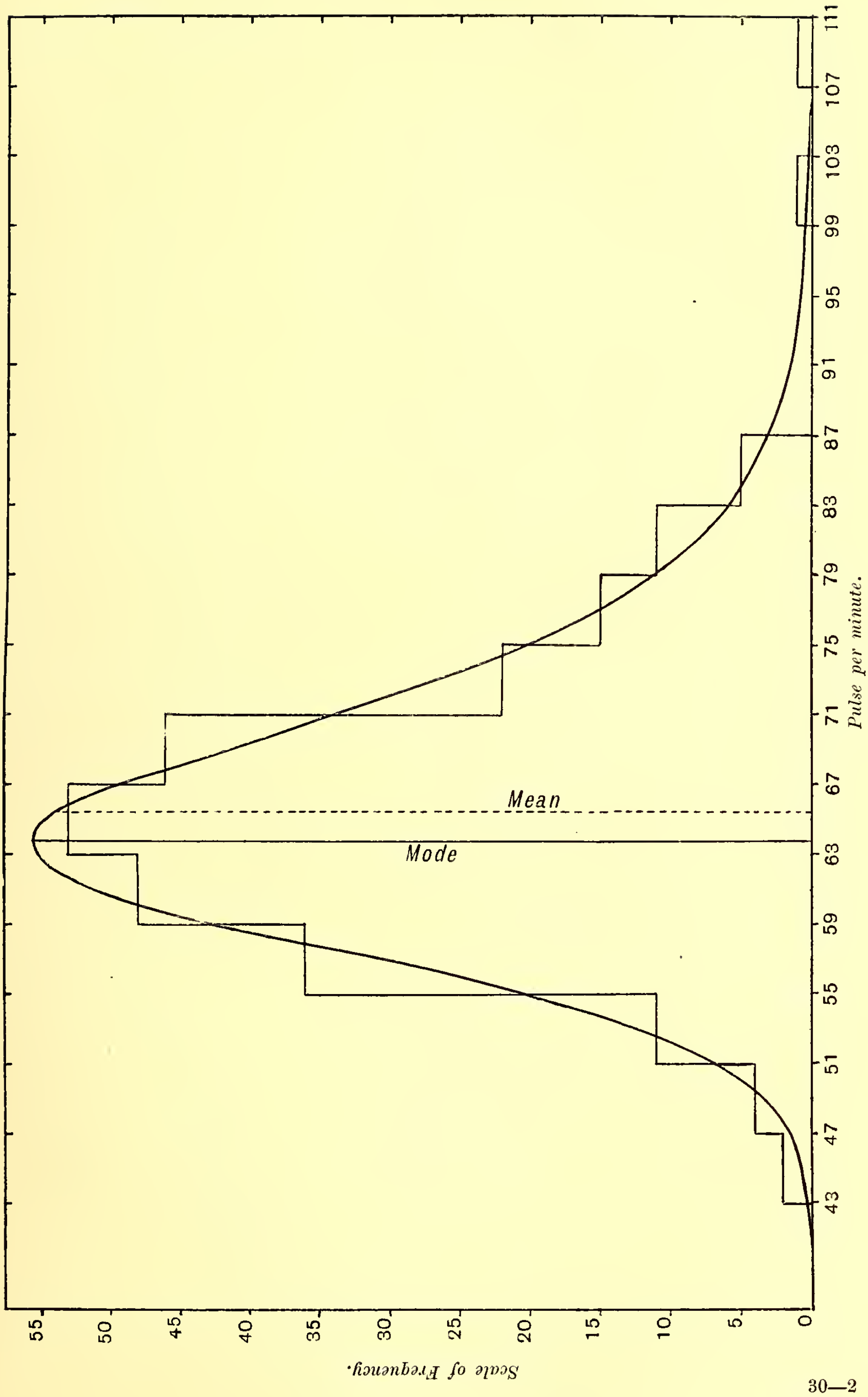

Pulse per minute.
43
47
51
55
59
63
67
71
75
79
83
87
91
95
99
103
107
111
Scale of Frequency.
0
5
10
15
20
25
30
35
40
45
50
55
Mean
Mode

FIG. 1.

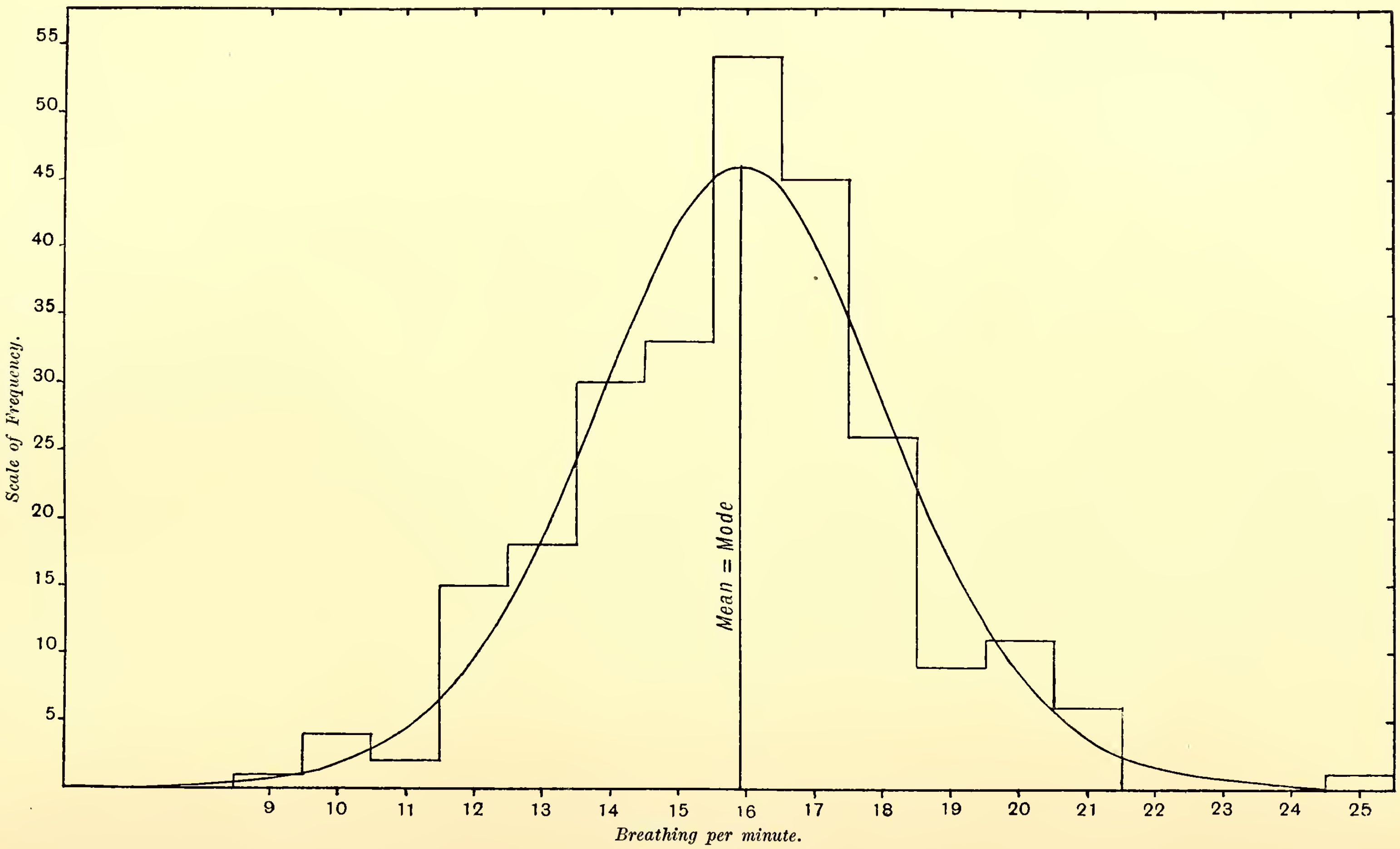

Scale of Frequency.
55
50
45
40
35
30
25
20
15
10
5
Mean = Mode
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
Breathing per minute.

FIG. 2.

III. The Tooth-Brush and Decayed Teeth.

By FRANK ROCK, M.D.

The use of the tooth-brush is generally regarded not merely as an essential of cleanliness but also as necessary in order to prevent decay of the teeth. The question as to whether there is any correlation between the use of the tooth-brush and the amount of decay is therefore an interesting one.

Below are two tables giving the condition of the teeth found at the medical inspection of two boys' and three girls' schools at Edmonton during the last two years. All the children are over thirteen years of age. The children in row *A* clean their teeth at least once a day; those in row *B* use a tooth-brush occasionally; this may mean once a week or once a fortnight or in fact any period that will allow a child's conscience to answer that it uses a tooth-brush occasionally; those in row *C* never use a tooth-brush.

TABLE I. *Boys.*

Number of Decayed Teeth.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
A	48	32	24	10	3	1	—	—	1	119
B	59	58	38	21	8	4	1	—	—	189
C	89	67	47	30	14	1	—	—	—	248
Totals	196	157	109	61	25	6	1	—	1	556

Mean number of decayed teeth: *A*, 1·1344; *B*, 1·3492; *C*, 1·2508; all the boys, 1·2626.

TABLE II. *Girls.*

Number of Decayed Teeth.

	0	*1*	*2*	*3*	*4*	*5*	*6*	Totals
A	86	44	56	27	13	2	1	229
B	66	51	36	23	5	2	—	183
C	38	29	13	10	3	2	—	95
Totals	190	124	105	60	21	6	1	507

Mean number of decayed teeth: *A*, 1·3319; *B*, 1·2131; *C*, 1·1263; all the girls, 1·2505.

The best way of grouping to form a two-row table would be to combine *B* and *C* together since the occasional use means practically very little use of the tooth-brush. This was done and the correlation* between the non-use of the tooth-brush and the number of decayed teeth was found to be for the boys +·074 and for the girls −·074. The correlation if a two-row table is formed by grouping *A* and *B* together is for boys +·004, and for girls −·069. The correlation in any case is trifling and in the case of the girls is unfavourable to the tooth-brush. However necessary for cosmetic purposes, it would appear from these data to exert no influence in the prevention of decay.

The probable error of these correlation coefficients is about ·03. They practically amount therefore to no correlation, and if they do not indicate any beneficial influence, at any rate they do not point to any harmful effect of the tooth-brush at the ages of thirteen and fourteen.

IV. Note on High Values of β_1 and β_2.

By GEORG DUNCKER, Ph.D.

Auf der Südseeexpedition der Hamburgischen wissenschaftlichen Stiftung hatte ich Gelegenheit zu den beiden folgenden Zählungen:

(1) St Matthias, Riffinsel bei Ekaliu, 22. IX. 08.

Armzahl von *Asterina exigua* Lam.—$n=600$.

V	4	5	6
f	6	582	12

(2) Neu Pommern, Matupi, 9. IV. 09.

Armzahl von *Archaster typicus* M. & Tr.—$n=902$.

V	4	5	6
f	2	895	5

Ihre Bestimmungswerthe sind

	1. Asterina	2. Archaster
M	5·01000	5·00333
σ	0·17292	0·08803
β_1	1·76048	4·76152
β_2	33·12965	128·47974
κ_1	50·26143	182·94327
κ_2	0·50934	1·20114

und die beschreibenden Kurven dementsprechend

1. $y=y_0(\cos\vartheta)^{2m}e^{-\nu\vartheta}$ (Pearson, 1895).
2. $y=y_0(x-a)^{q_2}x^{-q_1}$ (Pearson, 1901).

Hamburg, 28. I. 10.

---

* The method used was that of the two-rowed table suggested by Pearson, *Biometrika*, Vol. VII. p. 248.

V. Further Remarks on the Law of Ancestral Heredity*.

By KARL PEARSON, F.R.S.

This law is a *rule* for predicting the *average* value of a character in the offspring of ancestry with given characters.

It is based on the following assumptions:

(i) The proper prediction formula is the multiple regression formula.

(ii) The individual ancestors in any generation may be replaced by the mid-ancestor of this generation, i.e. an individual supposed to have, as his character, that linear function of the characters of all the ancestors in that generation which is most highly correlated with the character of the offspring.

(iii) The individual ancestral correlations decrease in geometrical progression. This is very close to the truth in practically all the races that have been statistically investigated.

The general multiple regression formulae are

$$x_0 - \bar{x}_0 = S\left\{c_p \frac{\sigma_0}{\Sigma_p}(X_p - \bar{X}_p)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

$$\Sigma_0 = \sigma_0 \sqrt{1 - S(c_p r_{po})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)},$$

where X_p is the character of the pth mid-ancestor, r_{po} the correlation of this ancestor and the offspring, while

$$\Sigma_p = \frac{\sigma_p}{2^{\frac{1}{2}p}} \sqrt{1 + \epsilon_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)},$$

$$r_{p-q} = 2^{\frac{1}{2}(p-q)} \rho_{p-q} / \sqrt{(1+\epsilon_p)(1+\epsilon_q)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)},$$

where ϵ_p is the assortative mating in the pth generation, σ_p is the variability of that generation and ρ_{p-q}=average correlation of the *individual* pth ancestor with the offspring of the qth generation.

Further we have to determine the c's

$$\left.\begin{array}{l} r_{01} = c_1 + c_2 r_{12} + c_3 r_{13} + \;\ldots\; + c_n r_{1n} \\ r_{02} = c_1 r_{21} + c_2 + c_3 r_{23} + \;\ldots\; + c_n r_{2n} \\ r_{03} = c_1 r_{31} + c_2 r_{32} + c_3 + \;\ldots\; + c_n r_{3n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)}.$$

Assume $r_{pq} = a\beta^{p-q}$, which will clearly be the case if ρ_{p-q} is of the form

$$\zeta\theta^{p-q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)},$$

or in accord with our condition (iii). We shall show that $c_p = \gamma\eta^p$ satisfies the above equations.

Substituting we have

$$\left.\begin{array}{l} a\beta = \gamma(\eta + \eta^2 a\beta + \eta^3 a\beta^3 + \;\ldots \\ a\beta^2 = \gamma(\eta a\beta + \eta^2 + \eta^3 a\beta + \;\ldots \\ a\beta^3 = \gamma(\eta a\beta^2 + \eta^2 a\beta + \eta^3 + \;\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)}.$$

---

* Reproduced from lecture notes on multiple regression.

Or, supposing the ancestry to ascend indefinitely by the same law,

$$\left.\begin{aligned} a\beta &= \gamma\left(\eta + \frac{\eta^2 a\beta}{1-\eta\beta}\right) \\ a\beta^2 &= \gamma\left(\eta a\beta + \eta^2 + \frac{\eta^3 a\beta}{1-\eta\beta}\right) \\ a\beta^3 &= \gamma\left(\eta a\beta^2 + \eta^2 a\beta + \eta^3 + \frac{\eta^4 a\beta}{1-\eta\beta}\right) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)}.$$

Multiply each equation by η and subtract it from the one below it. We find

$$\left.\begin{aligned} a\beta(\beta-\eta) &= \gamma\eta a\beta \\ a\beta^2(\beta-\eta) &= \gamma\eta a\beta^2 \\ \ldots\ldots\ldots\ldots &= \ldots\ldots \\ \text{or}\qquad \beta-\eta &= \gamma\eta \text{ in each case} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

Substitute this value of $\gamma\eta$ in the first equation and we have

$$\left.\begin{aligned} a\beta &= (\beta-\eta)\left(1 + \frac{\eta a\beta}{1-\eta\beta}\right) \\ a\beta &= \frac{(\beta-\eta)(1-\eta\beta)}{\eta^2-2\eta\beta+1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(x)}.$$

Hence

$$\Sigma = \sigma_0\sqrt{1 - S\left(\gamma a(\eta\beta)^p\right)} = \sigma_0\sqrt{1 - \frac{\gamma a\eta\beta}{1-\eta\beta}} = \sigma_0\sqrt{1 - \frac{(\beta-\eta)^2}{\eta^2-2\eta\beta+1}} = \sigma_0\sqrt{\frac{1-\beta^2}{\eta^2-2\eta\beta+1}} \quad \ldots\text{(xi)},$$

which determines the variability of the array with given ancestry.

Returning to Equation (x) we have

$$\eta^2 - \frac{\eta(1-2a\beta^2+\beta^2)}{\beta(1-a)} + 1 = 0,$$

or

$$\eta^2 - \eta\lambda + 1 = 0,$$

$$\eta = \frac{\lambda \pm \sqrt{\lambda^2-4}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xii)},$$

where it is necessary to take the root less than unity.

Application I. Galton's Rule. Sir Francis Galton took the "contributions" of each generation of ancestry to be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... Thus each parent contributed on an average $\frac{1}{4}$, each grandparent $\frac{1}{16}$, each great-grandparent $\frac{1}{64}$, etc.

Our equation, if we neglect assortative mating, is

$$\begin{aligned} x_0 - \bar{x}_0 &= \tfrac{1}{2}(X_1 - \bar{X}_1) + \tfrac{1}{4}(X_2 - \bar{X}_2) + \ldots \\ &= S\left\{c_p \frac{\sigma_0}{\Sigma_p}(X_p - \bar{X}_p)\right\} \\ &= S\left\{\gamma(\sqrt{2}\eta)^p (X_p - \bar{X}_p)\right\}, \end{aligned}$$

if we put $\sigma_p = \sigma_0$, i.e. suppose population stable.

$$\therefore\ \gamma = 1, \quad \sqrt{2}\eta = \tfrac{1}{2},$$

or by (ix)

$$\beta = 2\eta = 1/\sqrt{2}.$$

Therefore by (x)

$$a = \frac{\beta-\eta}{\beta}\,\frac{(1-\eta\beta)}{\eta^2-2\eta\beta+1} = \frac{1\frac{3}{4}}{2\frac{5}{8}} = \cdot 6.$$

Thus the mid-parental correlation $= a\beta^p = \cdot 6\left(\frac{1}{\sqrt{2}}\right)^p$.

The individual ancestral correlations are found from

$$r_{p-q}=2^{\frac{1}{2}(p-q)}\,\rho_{p-q},$$

or

$$\rho_{p-q}=\cdot 6\left(\frac{1}{\sqrt{2}}\right)^{p-q}\Big/(\sqrt{2})^{p-q}$$

$$=\cdot 6\,(\tfrac{1}{2})^{p-q}.$$

Hence the individual parental correlation is ·3 and the geometrical ratio $\frac{1}{2}$.

It is thus clear that Galton's Rule is not absolutely consistent with the Mendelian somatic correlations $\frac{1}{3}$, $\frac{1}{3}\times\frac{1}{2}$, etc., ..., although it is not very far removed from them.

The variability of an array of brothers from same ancestry is found from (xi) to be

$$\cdot 8944\sigma_0,$$

or, the reduction in variability by continued selection will not exceed 11 °/₀.

The correlation of brothers

$$=R=(\beta-\eta)/\sqrt{\eta^2-2\eta\beta+1}$$

$$=\cdot 4472.$$

Application II. Suppose the individual ancestral correlations follow the simple Mendelian somatic relation $\frac{1}{3}$, $\frac{1}{3}\times\frac{1}{2}$, etc.

Then

$$r_{p-q}=(\sqrt{2})^{p-q}\frac{2}{3}\times\frac{1}{2^{p-q}}$$

$$=\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)^{p-q}.$$

Hence

$$a=\frac{2}{3},\quad \beta=\frac{1}{\sqrt{2}}=\cdot 7071,$$

whence we deduce

$$\eta=\cdot 3100,\quad \gamma\eta=\cdot 3971.$$

Our series is therefore

$$x_0-\bar{x}_0=\cdot 5617\,(x_1-\bar{x}_1)+\cdot 2463\,(x_2-\bar{x}_2)+\cdot 1080\,(x_3-\bar{x}_3)+\cdot 0474\,(x_4-\bar{x}_4)+\text{etc.},$$

this giving slightly more influence to the parents and less to the ancestry than Galton's Rule.

The correlation of brothers is ·4896, giving

$$\Sigma_0=\sigma_0\times\cdot 8720,$$

or a reduction of variability of about 13 °/₀.

Not much stress, however, must be laid on these results for a single Mendelian unit character. They are the furthest from true linear regression of the mid-parent and theoretically only two grades of the character occur.

Application III. Suppose the correlations are given by the Mendelian gametic values $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.

We have

$$r_{p-q}=(\sqrt{2})^{p-q}\times\left(\frac{1}{2}\right)^{p-q}=\left(\frac{1}{\sqrt{2}}\right)^{p-q}.$$

Hence

$$a=1,\quad \beta=\frac{1}{\sqrt{2}}.$$

It follows that

$$\eta=0,\quad \gamma\eta=\beta=\frac{1}{\sqrt{2}}.$$

Hence the prediction formula reduces to

$$x_0-\bar{x}_0=X_p-\bar{X}_p,$$

or the parents suffice to determine the offspring. This is in complete accordance with the Mendelian idea of the gametic parental constitution determining the offspring.

Application IV. Return now to the case of a continuous variable for which the law of ancestral heredity was first developed. Observation seems to show that we have nearly $\frac{1}{2}$, $\frac{1}{2}\times\frac{2}{3}$, $\frac{1}{2}\times(\frac{2}{3})^2$, etc.

Hence $$r_{p-q}=(\surd 2)^{p-q}\tfrac{1}{2}\times(\tfrac{2}{3})^{p-q},$$

or $$a=\tfrac{1}{2},\quad \beta=\left(\frac{2\surd 2}{3}\right)=\cdot 9428,$$

whence $$\eta^2-2\cdot 12134\eta+1=0,$$

$$\eta=\cdot 70712=\frac{1}{\surd 2},\quad \gamma\eta=\cdot 2357,$$

$$\gamma=\cdot 3333,\quad \eta\surd 2=1\cdot 0000.$$

Thus the series becomes very nearly

$$x_0-\bar{x}_0=\tfrac{1}{3}(X_1-\bar{X}_1+X_2-\bar{X}_2+X_3-\bar{X}_3+\ldots).$$

This result is noteworthy; it shows that with the values found, three generations would suffice to produce offspring with the full selected character.

Here every individual ancestor provides on the average *half* the contribution of the ancestor one grade closer and not one quarter as in Galton's Rule. Hence by continual selection, we could advance a character *beyond* the selected value. How far such a selection is possible we do not know. The correlation with other characters would probably introduce counteracting selection.

We have in this case

$$\Sigma_0=\sigma_0\sqrt{1-R^2}=\sigma_0\sqrt{1-(\cdot 5773)^2},$$

whence the correlation for brother $=\cdot 5773$, and

$$\Sigma_0=\sigma_0\times\cdot 8165,$$

or we have an 18 °/ₒ reduction in variability by in-breeding.

Galton's Condition.

Galton made it a condition of his series that if *all* the ancestry were selected and of value h, the offspring should have the same deviation h.

In our case $$x_0-\bar{x}_0=\gamma\{\eta\surd 2(X_1-\bar{X}_1)+(\eta\surd 2)^2(X_2-\bar{X}_2)+\text{etc.}\}$$
$$=\gamma\eta\surd 2\,h/(1-\eta\surd 2),$$

if all the ancestry deviate by h. Hence

$$\gamma\eta\surd 2/(1-\eta\surd 2)=1.$$

This is the generalised Galton condition.

It is very doubtful, however, whether it ever holds. Supposing it to hold, we may select for n generations only, then

$$x-x_0=\gamma\{\eta\sqrt{2}h+(\eta\surd 2)^2h+\ldots+(\eta\surd 2)^n h\},$$

if the other mid-parents were non-selected.

Hence the offspring h' $$h'=\frac{\gamma\eta\surd 2/\{1-(\eta\surd 2)^n h\}}{1-\eta\surd 2},$$

$$h'=h\{1-(\eta\surd 2)^n\},$$

by the condition. Thus if $\eta\surd 2$ differ at all from unity, we rapidly get the full effect of selection, i.e. offspring $=h$. If, however, we stop selection at the nth generation, we have for the offspring of this generation with in-breeding

$$\begin{aligned}x-x_0&=\gamma[\eta\sqrt{2}h\{1-(\eta\surd 2)^n\}+(\eta\surd 2)^2h+(\eta\surd 2)^3h+(\eta\surd 2)^{n+1}h]\\&=\gamma\{\eta\sqrt{2}h+(\eta\surd 2)^2h+(\eta\surd 2)^3h+\ldots+(\eta\surd 2)^n h\}\\&=\frac{\gamma\eta\surd 2}{1-\eta\surd 2}\{1-(\eta\surd 2)^n\}h\\&=\{1-(\eta\surd 2)^n\}h=h',\end{aligned}$$

or, if selection be stopped at any stage, there will on in-breeding be *no* regression.

It is a remarkable fact that many biologists have accepted Galton's theory of regression, without seeing that there is no regression whatever on Galton's hypothesis after the first generation, at which selection is stopped! This misunderstanding arises from the common belief that a word carries its own definition and that it is not needful to study its algebraical significance.

Looked at as a whole it appears to me that the theory of multiple correlation is the natural manner in which to approach the theory of ancestral inheritance. The work done on man shows that for all sorts of types of measurable characters we have true linear regression, and that our correlations of *measurable* characters in all species hitherto dealt with are singularly constant. Further these correlations appear for the same grades of relationship to be the same for measurable and non-measurable characters. We are largely compelled to measure ancestral correlations by colour characters as no other data are available, but the equality of pigmental and of measurable-character correlations for grades we can compare gives us confidence in testing the decadence of resemblance on pigmental characters. And the law of geometrical decadence being once established we have a multiple regression formula which may be legitimately applied to all measurable characters.

The fact that the Mendelian gametic correlations approach in some respects those found by observation on populations, is not a justification of Mendelism. It is only an indication that for such special cases as "unit" characters, even if they exist, there is no absurdity in our geometrical law of decadence. Most biometricians, however, who have measured and observed characters in man or animals have not been able to classify into A and not-A, but believe that there are many grades of A and not-A and that probably every one of these are capable of selection and of inheritance. Thus they look to continuous or at least multiple variations of the germ-plasm with regard to any single character, and not to the mere presence or absence of a *single* determinant.

LITERATURE.

F. Galton: *Natural Inheritance*, 1889, pp. 133, 135—7.

Galton's correlation values (parental $\frac{1}{3}$, grandparental $\frac{1}{9}$, etc.) are not in accord with wider observation, and they are opposed to the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. of his own law of "contributions."

K. Pearson: *On the Law of Ancestral Heredity*, *Royal Soc. Proc.* Vol. 62, 1898, pp. 386—412.

This is the first attempt to apply the formulae of multiple regression to the problem of heredity. It endeavours to show how Galton's correlations must be brought into touch with his "contributions."

The paper is now inadequate because: (i) we now know much better values of the ancestral correlations—we had then only very few data; and (ii) there are one or two bad algebraical slips. These are corrected in

K. Pearson: *The Law of Ancestral Heredity*, *Biometrika*, Vol. II. p. 211.

But another stupid blunder is made in this paper on p. 224, where by an oversight a and r are given the individual and not the mid-parental values. I hope this has been finally put right in this note.

Galton contributed a paper, *R. Soc. Proc.* Vol. 61, p. 402, on the Ancestral Law in Basset Hounds, but this is really written from a somewhat different standpoint; that standpoint has been considered by me in a paper on the *Law of Reversion*, *R. Soc. Proc.* Vol. 66, p. 140.

There has been a great deal of perfectly idle criticism of the Law of Ancestral Heredity (Archdall Reid, Bateson, Darbishire) principally based on a crude application of Galton's original $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ... contributions to individuals with *selected* ancestry (Darbishire, Castle, etc.). The least that can be said of these criticisms is that the writers had not grasped the fundamental idea of correlation before they started to criticise the ancestral law.

VI. The Inheritance of Fertility.

By R. E. LLOYD, D.Sc. (Lond.), Professor of Biology, Medical College, Calcutta.

In a recent number of *Biometrika* Professor Pearson has criticised a short paper of mine which dealt with the fertility of rats*. This paper was in the main a record of observations which I knew to be accurate. No new principle was deduced by me from these observations, but I pointed out that they were opposed to a certain principle which had been enunciated by Professor Pearson and was now occupying a prominent position in a well known text-book. I mention this here since in summing up his criticism of my work and that of others Professor Pearson writes—"These are far from the only cases in which principles bearing on Evolution have been propounded on the basis of researches which prove in fact their negation." I do not understand this passage in reference to my own paper. I propounded no principle bearing on Evolution or any other subject, I merely opposed a principle which had been propounded by another.

Professor Pearson has justly shown that I overlooked or at any rate did not point out that there was a very sensible relation between the weight of a rat and the number of its offspring. From my figures he demonstrates clearly that on the average large rats are slightly more fertile than small ones. He also points out that this fact is appreciable in my tables even before they have received mathematical treatment. This is undoubtedly true.

On first reading the criticism, I thought that those who might read it without reference to the matter criticised, would receive the impression that I had written wholly in order to show that there was no correlation between weight and fertility in rats and that upon this demonstration I had set up some biological principle which was hardly worth mentioning.

The impression that my paper would appear in this light was confirmed by an abstract of the criticism which has just been published in the following words,—"Lloyd has maintained that there is no correlation between size and fertility in rats, but it is shown that his tables indicate quite sensible correlation."—I saw therefore that the main theme of my paper had become obscured owing to Professor Pearson's critical treatment of what I regard as a side issue.

The principle enunciated by Professor Pearson which was the subject of my remarks, is as follows—"Fertility is not uniformly distributed among all individuals but for stable races there is a strong tendency for the character of maximum fertility to become one with the character which is the type." I pointed out that my figures did not support this principle. My words are (A) "The maximum fertility of rats as measured by the number of young which they produce at a birth is not one with the character which is the type, as regards size" and again later on I say (B) "There is clear evidence that the largest and the smallest rats are quite as fertile as those of average size." Professor Pearson chooses the second sentence for criticism and shows that it contains an untruth, but surely it must be obvious to anyone reading my paper that its main theme is concentrated in the first sentence (A). In speaking of the relation between weight and fertility Professor Pearson says—"But it is not a relation in which the type rat is the most fertile." This was my view precisely, it was indeed the chief view expressed in my paper. I cannot therefore admit that there is justice in Professor Pearson's remark—"It will be seen at once that Captain Lloyd's view is not supported by his own data." By inspection of the table I concluded that rats of normal size were not the most fertile. Professor Pearson came to the same conclusion and further pointed out that large rats were slightly more fertile than small ones.

---

* *Records Indian Museum*, Vol. III. Pt. III.

1. The conclusion that there is a strong tendency for the character of maximum fertility to become one with the character which is the type is in my opinion unjustifiable in any case, because it seems that an individual cannot represent the type of its race as regards all its features. We can only speak of a typical individual when we are dealing with one measurable feature at a time. I use the word feature here because an attribute of an organism which can be measured is not necessarily a character in the Mendelian sense although it may be in certain cases, as for example in the case of total size. If then I am right in supposing that an individual cannot represent that notion which we call the type of a race, the idea that type individuals are more fertile than their fellows is erroneous.

2. Two questions have been raised in this discussion. (A) Are type rats the most fertile? (B) Are heavy rats the most fertile? The subject of fertility is of special interest; I hope therefore that I may be permitted to raise a third question which has a direct connection with the subject and with my own observations.

Professor Pearson inclines to the opinion that fertility is never inherited. He speaks of "The absence of inheritance in the case of fertility which seems to be a necessary foundation of the idea of Darwinian evolution." This conclusion is based upon observations which have shown that in mankind, mice, swine and hens, fertility was not being inherited at the time of investigation. I may mention that the same results may be obtained from the silk-moth *Attacus ricini*, if all the eggs of a 200 egg mother and of a 300 egg mother be raised,—both mothers being selected from the same brood—there will be no appreciable difference in the fertility of their offspring as measured by the number of their eggs. But these cases do not prove that differences of fertility are not inherited in any case.

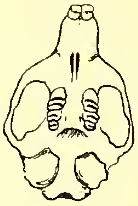
The fertility of a race may be measured by the average number of offspring produced by the mothers during their lifetime, although of course it is usually impossible to measure it in this manner. As estimated in this way, fertility varies among the many species from less than ten to many millions. I cannot understand how these differences of fertility have become established if, as Professor Pearson supposes, fertility is never inherited. It may be that there is some simple flaw in my reasoning but I have sought in vain for it. No one doubts that the differences of form which are characteristic of the various species have been attained through the inheritance of variations, i.e. appreciable differences between parent and offspring, and yet we are asked to believe that the various degrees of fertility, which are just as characteristic, have been attained in some other way, unimagined at present.

Since I cannot understand how specific differences can come into being otherwise than through the inheritance of variations, I conclude that fertility must be inherited on certain occasions. Hence, a demonstration such as that made by Raymond Pearl that in a certain case fertility is not inherited is to me a clear demonstration of the fact that variation is of two kinds, inheritable and uninheritable, i.e. mutation and fluctuation.

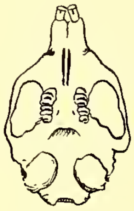
Let me illustrate this question by a particular case. There are in India two kinds of field rats known by the generic names Gunomys and Nesokia. Without referring to the many ways in which these two genera resemble one another, it may be said that they differ from one another in two respects. Gunomys produces larger litters than Nesokia, on the average in the proportion of 8 : 4. Nesokia has larger teeth than Gunomys. These facts are well known and may be verified from specimens deposited in the larger Museums.

The female Gunomys is equipped for the nourishment of a large family, along either side she has a continuous row of teats which extend from breast to groin. The Nesokia has teats on the breast and in the groin but not a continuous series along the sides. Gunomys extends throughout India and beyond its limits. Nesokia is confined to the north-western parts of the peninsula. In certain parts of the Punjab, as for example in the Amritzar district, the two genera live side by side in the same fields, though not of course in the same burrows. The fact that each kind has received a generic name of its own has enhanced the difference between them.

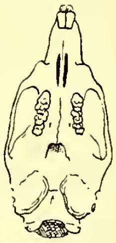
I will now dwell for a moment on their resemblances as they can be seen at Amritzar. I have elsewhere related of a certain observer, who although in the habit of handling large numbers of each kind daily, yet regarded them as of one race. Probably most people would have made the same mistake. The teats are hidden in the fur. The dental difference is not obvious until the skull has been extracted and cleaned. Gunomys is on the average larger than Nesokia but their ranges of variation overlap in this respect. The fur is very variable, but exactly the same varieties of tint and texture are to be found in both genera. In the investigations round Amritzar large numbers of the burrows of both kinds were opened up, and it was finally decided that there must be two races present because the nests were found to contain either few or many young ones. This conclusion was of course fully borne out when the number of the teats was noticed. I dwell on these points in order to demonstrate the remarkable resemblance between these two genera. This can be illustrated in another way. It is well known that among mammals the form of the skull is one of the most definite points for classification. The various domestic races even possess their own type of skull in many cases. But if we examine the skulls of these two genera we find a remarkable degree of resemblance. When looked at from above or from the side the two kinds cannot be distinguished from one another with certainty, but from below they can be recognised at a glance. The palatine foramen of Nesokia is about half as long as that of Gunomys. The roots of the incisor teeth come to an end on either side of the palatine foramen; in Gunomys they cause a slight narrowing of the posterior part of the foramina, but in Nesokia in which all the teeth are larger, they encroach so much upon these foramina as to abolish them posteriorly. These points are shown in the diagrams which have been traced from photographs. The great resemblance in general form between the two skulls on the left is obvious, the skull of a Mus is shown in contrast.



Nesokia



Gunomys



Mus

Except for the small difference in the teeth and palatine foramen, the difference in fertility is the only essential point of distinction between the two genera. No one who has examined the skulls of these animals will doubt that the two kinds are very closely related genetically. As regards the shortness of the palatine foramen, Nesokia stands alone among the different genera of the Muridae; this is good reason for supposing that Nesokia was derived from Gunomys, a view which is supported by their Geographical distribution. But for the sake of our argument it is unnecessary to press this point. Let us suppose that the Gunomys was derived from the Nesokia or both from a common stock. In any case how can they have arrived at their present condition unless fertility was inherited? The more I think over this case, the surer I feel that differences of fertility must have been inherited. Can Professor Pearson show us the way out of the difficulty?

3. We do not know how the one genus was derived from the other. No one observed the event or events which led to the establishment of the newer genus, but we can do more than guess at the manner of them. Our imagination may be guided by certain knowledge of other events which have been observed. We know that a pair of guinea-pigs in captivity may give rise to a hundred or more healthy looking members of their own kind in a few years. We also

know that a few rabbits can stock a continent with their kind. May not, therefore, all the Nesokias be derived from an original pair or two which were in the first case the offspring of Gunomys?

The supposition that every species was derived from a single pair is so ancient as to appear ridiculous, but the scientist can only say that there is no evidence in favour of it. But is there no evidence in favour of it? If it can be shown that there are in the world groups of animals of very limited numerical strength (less than a hundred) each member of which possesses certain special characters uniting it with its group fellows and separating it from all other animals, the demonstration will in my opinion afford evidence in favour of this supposition. I have shown elsewhere* that such groups not only exist, but are common, at least among the rats of India. In order to obtain knowledge of these groups it is necessary to examine animals in very large numbers and over wide areas. At the present day such groups are overlooked because it is the custom of naturalists to search for new species. A new species is in itself considered to be an interesting thing. A type specimen is chosen as a representative of the species and the supposition is thereby made that this type specimen is one of an unknown but large number of like animals. But the numerical strength of the "species" is never enquired into, it may not be large, it may be ten, fifty or a hundred.

A new species may be confined to a single house or to the corner of a field or even to a single nest. Rare species are indeed plentiful. It may be asked—"What has all this to do with the question of fertility?" I will endeavour to show the connection. A small group can only become a large one when production is in excess of elimination. A new group will grow up among an old group either because its new features are of life-saving value to it and tend to reduce elimination, or because the new group has a higher rate of production than the old one. Since the distinguishing marks of species do not as a rule appear to be of life-saving value to their possessors, I conclude that in many cases the new groups must grow in numbers simply because they have a higher rate of production, but this conclusion is untenable if we are to hold with Professor Pearson that differences of fertility are never inherited.

VII. Remarks on Professor Lloyd's Note on Inheritance of Fertility.

By KARL PEARSON, F.R.S.

Professor Lloyd says that no new principle was deduced by him from his observations, and again that he "propounded no principle bearing on Evolution or any other subject." Either he has overlooked what he himself wrote, or else he must have very vague ideas of what does bear on Evolution. Yet he wrote p. 262 of his memoir: "The result obtained was quite unexpected [presumably therefore it was *new*]. The maximum fertility of rats (as measured by the number of young which they produce at a birth) is not one with the character which is the type as regards size. In other words, gigantic and dwarfed rats are just as fertile as common rats of average size."

Thus Professor Lloyd himself says that as far as size was concerned there was no "maximum fertility." Professor Lloyd used this result to combat the principle that, if there were a maximum fertility, it must be associated with the modal value of the character, or the race could not be stable. How where there was in his opinion no maximum fertility observable in his experiments, he could use its non-existence to combat a statement of where it would occur, if it existed, I fail to understand.

He further states that he laid no stress on this (B) absence of differential fertility in rats, and that he did lay stress on (A) the maximum fertility of rats not being associated with the type as far as size is concerned. As he had, according to his own interpretation of his

---

* *Records Indian Museum*, Vol. III. Pt. I. and Vol. V. Pt. II.

observations, shown that there was no maximum fertility at all, it is hard to comprehend what value he set on his observations. Apparently, he desired to show that a maximum fertility could exist not associated with the type character. That I take it would be a new principle, and one which I should have said would have an enormous bearing on Evolution. As a matter of fact in the "Conclusions" given in his memoir (p. 264), there is no reference to (A), but the result (B) he emphasises by *italics*.

"There is clear evidence that the largest and smallest rats *are quite as fertile as those of average size*" (p. 264). As there was "clear" evidence of the exact opposite of this statement in his observations, and as the age and number of litters of his rats were not given, it was quite impossible to determine from Captain Lloyd's data, whether (i) there was any maximum fertility at all associated with size and (ii) if there were, with what size it was associated.

Professor Lloyd drew the very definite conclusion that fertility was not related to weight in his rats; Fraülein Hanel drew the very definite conclusion that there was no inheritance within the "pure line" in her *Hydra*. Both of these conclusions were erroneous, as the most elementary statistical examination would have shown either of them. I used both to illustrate the point thrust on me daily by the examination of many memoirs that biology cannot safely do without biometry. Professor Lloyd states that some principle enunciated by me is "now occupying a prominent position in a well-known text-book." I cannot be responsible for that text-book, whichever it may be, nor how the principle may be stated. The essential contributions I have made to this supposed principle may be summed up as follows.

(i) Genetic selection, i.e. fertility correlated with a somatic character will modify natural selection, unless the modal somatic character exhibits the maximum fertility. A progressive change in type would follow any other association until the modal value became that of maximum fertility. (*Phil. Trans.* Vol. 187, A, 1896, p. 258; Vol. 192, A, 1899, p. 258, etc.)

I cannot see any flaw in this argument whatever.

(ii) Genetic selection either does not exist or if it does there is instability in the race.

In the *Grammar of Science* (2nd Edition, 1900, p. 440 *et seq.*) I referred to two species of flowers in which I had found the modal capsules to contain the greatest bulk of fertile seed. If I had found non-modal capsules to contain such bulk of fertile seed, I should, having shewn that the character I was dealing with was inherited, have argued that the plant in question was changing or could change its type independently of natural selection, i.e. that genetic selection would at least modify if it did not overmaster natural selection. If Professor Lloyd had been right in asserting that there was no relation between fertility and weight in his rats then there could be no genetic selection, and what he calls my principle could not come into play. For the basis of that principle lies in the words: "If fertility be correlated with any organ or character." Actually Professor Lloyd's data showed a considerable correlation between weight and size of litter; and if this had been correct then genetic selection would have come into play and he would have reached an important result—just the reverse of what he himself drew from his own statistics!—As a matter of fact, I think his material was vitiated because he had not inquired into the age factor and the correlation of age and size of litter in his rats. There was nothing in his material which would demonstrate or refute the principle that when fertility-differentiation exists and is correlated with somatic character, then the modal character must be associated with the maximum fertility, or the race will lack stability in type, until this association is attained. Professor Lloyd remarks:

> "The conclusion that there is a strong tendency for the character of maximum fertility to become one with the character which is the type is in my opinion unjustifiable in any case, because it seems that an individual cannot represent the type of its race as regards all its features. We can only speak of a typical individual when we are dealing with one measurable feature at a time."

I don't think I ever read a passage which shows greater need for biometric training in a biologist! Can Professor Lloyd have the least conception of what are the leading features of a multiple frequency surface? Has he never heard of the "mean man" of Quetelet, or of Edgeworth's defence of that "mean man's" actuality? The uniqueness of the mode in most multiple frequency surfaces is a well-established fact, and if Professor Lloyd will strive to see what follows a differential fertility correlated with a non-modal somatic character, he will quickly discover how few generations suffice to make that somatic character the modal value.

Professor Lloyd asserted on the basis of his rats that there was no relation of fertility to weight, he now says that I agree with him that there was no relation of modal weight to fertility. I regret that I do nothing of the kind. Knowing that age is related to weight and to size of litter I am quite unable to assert that his data prove or disprove anything at all with regard to fertility. I am only in a position to say that I have not, hitherto, been able to find a *marked* inheritance of fertility *within the race*, and begin to doubt its existence*. Professor Lloyd now turns round and leaves the question of fertility within the race which he had been discussing and to which his data applied, and says that in two races, which he himself differentiates by the *number of teats* with which they are provided, one has a greater fertility than the other. That is to say he passes from an intraracial to an interracial problem and from weight of rat to number of teats without apparently noticing the jump. Fertility pure and simple may be correlated with many things. Net fertility in man is correlated with the size of the pelvis, but because the pelvis is a character transmitted by heredity, it does not follow that fertility pure and simple is an hereditary character. The slight intensity of the inheritance of fertility such as we find intraracially in man and other mammals is quite compatible with its being only indirectly transferred because it is correlated with directly inherited characters. If Professor Lloyd wishes to meet my point, he should correlate fertility intraracially with the number of teats and then investigate whether this correlation is zero, or whether the modal number of teats is associated with the maximum fertility. If neither of these things be true, then I am quite sure that he will discover that his race is unstable and rapidly changing its type as far as teats are concerned. But a simple statement that two races of rats have different type fertilities does not seem to me to have any application to what he calls my "principle"—the less so when he tells us that their modal number of teats differ significantly. The appearance of additional teats may be the character, or one of the characters, which may lead to increased size of litter, but this has nothing to do with the intraracial inheritance of fertility pure and simple. It surely confirms my point that instability follows association of fertility with a non-modal character.

---

* Many investigations have shown some inheritance of fertility: see for example my own paper with Alice Lee and Bramley Moore of 1899 (*Phil. Trans.*, Vol. 192, pp. 257—330) on fertility in man and in the thoroughbred race horse. But later work by others on man, mice, swine, poultry, etc. seems to me to indicate that fertility is not directly inherited, but only to a secondary degree as being correlated with inherited physical characters. The sort of values one finds for the heredity of fertility range from ·05 to ·15. I think that in 1899 I laid too much stress on the possibility of the *direct* inheritance of fertility, but this was an inference formed before I had determined the high parental correlation of 0·5 for physical characters. I thought then that disturbing factors might possibly reduce Galton's value of $\frac{1}{3}$ for the parental correlation to the low values we had found in the case of man and horse. I do not think they could reduce $\frac{1}{2}$ to those values.

VIII. On the Probability that two Independent Distributions of Frequency are really Samples from the same Population.

By KARL PEARSON, F.R.S.

(1) In a memoir contributed to the *Phil. Mag.* 1900 (Vol. 50, p. 157) I have dealt with the problem of the probability that a given distribution of frequency was a sample from a *known* population. That investigation was the basis of my treatment of the "goodness of fit" of theory and frequency samples. The present problem is of a somewhat different kind, but is essentially as important in character. We have two samples, and *a priori* they may be of the same population or of different populations; we desire to find out what is the probability that they are random samples of the same population. This population is one, however, of which we have no *a priori* experience. It is quite easy to state innumerable problems in which such knowledge is desirable. We have two records of the number of rooms in houses where (i) a case of cancer has occurred, (ii) a case of tuberculosis has occurred; the number of cases of each disease may be quite different, and we may not be acquainted with the frequency distribution of the number of rooms in the given district. What is the chance that there is a significant difference in the tuberculosis and the cancer houses? Or again, we have a frequency distribution of the interval in days between bite and onset of rabies in two populations of bitten persons (i) who have been and (ii) who have not been inoculated in the interval. What is the probability that the inoculation has modified the interval? Many other illustrations will occur to those who are dealing with statistics, but the above will suffice to indicate the nature of the problems I have in view.

(2) Let the population from which the two samples, if undifferentiated, are supposed to be drawn be given by the class-frequencies

$$\mu_1, \quad \mu_2, \quad \mu_3, \quad \mu_4 \ldots \mu_p, \quad \mu_q \ldots \mu_s,$$

the total population being M.

Let the samples be given by the frequencies in the same classes:

									Total
1st Sample	f_1	f_2	f_3	......	f_p	f_q	......	f_s	N
2nd Sample	f_1'	f_2'	f_3'	......	f_p'	f_q'	......	f_s'	N'

where the totals N and N' differ as widely or as little as we please. Let $\Sigma_1, \Sigma_2, \Sigma_3 \ldots \Sigma_p, \Sigma_q \ldots \Sigma_s$ be the standard deviations of the frequencies of the first sample, $\Sigma_1', \Sigma_2', \Sigma_3' \ldots \Sigma_p', \Sigma_q' \ldots \Sigma_s'$ be the standard deviations of the frequencies of the second sample, and R_{pq}, R_{pq}' be the correlations of the pth and qth frequencies of the two samples. Now the two samples are supposed to be absolutely independent. Hence there will be no correlation between any deviation in any frequency of the first row and the deviation of any frequency in the second row. Further the two frequency distributions being by hypothesis random samples of the population M, we have:

$$\Sigma_p^2 = N \frac{\mu_p}{M}\left(1 - \frac{\mu_p}{M}\right), \quad \Sigma_p'^2 = N' \frac{\mu_p}{M}\left(1 - \frac{\mu_p}{M}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

$$\Sigma_p \Sigma_q R_{pq} = -N \frac{\mu_p}{M}\frac{\mu_q}{M}, \quad \Sigma_p' \Sigma_q' R_{pq}' = -N' \frac{\mu_p}{M}\frac{\mu_q}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}.$$

Now consider the system of variables obtained by reducing the frequencies of the two samples N, N' to a common standard total n (e.g. to per milles or per cents.), and subtracting the differences of each class-frequency.

Let the new system of variables be :

$$x_p = n\left(\frac{f_p}{N} - \frac{f_p'}{N'}\right), \quad x_q = n\left(\frac{f_q}{N} - \frac{f_q'}{N'}\right) \text{ ....(iii).}$$

Now on the hypothesis of both frequencies being random samples of the *same* population :

$$\frac{\bar{f}_p}{N} = \frac{\bar{f}_p'}{N'} = \frac{\mu_p}{M},$$

$$\frac{\bar{f}_q}{N} = \frac{\bar{f}_q'}{N'} = \frac{\mu_q}{M}$$

and the mean values, $\bar{x}_p$ and $\bar{x}_q$ will be zero.

Further let σ_p and σ_q be the standard deviations of x_p and x_q, and r_{pq} the correlation of the latter pair. Then bearing in mind what has been said about the independence of the frequencies in the two series, we have at once :

$$\sigma_p^2 = n^2\left(\frac{\Sigma_p^2}{N^2} + \frac{\Sigma_p^{2'}}{N'^2}\right) \text{ ....(iv),}$$

$$\sigma_p \sigma_q r_{pq} = n^2\left(\frac{\Sigma_p \Sigma_q R_{pq}}{N^2} + \frac{\Sigma_p' \Sigma_q' R_{pq}'}{N'^2}\right) \text{ ....(v).}$$

Whence we find from (i) and (ii)

$$\sigma_p^2 = n^2\left(\frac{1}{N} + \frac{1}{N'}\right)\frac{\mu_p}{M}\left(1 - \frac{\mu_p}{M}\right) \text{ ....(vi),}$$

$$\sigma_p \sigma_q r_{pq} = -n^2\left(\frac{1}{N} + \frac{1}{N'}\right)\frac{\mu_p \mu_q}{M^2} \text{ ....(vii).}$$

Now write

$$\mu_p' = n^2\left(\frac{1}{N} + \frac{1}{N'}\right)\frac{\mu_p}{M} \text{ ....(viii),}$$

$$\mu_q' = n^2\left(\frac{1}{N} + \frac{1}{N'}\right)\frac{\mu_q}{M} \text{ ....(ix),}$$

and

$$M' = n^2\left(\frac{1}{N} + \frac{1}{N'}\right) \text{ ....(x).}$$

Then

$$\mu_p'/M' = \mu_p/M,$$

and since $S_1^s(\mu_p) = M$, it follows that $S_1^s(\mu_p') = M'$, or M' is the total population of a frequency distribution

$$\mu_1', \ \mu_2', \ \mu_3', \ \ldots \ \mu_p', \ \mu_q', \ \ldots \ \mu_s'.$$

Further

$$\sigma_p^2 = \mu_p'\left(1 - \frac{\mu_p'}{M'}\right) \text{ ....(xi),}$$

$$\sigma_p \sigma_q r_{pq} = -\frac{\mu_p' \mu_q'}{M'} \text{ ....(xii).}$$

Now (xi) and (xii) are absolutely identical with the type-standard-deviation and type-correlation of the frequency distribution of the system $x_1, x_2, x_3, \ldots x_p, x_q, \ldots x_s$ as measured from a theoretical system $\mu_1', \mu_2', \ldots \mu_p', \mu_q', \ldots \mu_s'$. They agree in form with equations (vii) and (viii) of my paper "On the Criterion that a given System of Deviations from the Probable in the case of a Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling," *Phil. Mag.*, Vol. 50, 1900, p. 161.

Hence we have only to form

$$\chi^2 = S_1^s\left(\frac{x_p^2}{\mu_p'}\right) \text{ ....(xiii),}$$

and having determined its value use Palin Elderton's Tables in *Biometrika*, Vol. I. p. 155.

Clearly
$$\chi^2 = S_1^s \left\{ \frac{n^2 \left(\frac{f_p}{N} - \frac{f_p'}{N'} \right)^2}{n^2 \left(\frac{1}{N} + \frac{1}{N'} \right) \frac{\mu_p}{M}} \right\}$$

$$= S_1^s \left\{ \frac{\left(\frac{f_p}{N} - \frac{f_p'}{N'} \right)^2}{\left(\frac{1}{N} + \frac{1}{N'} \right) \frac{\mu_p}{M}} \right\} \quad \text{..(xiv).}$$

It remains, exactly as in my paper referred to (pp. 164—6), to select the most reasonable value for μ_p/M, the proportional value of the pth frequency class in the population from which both samples are by hypothesis selected.

Now the best hypothesis as to the constitution of this population, on the assumption that both frequencies are random samples of it, will be that its pth frequency class is that indicated by the combined two samples, i.e. that

$$f_p + f_p' \text{ is proportional to } \mu_p, \quad \text{or} \quad (f_p + f_p')/(N + N') = \mu_p/M.$$

Substituting this value we find

$$\chi^2 = S_1^s \left\{ \frac{NN' \left(\frac{f_p}{N} - \frac{f_p'}{N'} \right)^2}{f_p + f_p'} \right\} \quad \text{....................................(xv).}$$

The calculation of χ^2 now presents no difficulties in any actual case.

(3) *Illustration I.* Let us inquire whether hair colour exercises a differential selection with regard to the incidence of scarlet fever and measles.

The following data are provided by Dr Macdonald, *Biometrika*, Vol. VIII. p. 28, for all scarlet fever and measles cases:

Hair Colour.

		Black	Dark	Medium	Fair	Red		Totals
Scarlet Fever	(i)	12	289	1109	360	94	f	1864
Measles ...	(ii)	0	85	367	184	25	f'	661
(i)+(ii) ...	(iii)	12	374	1476	544	119	$f+f'$	2525
(i)/1864 ...	(iv)	·0064	·1551	·5950	·1931	·0504	f/N	1·0000
(ii)/661 ...	(v)	·0000	·1286	·5552	·2784	·0398	f'/N'	1·0000
(iv) − (v) ...	(vi)	+ ·0064	+ ·0265	+ ·0398	− ·0853	+ ·0126	$f/N - f'/N'$	—
Square of (vi)	(vii)	·000,041	·000,702	·001,584	·007,276	·000,159	$(f/N - f'/N)^2$	—
(vii) ÷ (iii) ...	(viii)	·000,0034	·000,0019	·000,0011	·000,0134	·000,0013	$\frac{(f/N - f'/N)^2}{f+f'}$	·000,0211

Therefore
$$\chi^2 = NN' \times {\cdot}000{,}0211$$
$$= 1864 \times 661 \times {\cdot}000{,}0211$$
$$= 26{\cdot}00.$$

P from the tables (*Biometrika*, Vol. I. p. 155) is about ·000,03. In other words the odds are more than 33,000 to 1 against the occurrence of two such divergent samples of hair colour if they were *random* samples from the same population. I think we can conclude that they are really differentiated samples, or that scarlet fever and measles do not attack indifferently all individuals whatever their hair pigmentation; or, that scarlet fever and measles are differential in their selection. (I suppose, of course, *that no personal equation actually existed between the pigmentation recorders for measles and scarlet fever.*)

Illustration II. In the Scarlet Fever Cases, Dr Macdonald has added together the Belvidere and Ruchill contingents, the measles being based solely on Ruchill returns. I next asked whether this might be the source of the differentiation observed. The following table gives the work, when we take Ruchill scarlet fever and measles cases only. Thus comparing the two diseases at one hospital, we find

$$\chi^2 = 661 \times 1295 \times {\cdot}000{,}0301 = 25{\cdot}77,$$

or the improbability is precisely the same. The observed result is therefore not due to the combination of the Belvidere and Ruchill observations.

Hair Colour.

		Black	Dark	Medium	Fair	Red		Totals
Scarlet Fever	(i)	8	214	749	251	73	f	1295
Measles ...	(ii)	0	85	367	184	25	f'	661
(i)+(ii) ...	(iii)	8	299	1116	435	98	$f+f'$	1956
(i)/1295 ...	(iv)	·0062	·1652	·5784	·1938	·0564	f/N	1·0000
(ii)/661 ...	(v)	·0000	·1286	·5552	·2784	·0378	f'/N'	1·0000
(iv) − (v) ...	(vi)	+ ·0062	+ ·0366	+ ·0232	− ·0846	+ ·0186	$f/N - f'/N'$	0·0000
(iv) − (v)^2 ...	(vii)	·000,041	·001,340	·000,538	·007,157	·000,346	$(f/N - f'/N')^2$	—
(vii)/(iii) ...	(viii)	·000,0051	·000,0045	·000,0005	·000,0165	·000,0035	$\dfrac{(f/N - f'/N')^2}{f+f'}$	·000,0301

Illustration III. One further point occurred to me as worthy of investigation in Dr Macdonald's data. To what extent is it desirable to club Boys and Girls together in an inquiry of this kind?

For Scarlet Fever in Table III. p. 23, we have:

Hair Colour.

		Black	Dark	Medium	Fair	Red		Totals
Girls ...	(i)	3	137	389	138	38	f	705
Boys ...	(ii)	5	77	360	113	35	f'	590
(i)+(ii) ...	(iii)	8	214	749	251	73	$f+f'$	1295
(i)/705 ...	(iv)	·0043	·1943	·5518	·1957	·0539	f/N	1·0000
(ii)/590 ...	(v)	·0085	·1305	·6102	·1915	·0593	f'/N'	1·0000
(iv) − (v) ...	(vi)	− ·0042	+ ·0638	− ·0584	+ ·0042	− ·0054	$f/N - f'/N'$	0·0000
(iv) − (v)^2 ...	(vii)	·000,0176	·004,0704	·003,4106	·000,0176	·000,0292	$(f/N - f'/N')^2$	—
(vii) ÷ (iii) ...	(viii)	·000,0022	·000,0200	·000,0046	·000,0001	·000,0004	$\dfrac{(f/N - f'/N')^2}{f+f'}$	·000,0273

Hence $$\chi^2 = 705 \times 590 \times \cdot 000{,}0273 = 11\cdot 36.$$

This gives us $P = \cdot 0265$, or the odds are about 37 to 1 against the boys and girls being random samples of the same material. These are nothing like the odds obtained in the scarlet fever and measles differentiation of pigmentation; but they are sufficiently considerable to render it desirable in future investigations to keep the sexes separate.

The above illustrations were simply selected because I happen to have Dr Macdonald's results before me, and it seemed desirable to ascertain whether there was a selective disease incidence by a method which would not appeal to statistics of the general population made for another purpose. The main point brought out is that the excess incidence of measles over scarlet fever in the persons recorded as fair-haired is much beyond the limits of any random sampling differences.

Many other problems to which the method can be applied will occur to the biometrician, who is "in active practice."

IX. On a Correction to be made to the Correlation Ratio η.

By KARL PEARSON, F.R.S.

It is well known that as the square of the correlation ratio always involves the sum of the squares of differences of means, it must always take a positive value. Hence its mean value even, when its value for an indefinitely large population would be zero, must for finite samples be positive. Thus such values as ·05 to ·10 of η may not denote small but significant values of the correlation; they may denote solely the measure of η's mean value for actual zero correlation. The observed value of η ought to be compared with

$$\bar{\eta} \pm E_\eta,$$

where $\bar{\eta}$ is the mean value of η for zero correlation and E_η the probable error of η. E_η has been determined by me for any value of η and $= \cdot 67449/\sqrt{N}$, when there is no correlation*.

Let n_{x_p} be the total frequency of the population falling in the subrange, centred at x_p, and let $\bar{y}_{x_p}$ be the mean value of all the y-characters associated with this subgroup of x's. Let $\bar{y}$ be the mean, and σ_y the standard-deviation of all the y-characters. Then by definition:

$$\eta^2 = \frac{S\{n_{x_p}(\bar{y}_{x_p} - \bar{y})^2\}}{N\sigma_y^2} = \frac{\sigma_M^2}{\sigma_y^2},$$

$$\delta(\eta^2) = \frac{\sigma_M^2 + \delta\sigma_M^2}{\sigma_y^2 + \delta\sigma_y^2} - \frac{\sigma_M^2}{\sigma_y^2}$$

$$= \frac{\delta\sigma_M^2 \sigma_y^2 - \sigma_M^2 \delta\sigma_y^2}{\sigma_y^2(\sigma_y^2 + \delta\sigma_y^2)}$$

or, neglecting products of small quantities,

$$= \frac{\delta\sigma_M^2}{\sigma_y^2} - \frac{\sigma_M^2}{\sigma_y^2}\frac{\delta\sigma_y^2}{\sigma_y^2}$$

$$= \frac{\delta(\sigma_M^2)}{\sigma_y^2} - \eta^2\frac{\delta\sigma_y^2}{\sigma_y^2}.$$

Now the variation of σ_y^2 can either be positive or negative; hence if we take the mean values of both sides of the above equation

$$\text{Mean}\ \delta(\eta^2) = \frac{\text{Mean}\ \delta(\sigma_M^2)}{\sigma_y^2}.$$

Now if σ_M^2 be sufficiently large, η fairly large, $\delta(\sigma_M^2)$ may be either positive or negative with random sampling, but if η be either zero or unity, the variation of σ_M^2 must always be wholly

---

* "On the General Theory of Skew Correlation and Non-linear Regression." *Drapers' Company Research Memoirs*, 1905, Dulau and Co.

positive or negative, i.e. the mean value of $\delta\sigma_M^2$ must be a positive quantity or a negative quantity and cannot be zero.

Let $\bar{y}_{x_\rho}$, n_{x_ρ} and $\bar{y}$ represent the values in an indefinitely large population. Let $n_{x_\rho}+\delta n_{x_\rho}$, $\bar{y}_{x_\rho}+\delta\bar{y}_{x_\rho}$, $\bar{y}+\delta\bar{y}$ represent the values in any random sample, then

$$\begin{aligned} N\delta\sigma_M^2 &= S\{(n_{x_\rho}+\delta n_{x_\rho})(\bar{y}_{x_\rho}+\delta\bar{y}_{x_\rho}-\bar{y}-\delta\bar{y})^2\}-S\{n_{x_\rho}(\bar{y}_{x_\rho}-\bar{y})^2\} \\ &= S\{n_{x_\rho}(\delta\bar{y}_{x_\rho}-\delta\bar{y})^2\}+2S\{n_{x_\rho}(\bar{y}_{x_\rho}-\bar{y})(\delta\bar{y}_{x_\rho}-\delta\bar{y})\} \\ &\qquad +S\{\delta n_{x_\rho}(\bar{y}_{x_\rho}-\bar{y})^2\}+2S\{\delta n_{x_\rho}(\delta\bar{y}_{x_\rho}-\delta\bar{y})(\bar{y}_{x_\rho}-\bar{y})\}. \end{aligned}$$

Sum for all values and divide by number m of such values, then remembering:

(i) That the sum of all linear variations is zero, i.e. $\Sigma(\delta n_{x_\rho})/m=\Sigma(\delta\bar{y}_{x_\rho})/m=\Sigma(\delta\bar{y})/m=0$, and,

(ii) That there is no correlation between the number in any array and the mean of that array or between the mean of all the y's and the number in any x-array.

We have: $$N\times\text{mean value of }\delta(\sigma_M^2)=S\left\{n_{x_\rho}\frac{\Sigma(\delta\bar{y}_{x_\rho}-\delta\bar{y})^2}{m}\right\}.$$

Now: $$\frac{\Sigma(\delta\bar{y}_{x_\rho})^2}{m}=\frac{\sigma^2_{y_{x_\rho}}}{n_{x_\rho}},$$

$$\Sigma\frac{(\delta\bar{y})^2}{m}=\frac{\sigma_y^2}{N},$$

$$\frac{\Sigma(\delta\bar{y}_{x_\rho}\delta\bar{y})}{m}=\frac{\sigma^2_{y_{x_\rho}}}{N}.$$

The first two simply represent the square of the standard deviations of the mean of an array of y's at x_ρ containing n_{x_ρ} individuals, and of the y mean of the whole population containing N individuals. The last result is proved in the footnote below*. Hence substituting these values we have:

$$N\times\text{mean value of }\delta(\sigma_M^2)=S(\sigma^2_{y_{x_\rho}})+S\left(n_{x_\rho}\frac{\sigma_y^2}{N}\right)-2S\frac{(n_{x_\rho}\sigma^2_{y_{x_\rho}})}{N}.$$

---

* We have: $$n_{x_\rho}\bar{y}_{x_\rho}=n_{x_\rho y_1}y_1+n_{x_\rho y_2}y_2+\dots+n_{x_\rho y_s}y_s+\dots,$$
$$N\bar{y}=n_{y_1}y_1+n_{y_2}y_2+\dots+n_{y_s}y_s+\dots.$$

Hence: $$n_{x_\rho}\delta\bar{y}_{x_\rho}=\delta n_{x_\rho y_1}y_1+\delta n_{x_\rho y_2}y_2+\dots+\delta n_{x_\rho y_s}y_s+\dots-\bar{y}_{x_\rho}\delta n_{x_\rho},$$
$$N\delta\bar{y}=\delta n_{y_1}y_1+\delta n_{y_2}y_2+\dots+\delta n_{y_s}y_s+\dots.$$

Of if m be the number of samples:

$$\begin{aligned} n_{x_\rho}N\frac{\Sigma(\delta\bar{y}_{x_\rho}\delta\bar{y})}{m} &= -\bar{y}_{x_\rho}S_s\left\{y_s\frac{\Sigma(\delta n_{y_s}\delta n_{x_\rho})}{m}\right\} \\ &\quad +S_s\left\{y_s^2\frac{\Sigma(\delta n_{y_s}\delta n_{x_\rho y_s})}{m}\right\} \\ &\quad +S_{ss'}\left\{\frac{y_s y_{s'}\Sigma(\delta n_{y_s}\delta n_{x_\rho y_{s'}}+\delta n_{y_{s'}}\delta n_{x_\rho y_s})}{m}\right\}. \end{aligned}$$

But †:

$$\Sigma\left(\frac{\delta n_{y_s}\delta n_{x_\rho}}{m}\right)=n_{x_\rho y_s}-\frac{n_{y_s}n_{x_\rho}}{N},$$

$$\Sigma\left(\frac{\delta n_{y_s}\delta n_{x_\rho y_s}}{m}\right)=n_{x_\rho y_s}\left(1-\frac{n_{y_s}}{N}\right),$$

$$\Sigma\left(\frac{\delta n_{y_s}\delta n_{x_\rho y_{s'}}}{m}\right)=-\frac{n_{x_\rho y_{s'}}n_{y_s}}{N}.$$

† See *Drapers' Research Memoirs*, "Skew Correlation," p. 12, and *Biometrika*, Vol. v. pp. 191, 192.

But :
$$S(n_{x_\rho}\sigma_y{}^2)/N=\sigma_y{}^2\times S(n_{x_\rho})/N=\sigma_y{}^2,$$
$$S(n_{x_\rho}\sigma^2{}_{y_{x_\rho}})/N=(1-\eta^2)\,\sigma_y{}^2$$
$$=\sigma_y{}^2,\ \text{if }\eta\text{ be zero,}$$
$$S(\sigma^2{}_{y_{x_\rho}})=\sigma^2{}_{y_{x_1}}+\sigma^2{}_{y_{x_2}}+\sigma^2{}_{y_{x_3}}+\ldots+\sigma^2{}_{y_{x_\kappa}},\ \text{if there be }\kappa\text{ arrays,}$$
$$=\kappa\sigma_y{}^2,\ \text{if }\eta\text{ be zero,}$$
for all the arrays will be samples of the general population.

Thus, for a system without correlation
$$N\times\text{mean value of }\delta(\sigma_M{}^2)=\kappa\sigma_y{}^2+\sigma_y{}^2-2\sigma_y{}^2$$
$$=(\kappa-1)\,\sigma_y{}^2.$$
$$\text{Mean value of }\eta^2=\frac{\kappa-1}{N},\ \text{for true }\eta=0.$$
$$\text{Or, mean value of }\eta=\sqrt{\frac{\kappa-1}{N}}.$$
Hence no value of η is really significant if it does not differ sensibly from
$$\sqrt{\frac{\kappa-1}{N}}\pm{\cdot}67449\,\frac{1}{\sqrt{N}}.$$
If we do not suppose η zero, the mean addition to the value of η^2 will be
$$N\times\delta(\eta^2)=\kappa\,\frac{S(\sigma^2{}_{y_{x_\rho}})}{\kappa}\,\frac{1}{\sigma_y{}^2}+1-2(1-\eta^2).$$

The question arises as to what value should be given to the mean of the *unweighted* standard deviations squared of the arrays. We shall not err much in this small order expression, if we give it the value it would take if the arrays were weighted, or if the distribution were homoscedastic, i.e. $(1-\eta^2)\,\sigma_y{}^2$. Accordingly we have :
$$\delta(\eta^2)=\{(\kappa-2)(1-\eta^2)+1\}/N.$$
Or :
$$\text{Mean observed value of }\eta^2=\text{true value of }\eta^2+\frac{(\kappa-2)(1-\eta^2)+1}{N}.$$
Or :
$$\eta^2=\bar\eta^2-\frac{(\kappa-2)(1-\eta^2)+1}{N}\ ;$$
we see therefore that the correction for $\bar\eta^2$ rapidly diminishes as η approaches unity. Of course we have, if we prefer, instead of writing η^2 on the left, the observed value
$$\eta^2=\{\bar\eta^2-(\kappa-1)/N\}/\{1-(\kappa-2)/N\},$$
where we shall put for $\bar\eta^2$, the mean value, the observed value as most probably coincident with that mean value.

Hence, substituting we find :
$$n_{x_\rho}N\,\frac{\Sigma(\delta\bar y_{x_\rho}\delta\bar y)}{m}=-\bar y^2{}_{x_\rho}n_{x_\rho}+\frac{n_{x_\rho}}{N}\,\bar y_{x_\rho}N\bar y$$
$$+S_s(y_s{}^2n_{x_\rho y_s})-S_s\,\frac{y_sn_{x_\rho y_s}S_{s'}(n_{y_{s'}y_{s'}})}{N}$$
$$=n_{x_\rho}(-\bar y^2{}_{x_\rho}+\bar y_{x_\rho}\bar y-\bar y_{x_\rho}\bar y)+S_s(y_s{}^2n_{x_\rho y_s})$$
$$=S_s\{n_{x_\rho y_s}(y_s{}^2-\bar y^2{}_{x_\rho})\}=S_s\{n_{x_\rho y_s}(y_s-\bar y_{x_\rho})^2\}$$
$$=n_{x_\rho}\sigma^2{}_{y_{x_\rho}},$$
$$\therefore\ \frac{\Sigma(\delta\bar y_{x_\rho}\delta\bar y)}{m}=\frac{\sigma^2{}_{y_{x_\rho}}}{N},$$
which had to be proved.

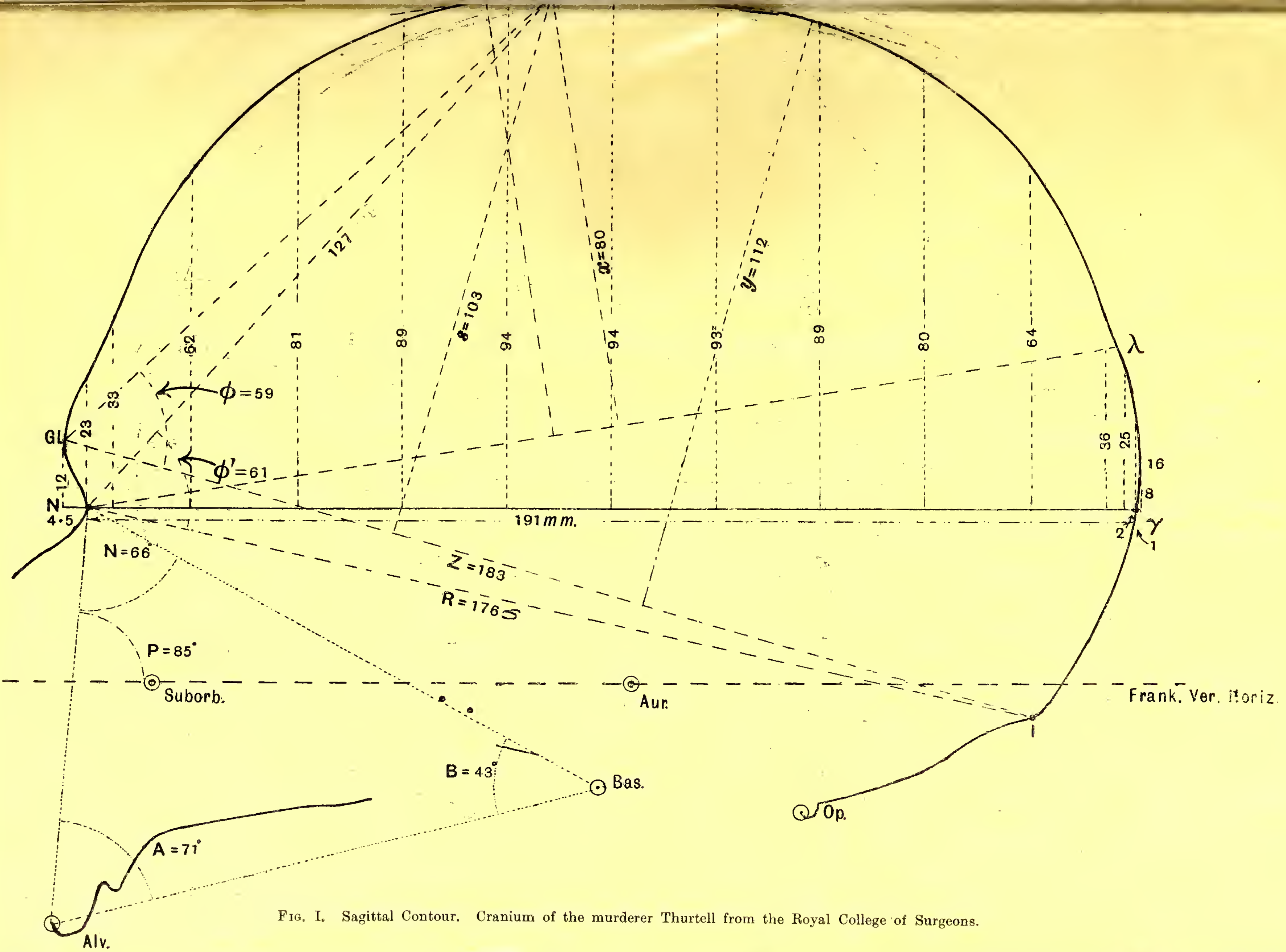


Fig. I. Sagittal Contour. Cranium of the murderer Thurtell from the Royal College of Surgeons.

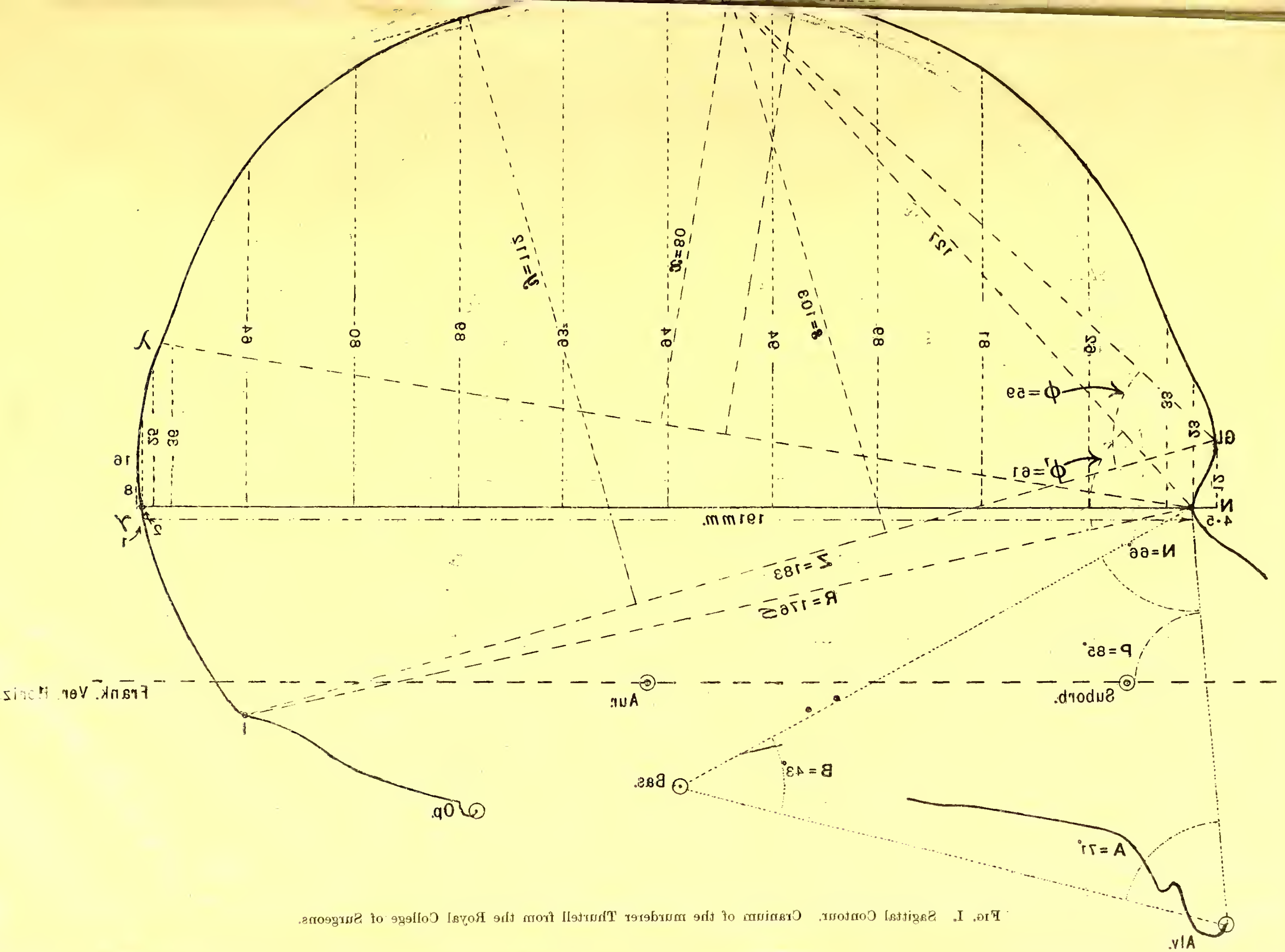


Fig. I. Sagittal Contour. Cranium of the murderer Thurtell from the Royal College of Surgeons.

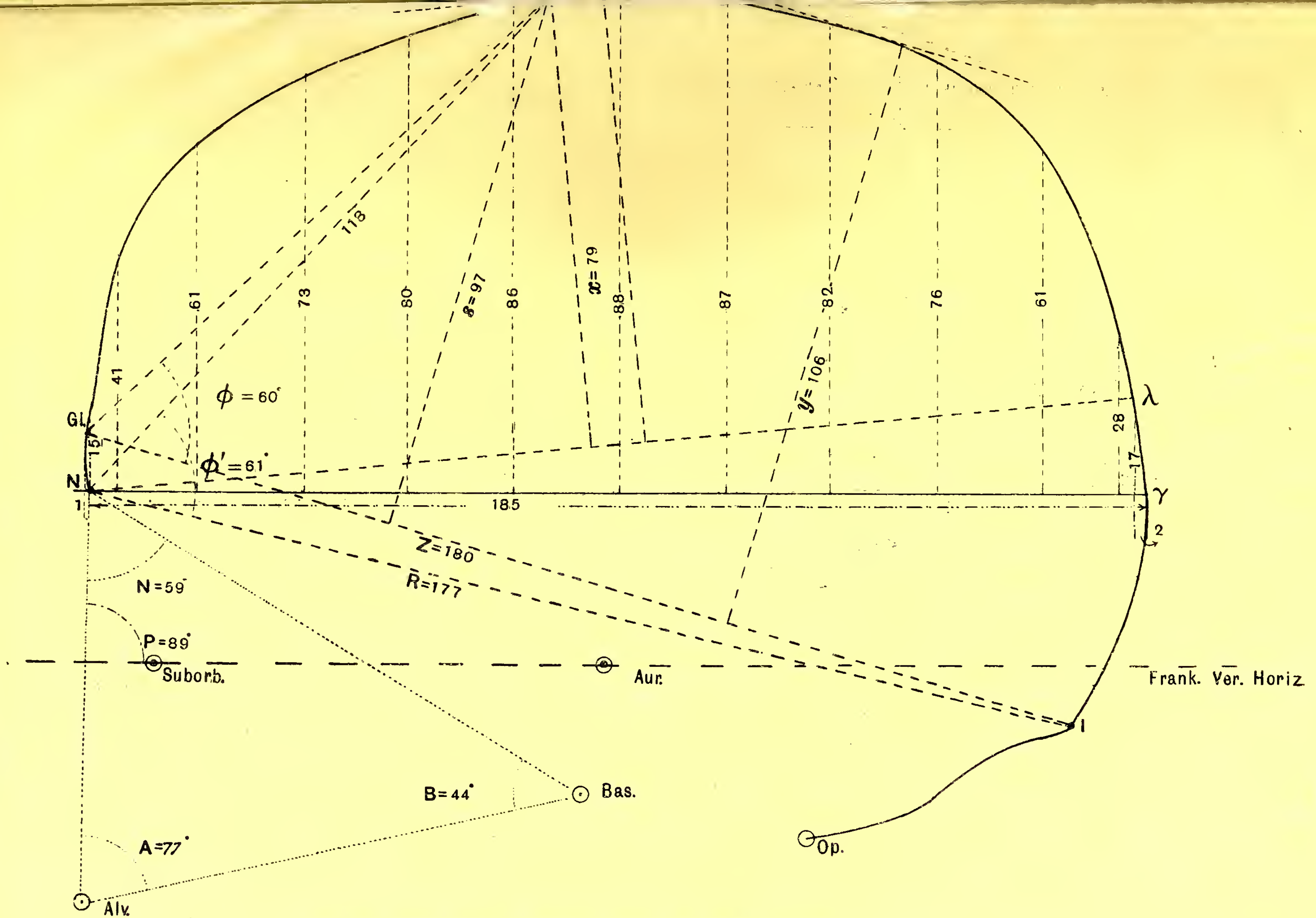


Fig. II. Sagittal Contour. Egyptian Cranium, E. 530 from Biometric Laboratory.

N.B. In splining in the contour the draughtsman has placed the protuberance *below* the true glabella of the original drawing.

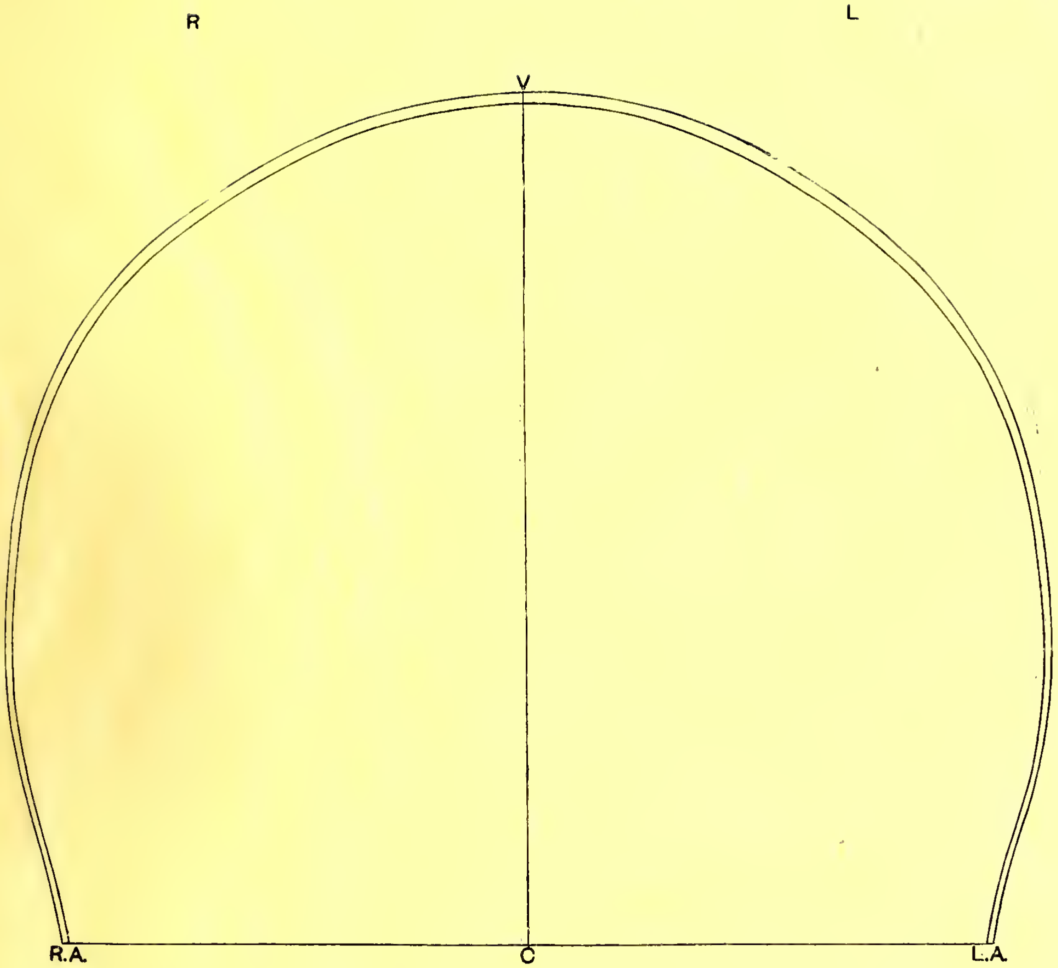


Fig. III. English Type Vertical Contour, 17th Century, showing range of variation.

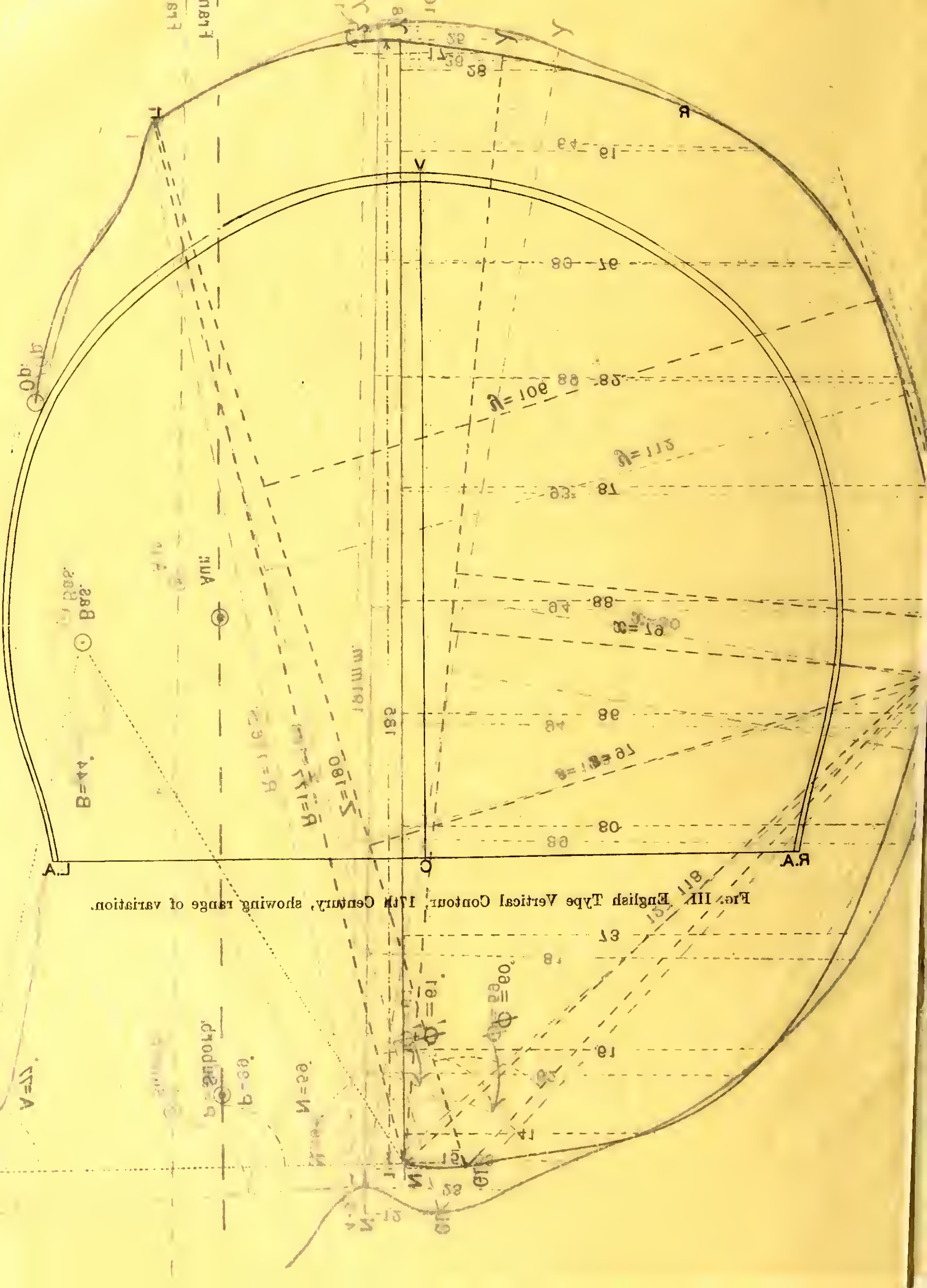


FIG. III. English Type Vertical Contour, 17th Century, showing range of variation.

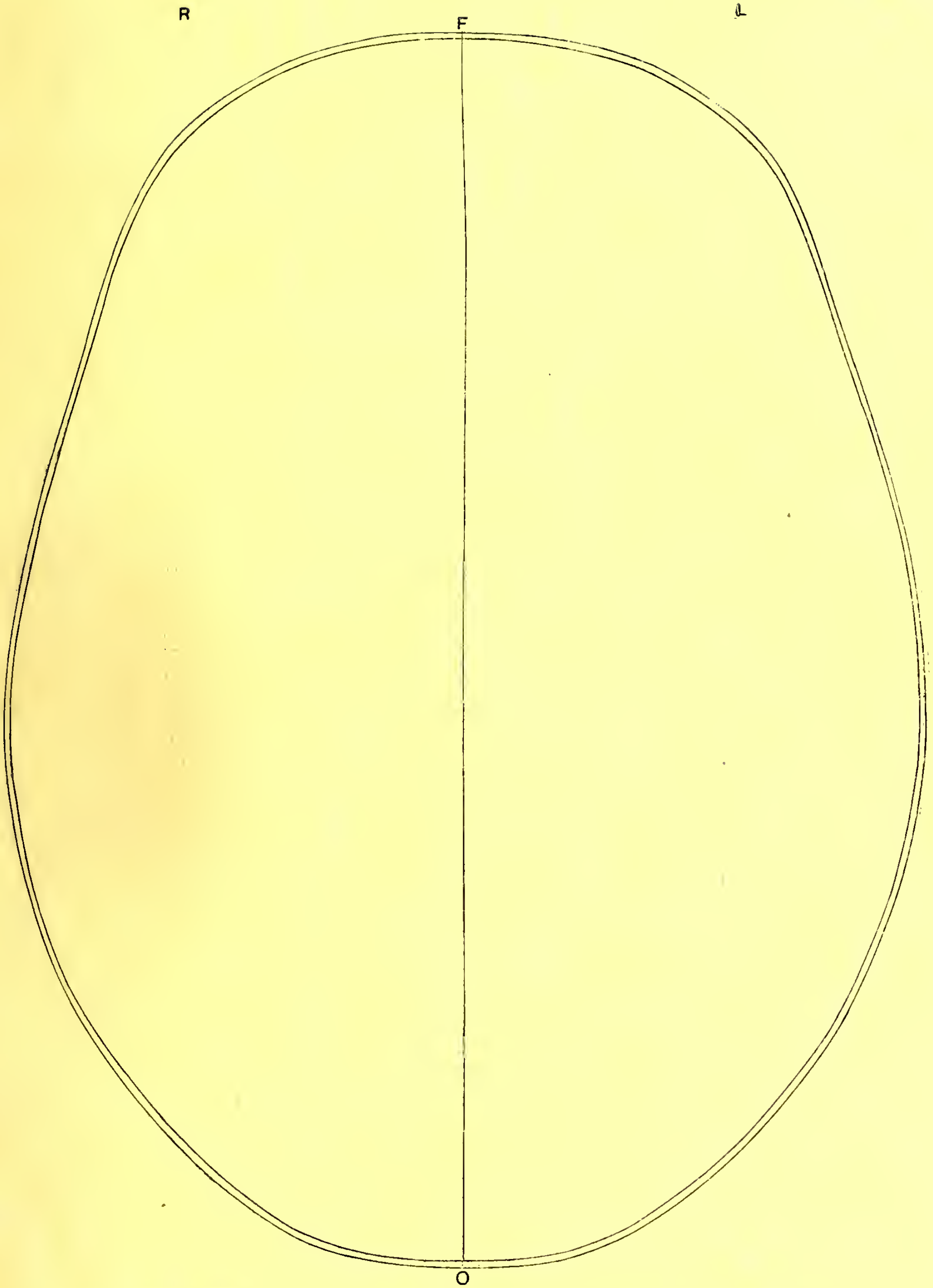


Fig. IV. English Type Horizontal Contour, 17th century, showing range of variation.

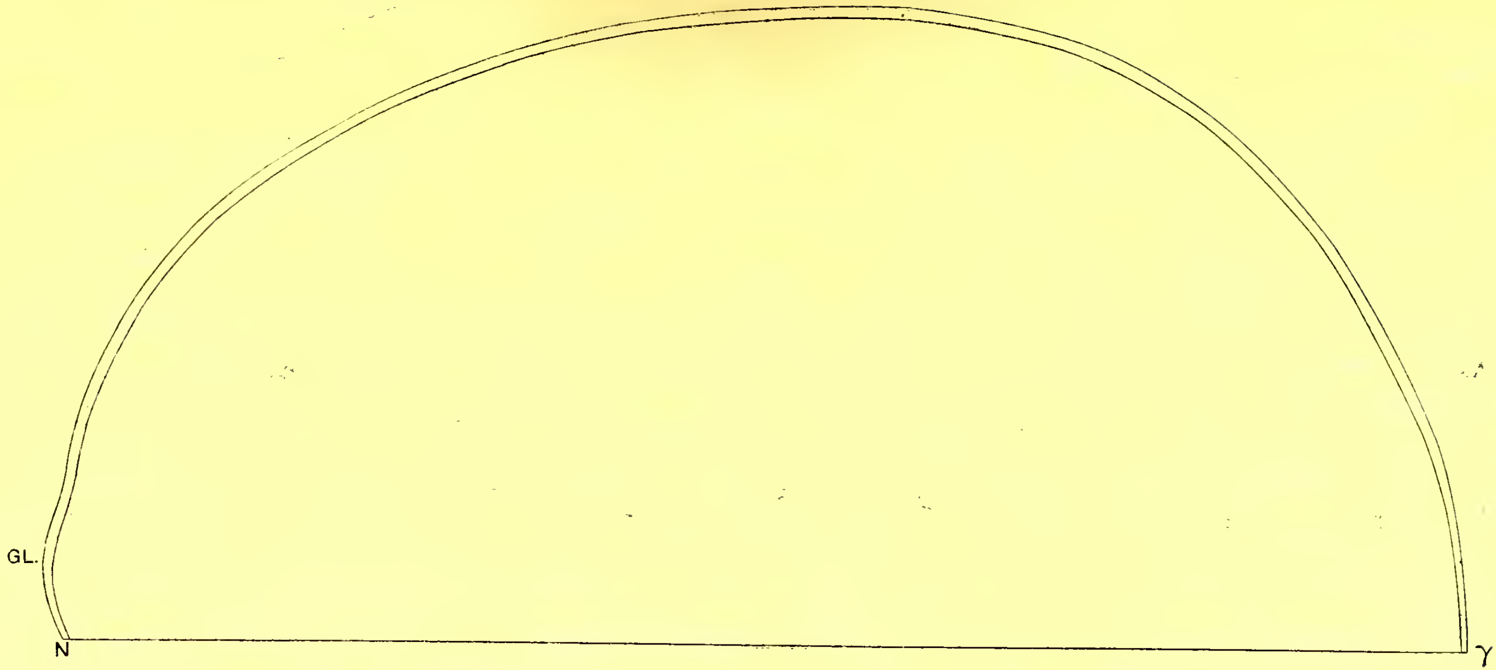


Fig. V. English Type Sagittal Contour, 17th century, showing range of variation.
]

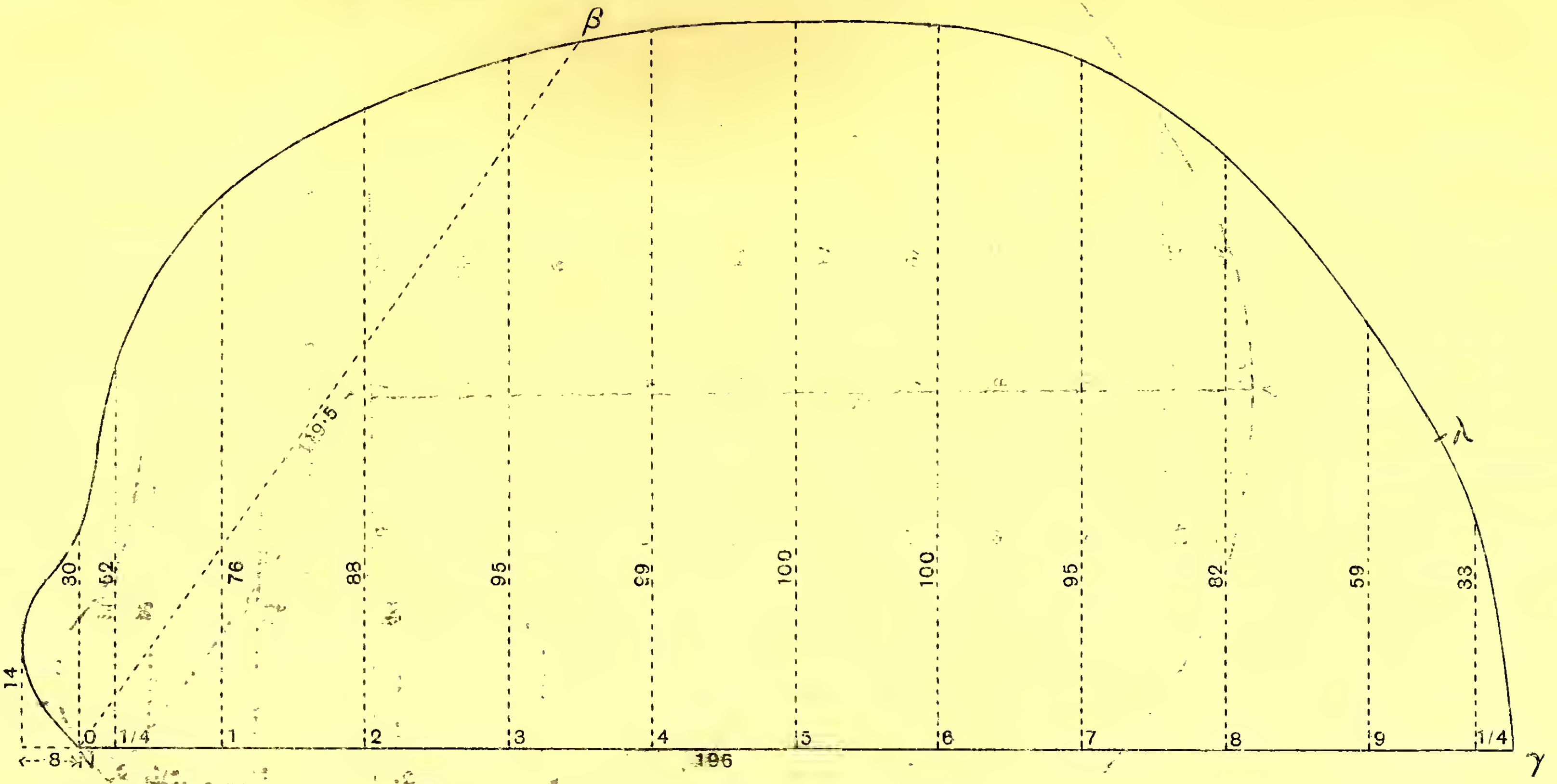


Fig. VI. Cro-Magnon. Sagittal Contour.

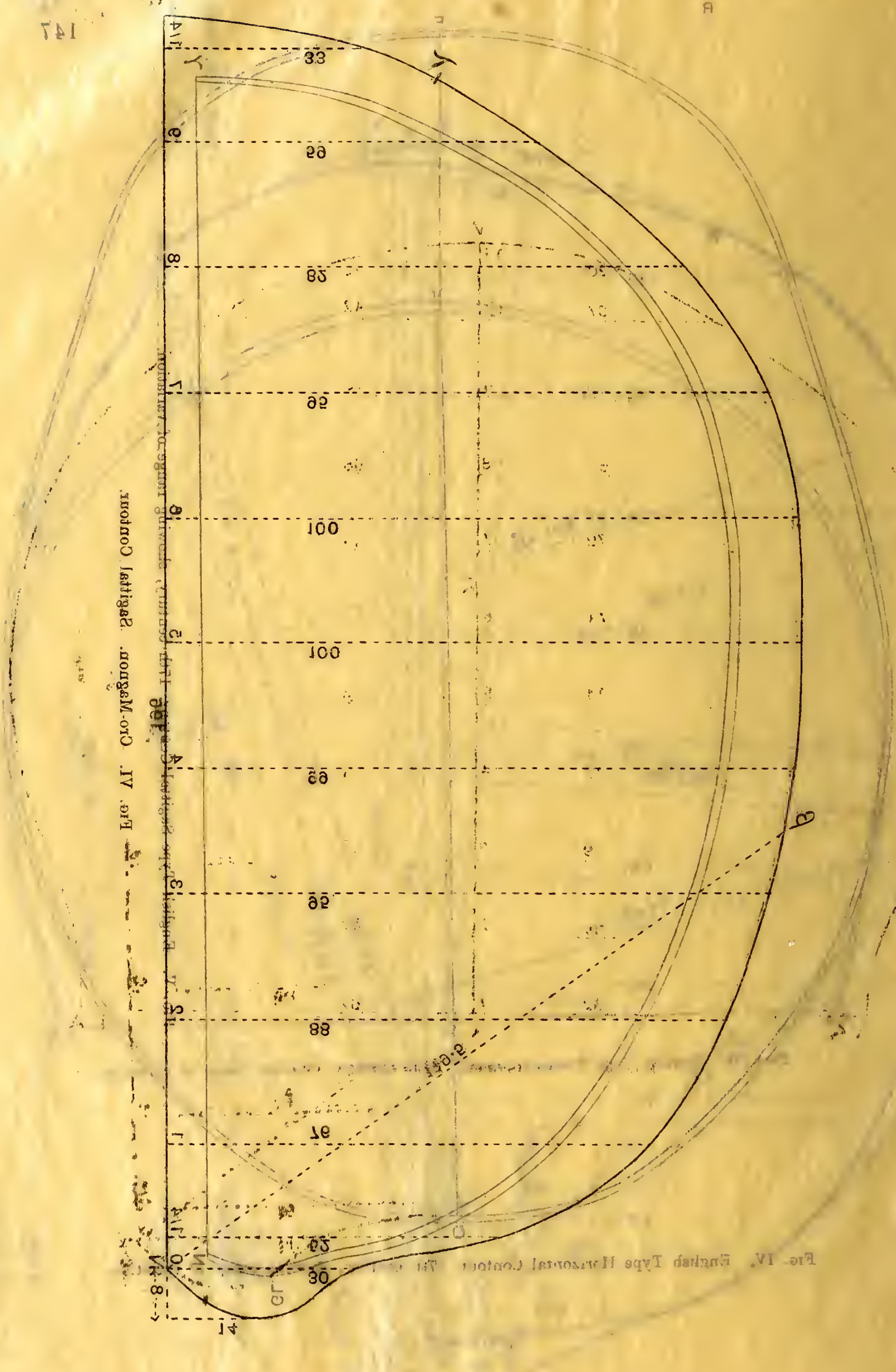


Fig. VI. Cro-Magnon. Sagittal Contour.

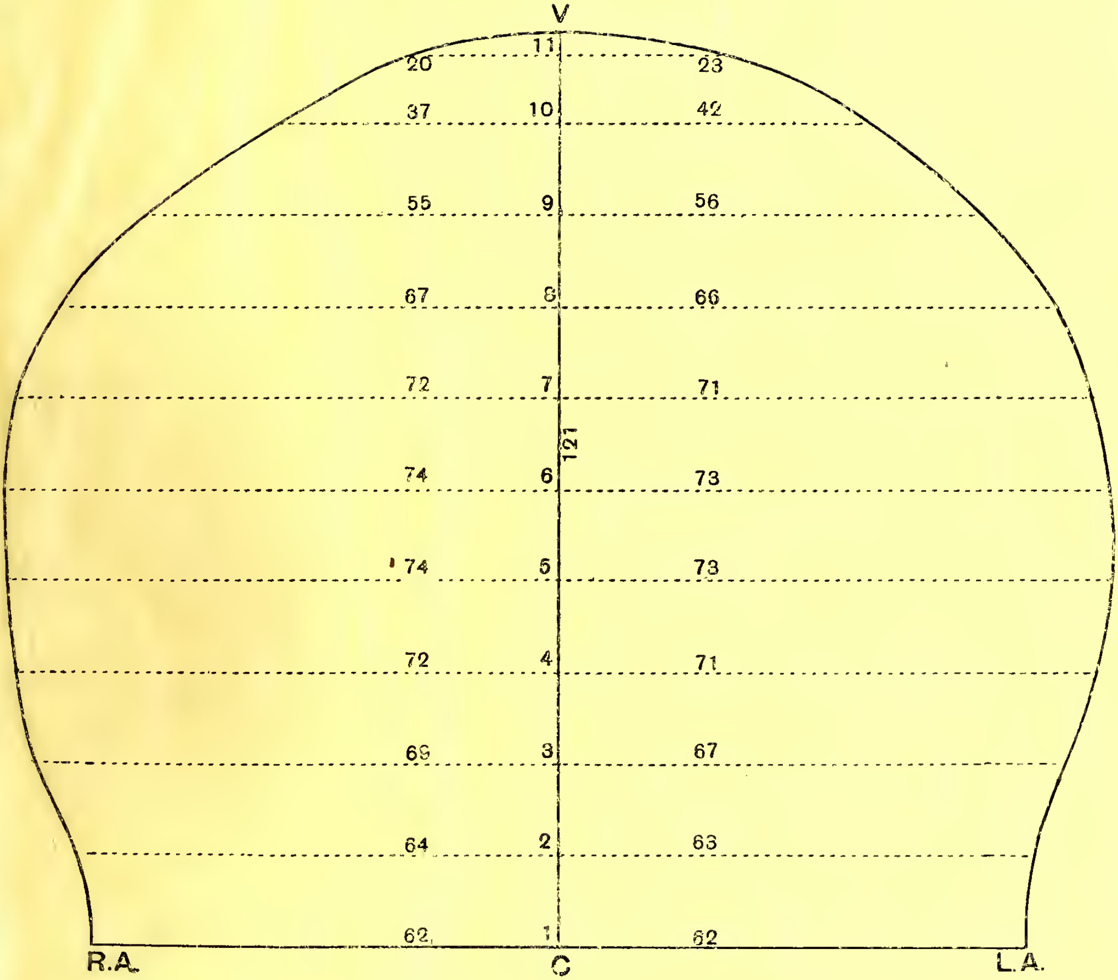


Fig. VII. Cro-Magnon. Transverse Contour.

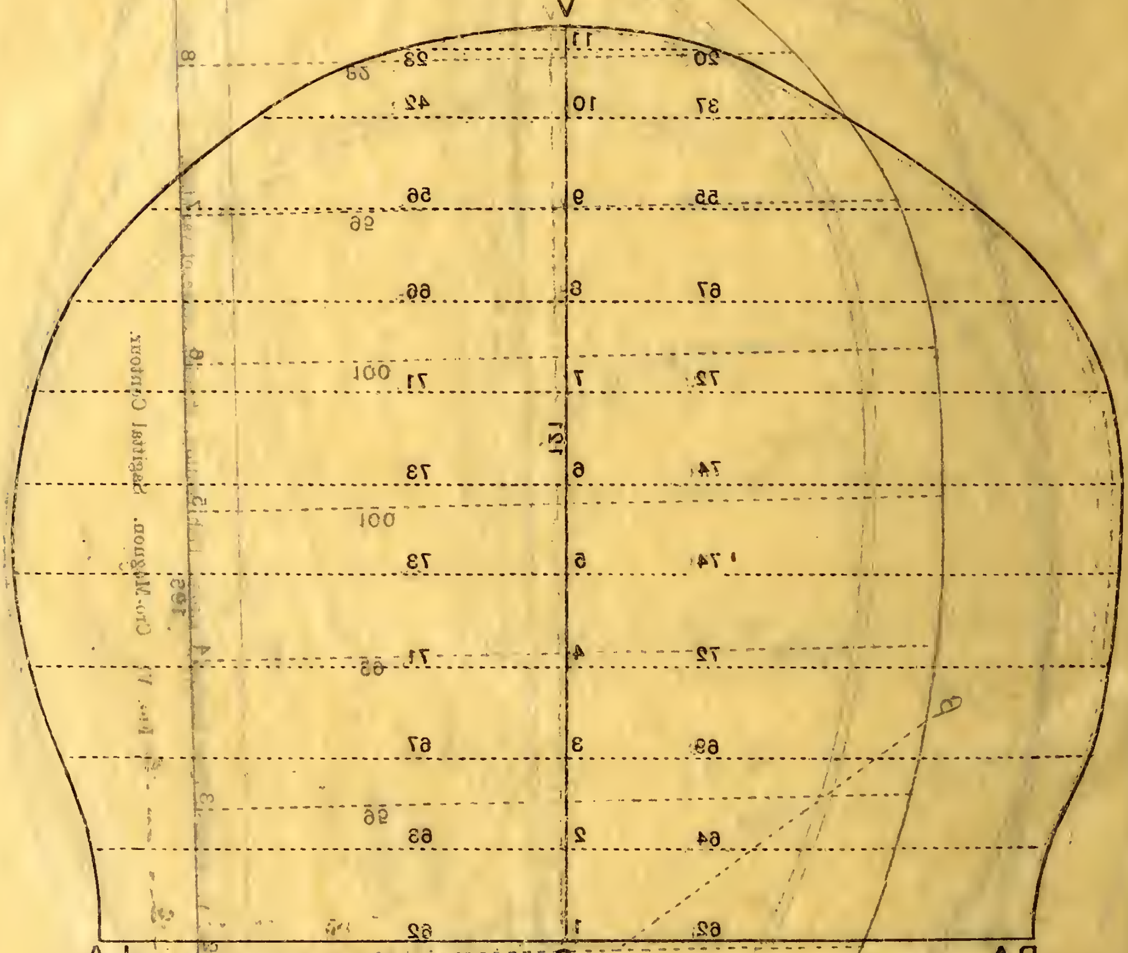


Fig. VII. Cro-Magnon. Transverse Contour.

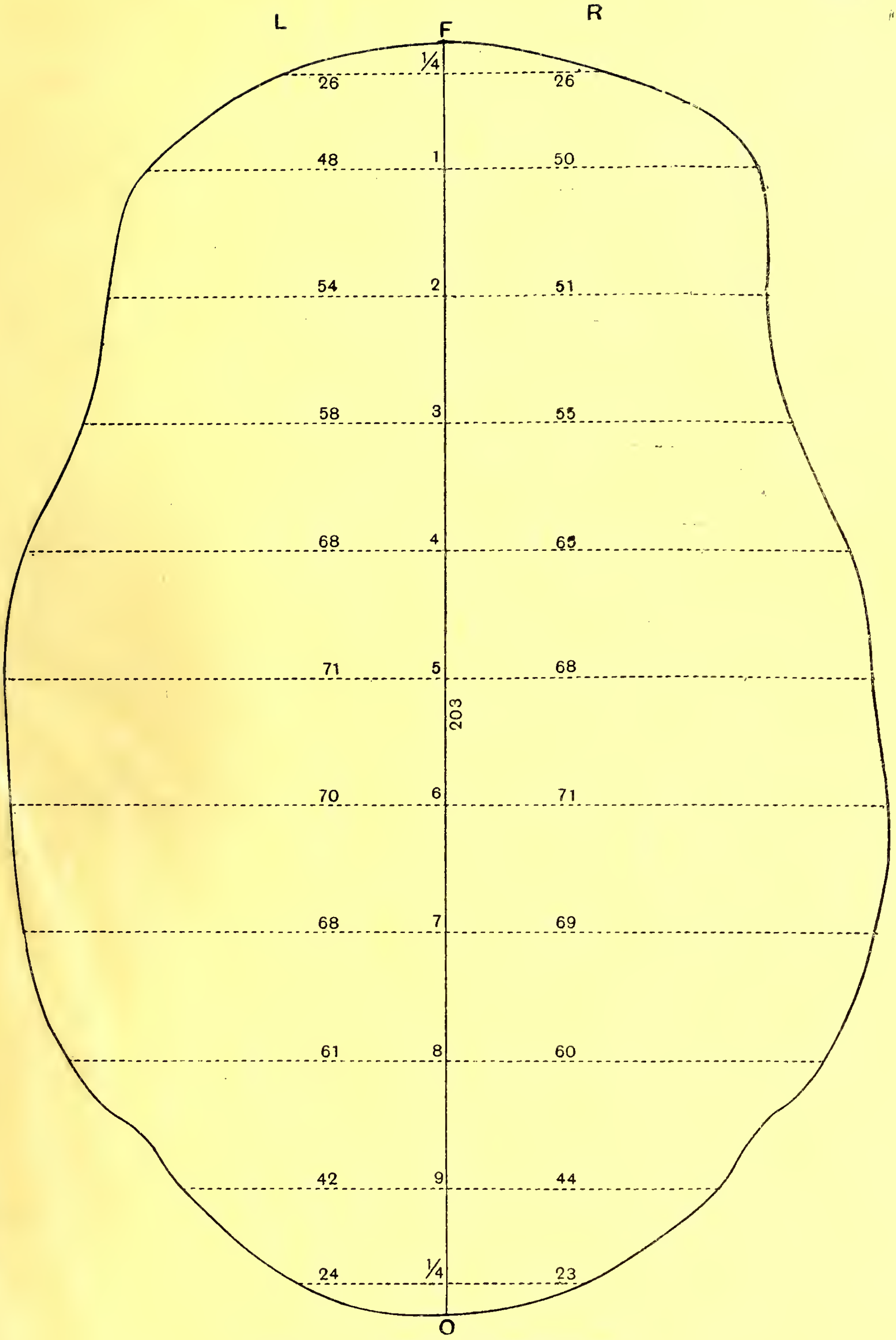


Fig. VIII. Cro-Magnon. Horizontal Contour.

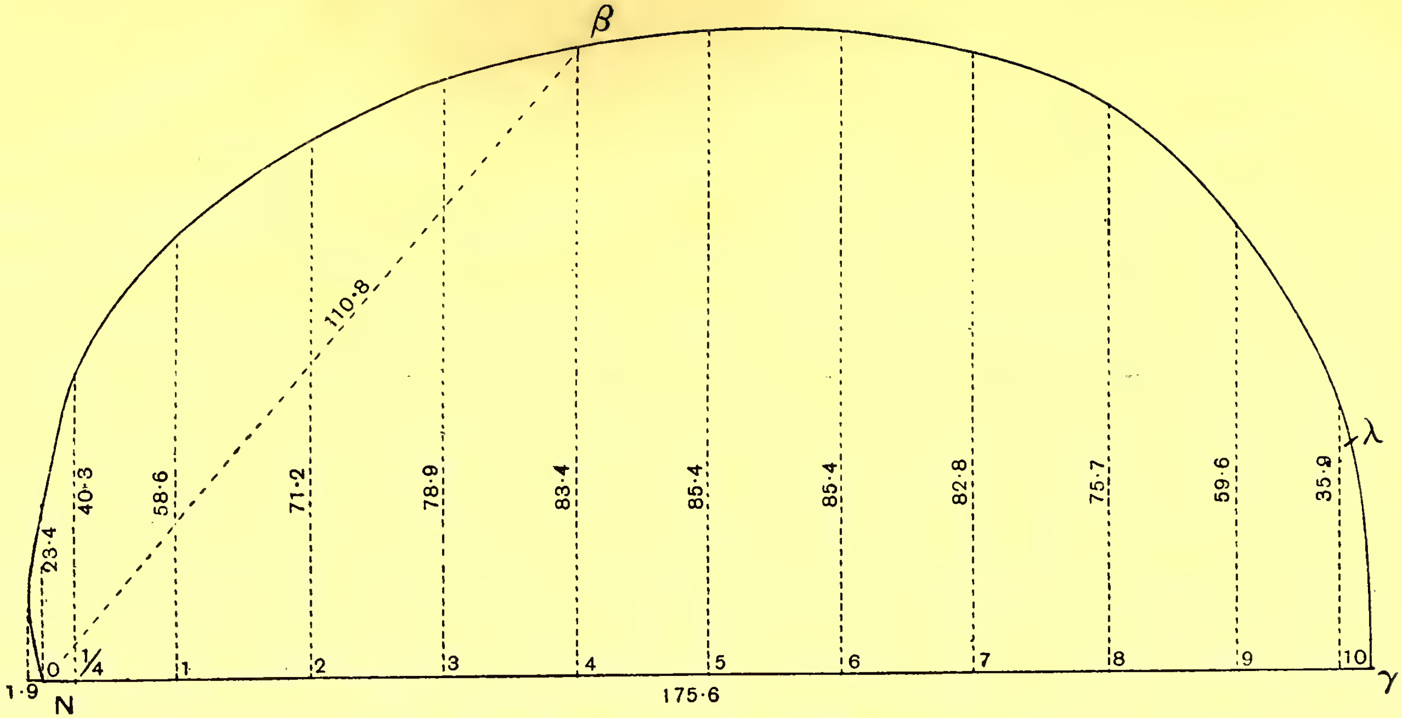


Fig. IX. Congo Crania. Fernand Vaz, 1864. Sagittal Contour.

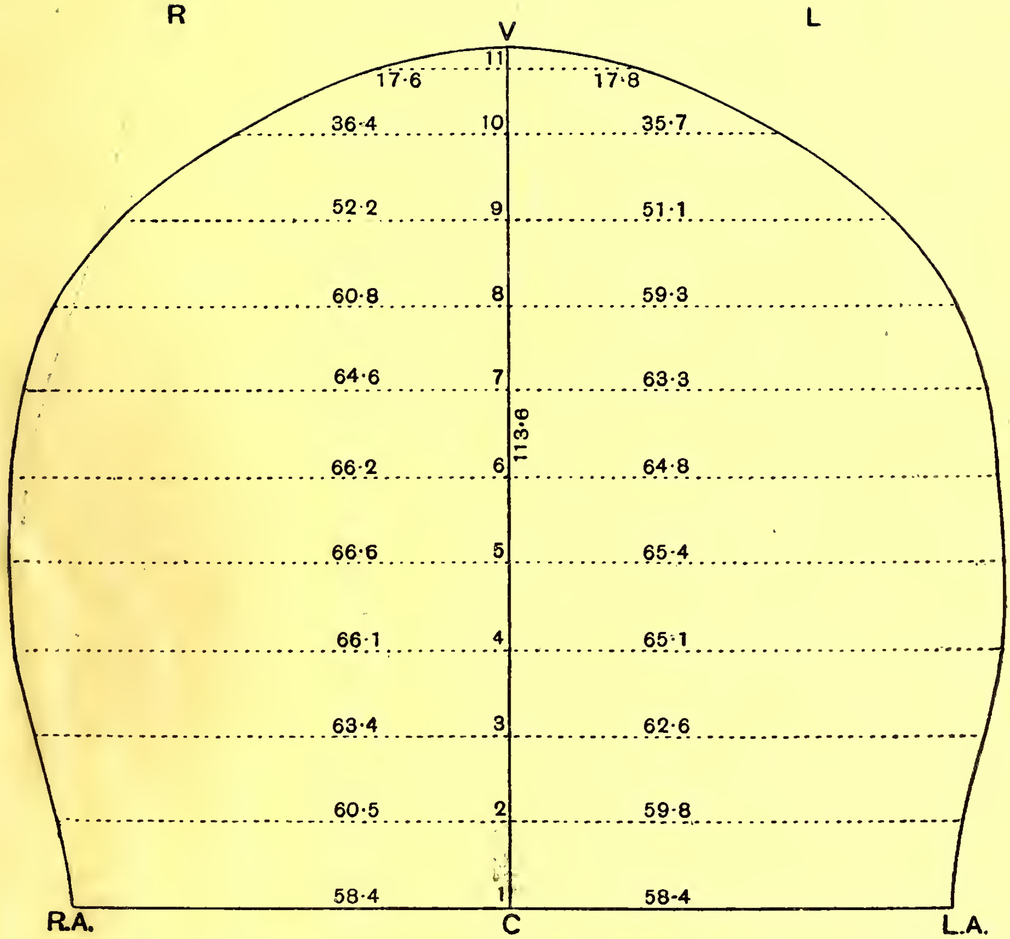


Fig. X. Congo Crania. Fernand Vaz, 1864. Transverse Contour.

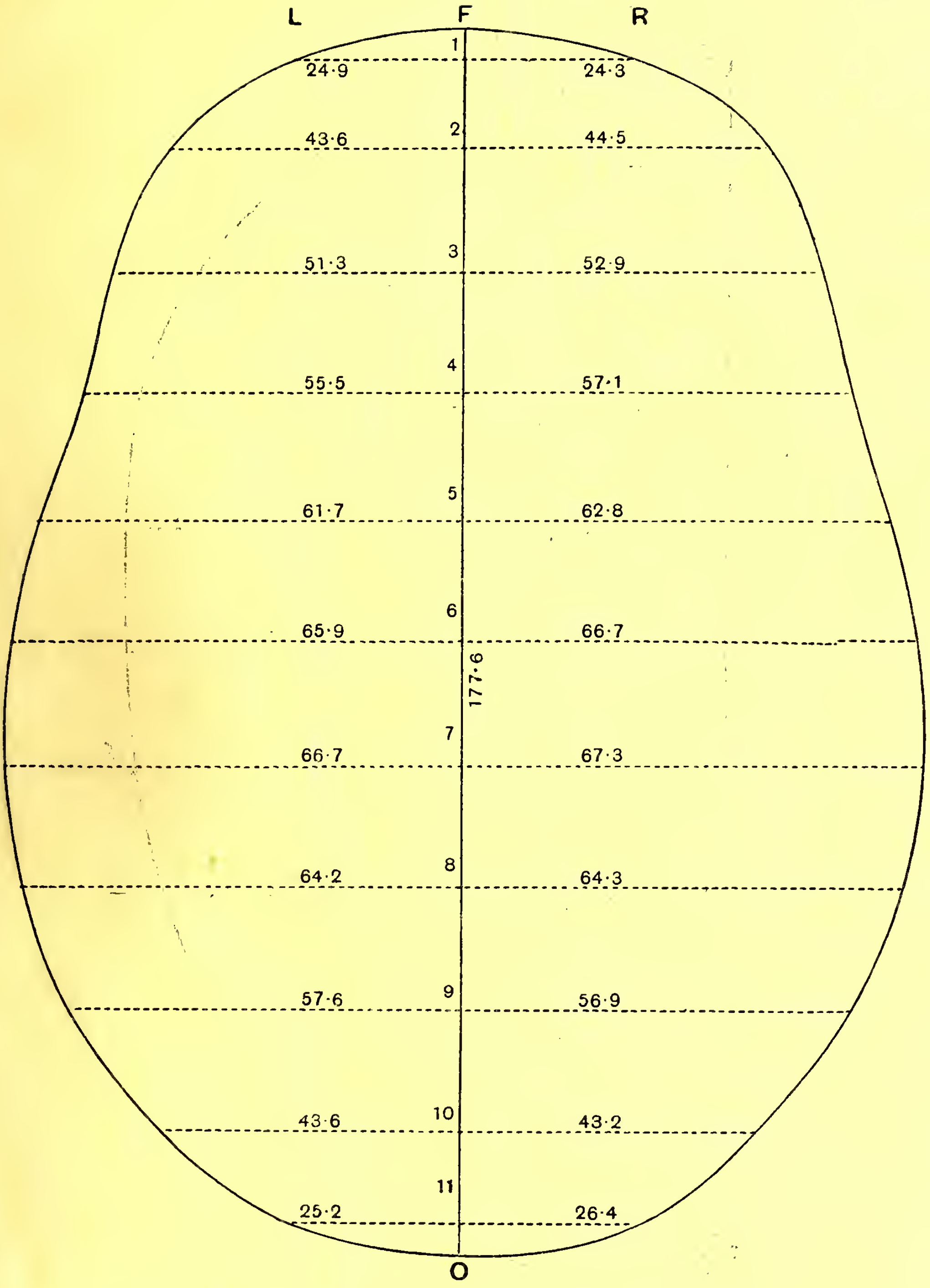


FIG. XI. Congo Crania. Fernand Vaz, 1864. Horizontal Contour.

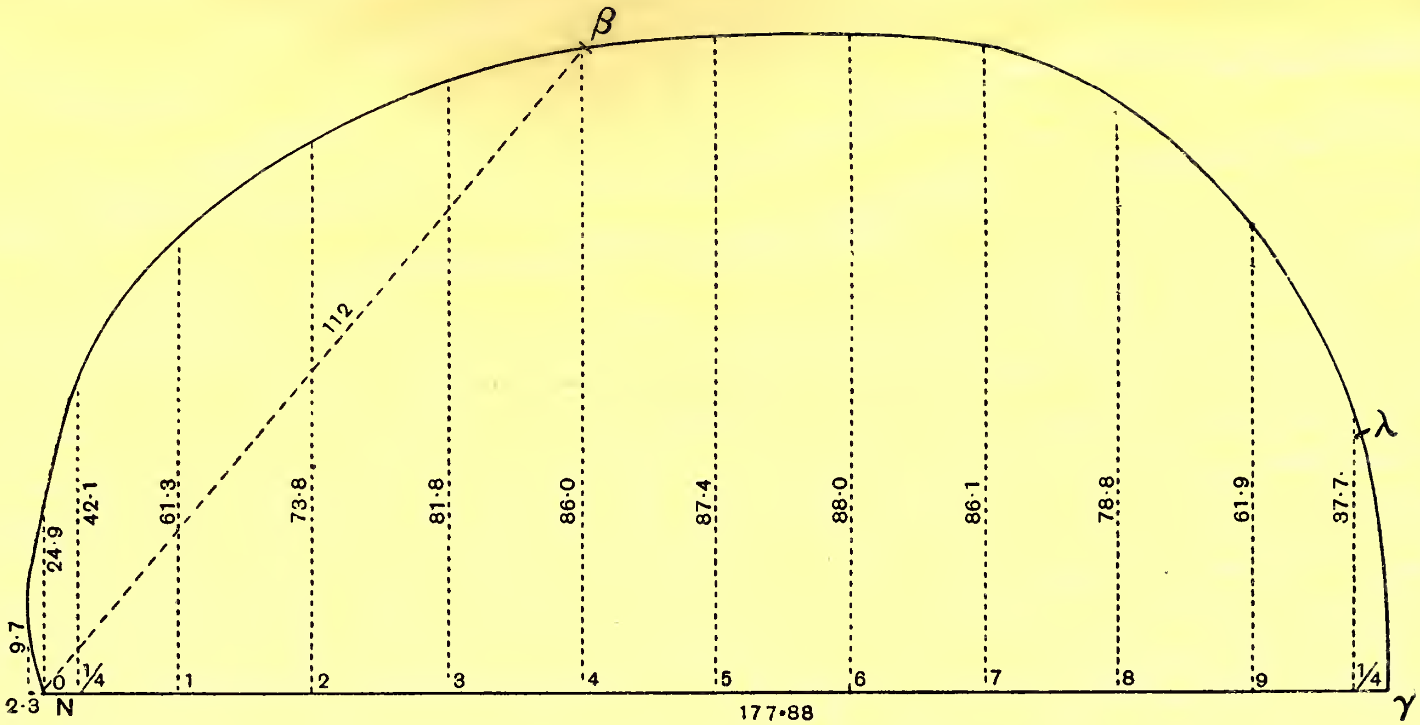


Fig. XII. Congo Crania. Fernand Vaz, 1880. Sagittal Contour.

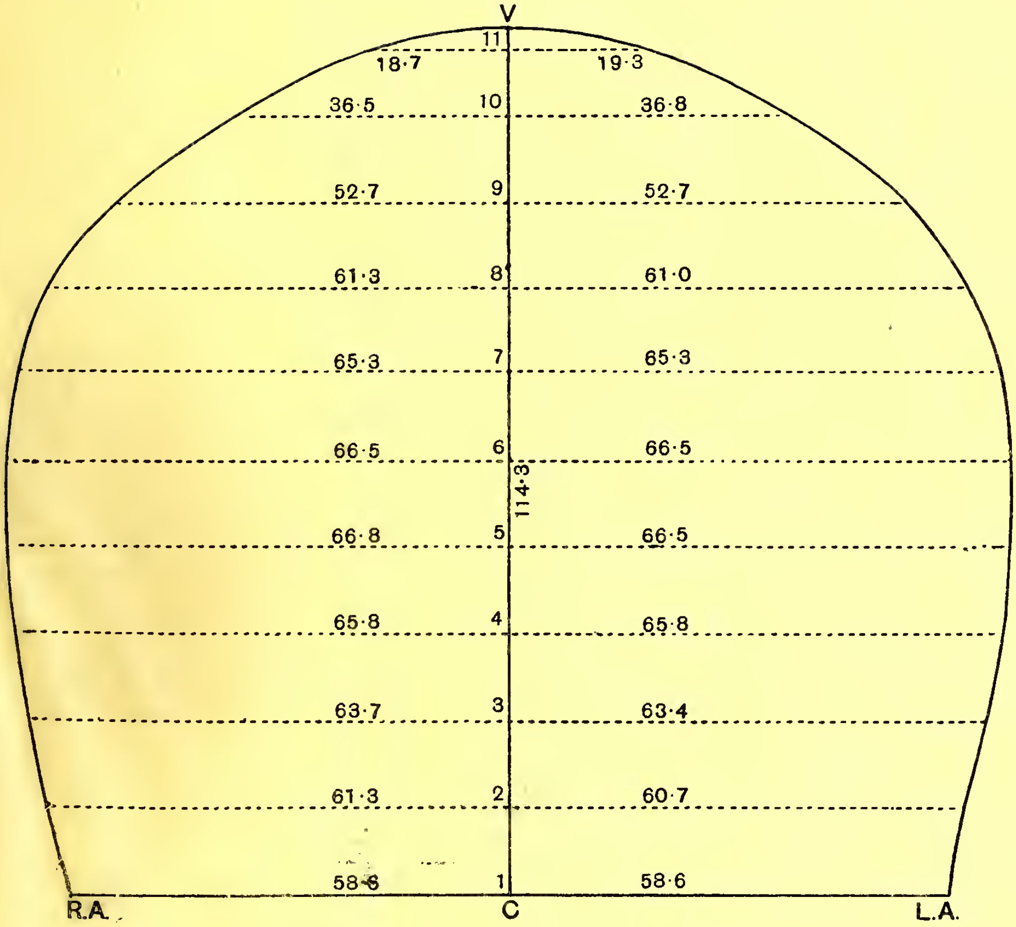


Fig. XIII. Congo Crania. Fernand Vaz, 1880. Transverse Contour.

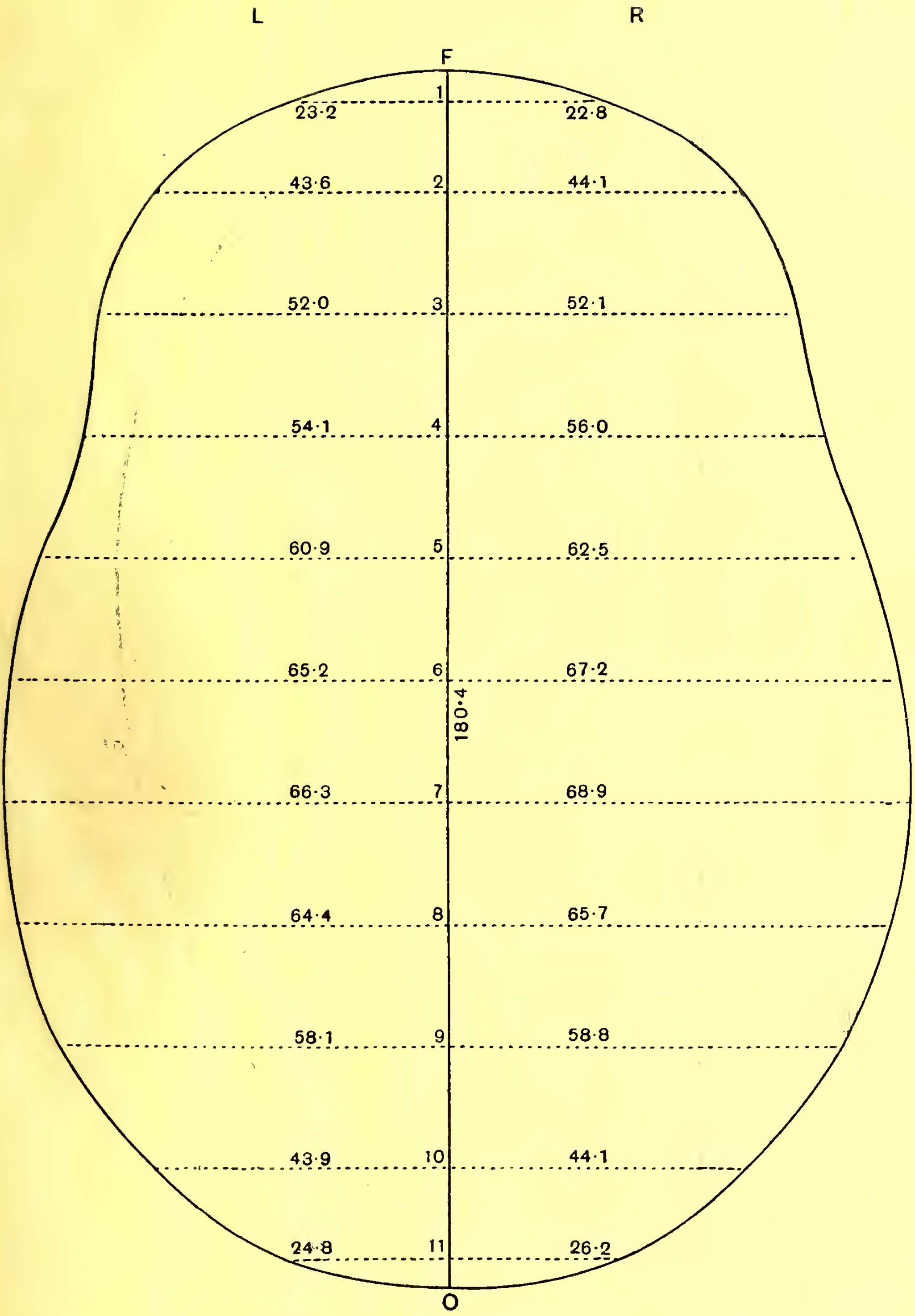


Fig. XIV. Congo Crania. Fernand Vaz, 1880. Horizontal Contour.

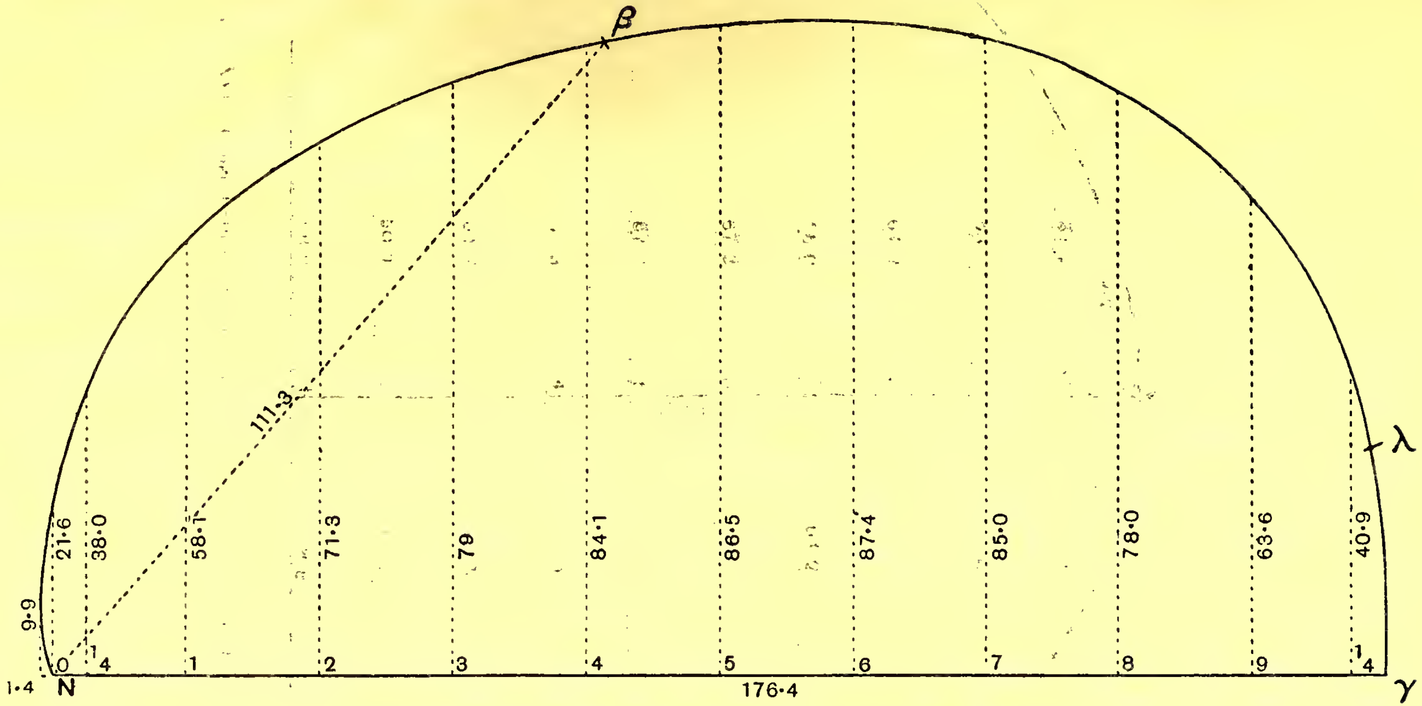


Fig. XV. Congo Crania. Batetelu Race (R.C. of S.). Sagittal Contour.

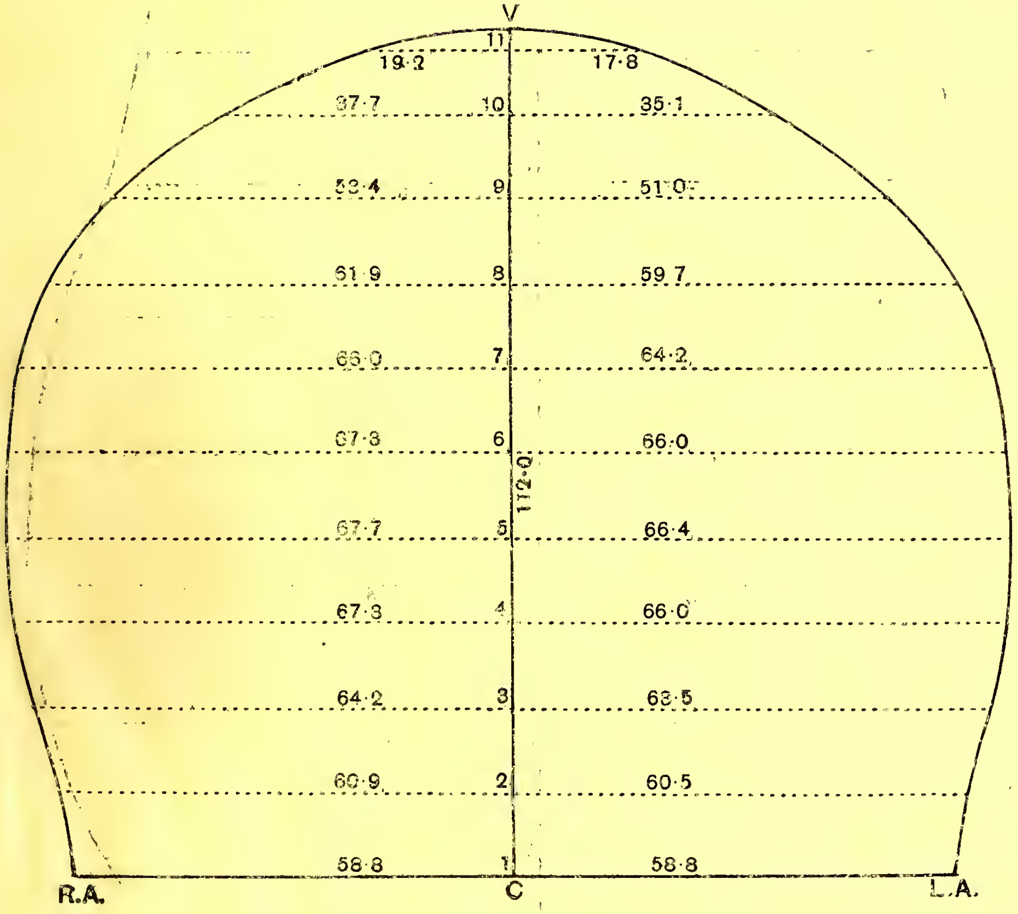


Fig. XVI. Congo Crania. Batetelu Race (R. C. of S.). Transverse Contour.

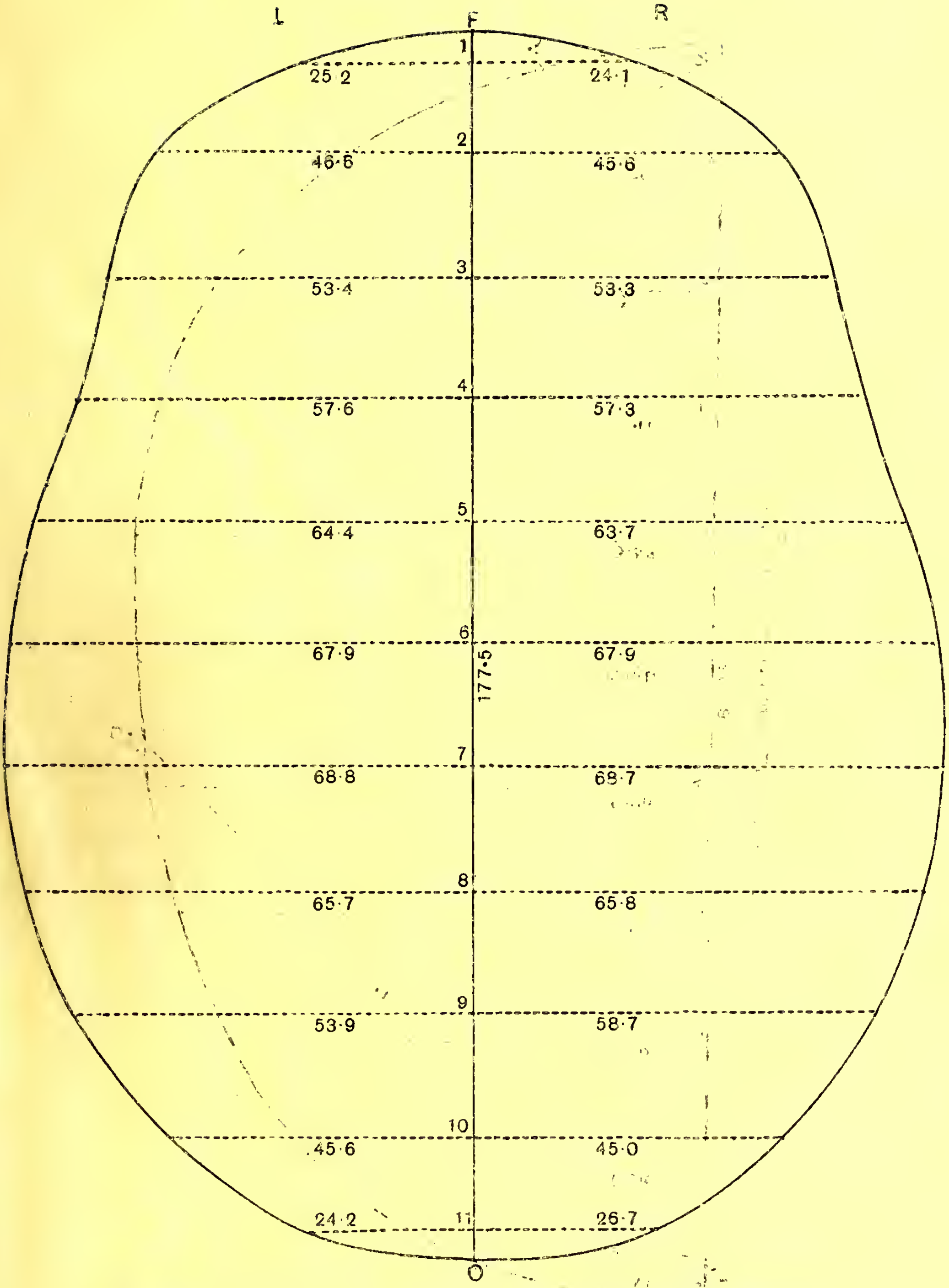


Fig. XVII. Congo Crania. Batetelu Race (R. C. of S.). Horizontal Contour.

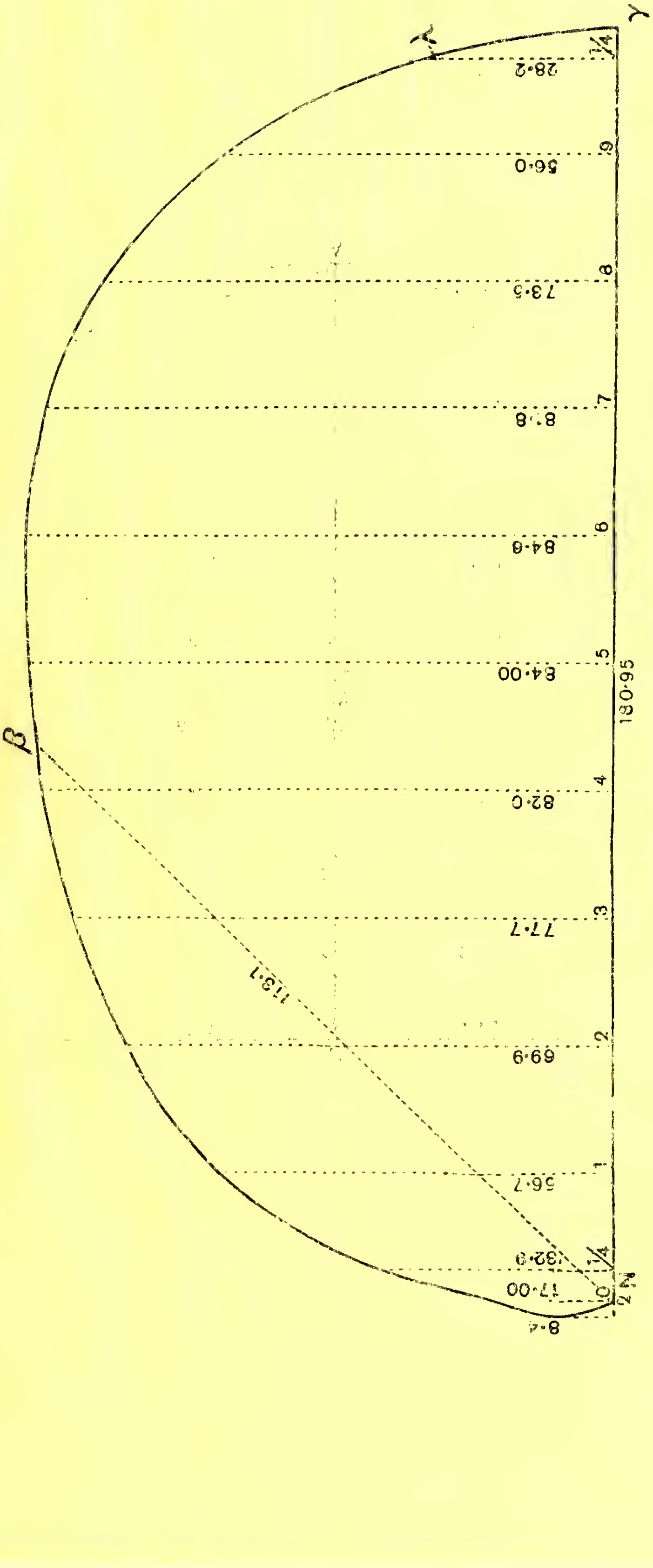


Fig. XVIII. Egyptian Crania. Sagittal Contour.

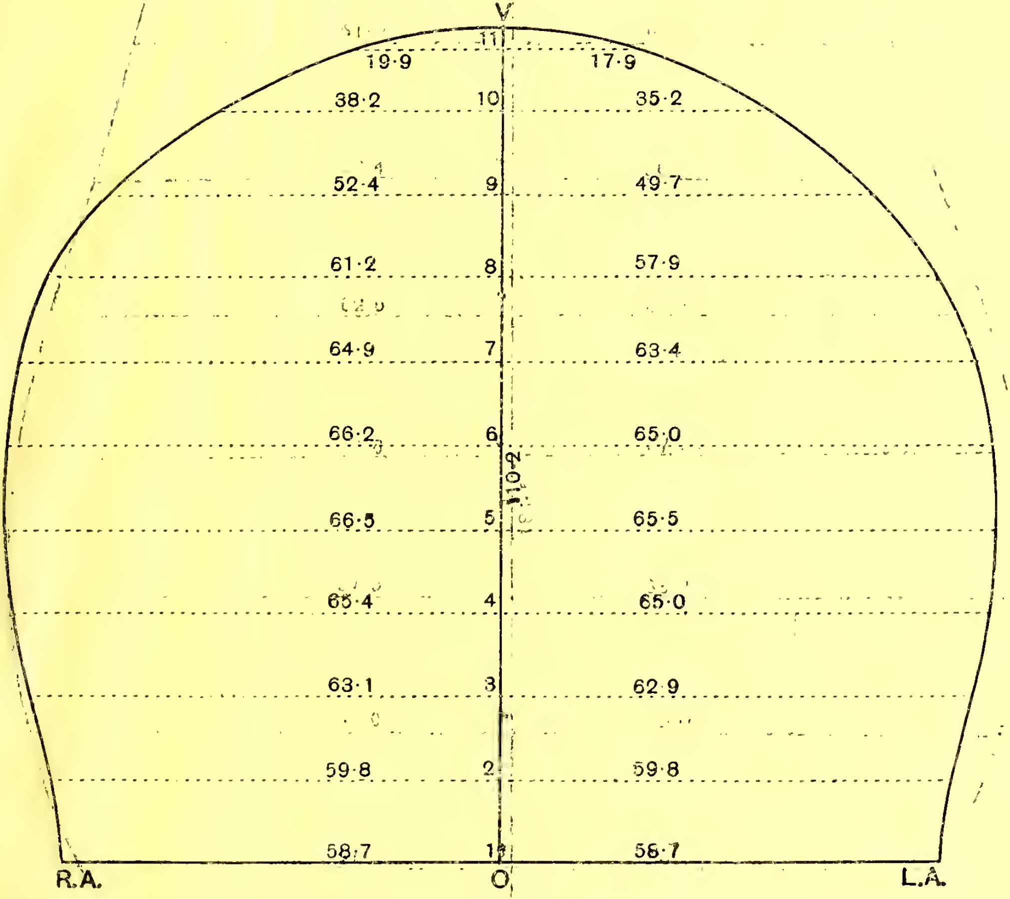


Fig. XIX. Egyptian Crania. Transverse Contour.

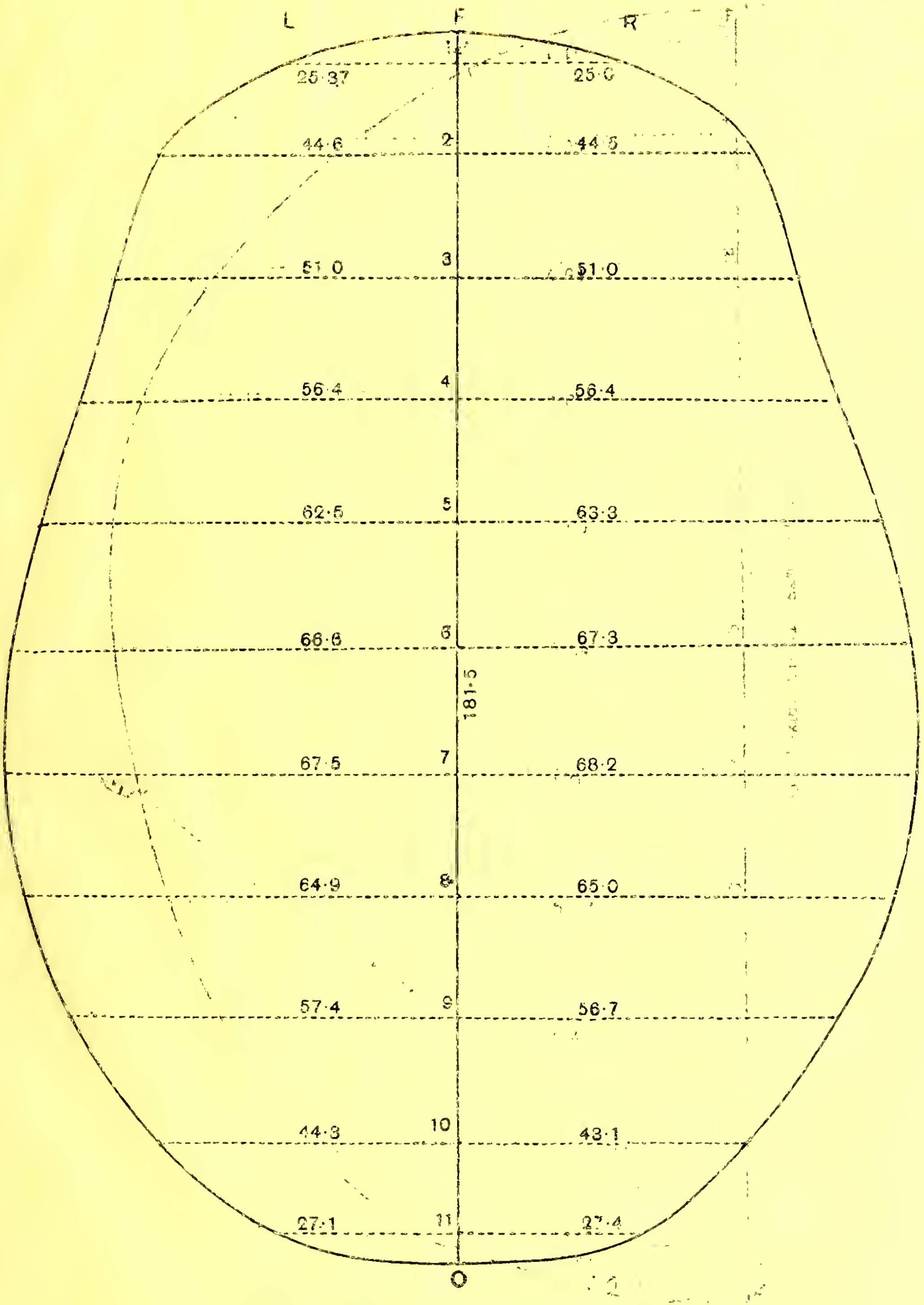


FIG. XX. Egyptian Crania. Horizontal Contour.

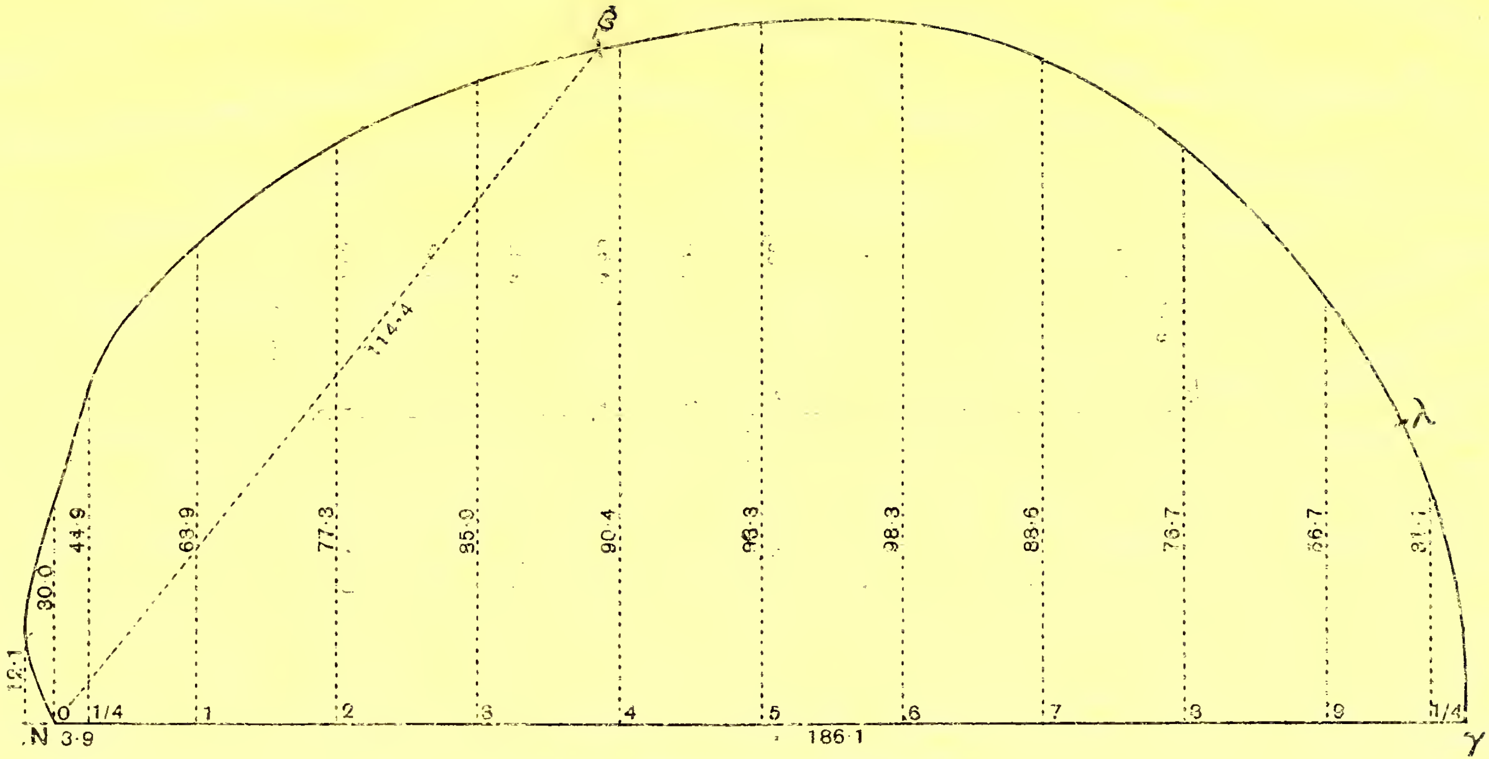


Fig. XXI. Eskimo Crania. Sagittal Contour.

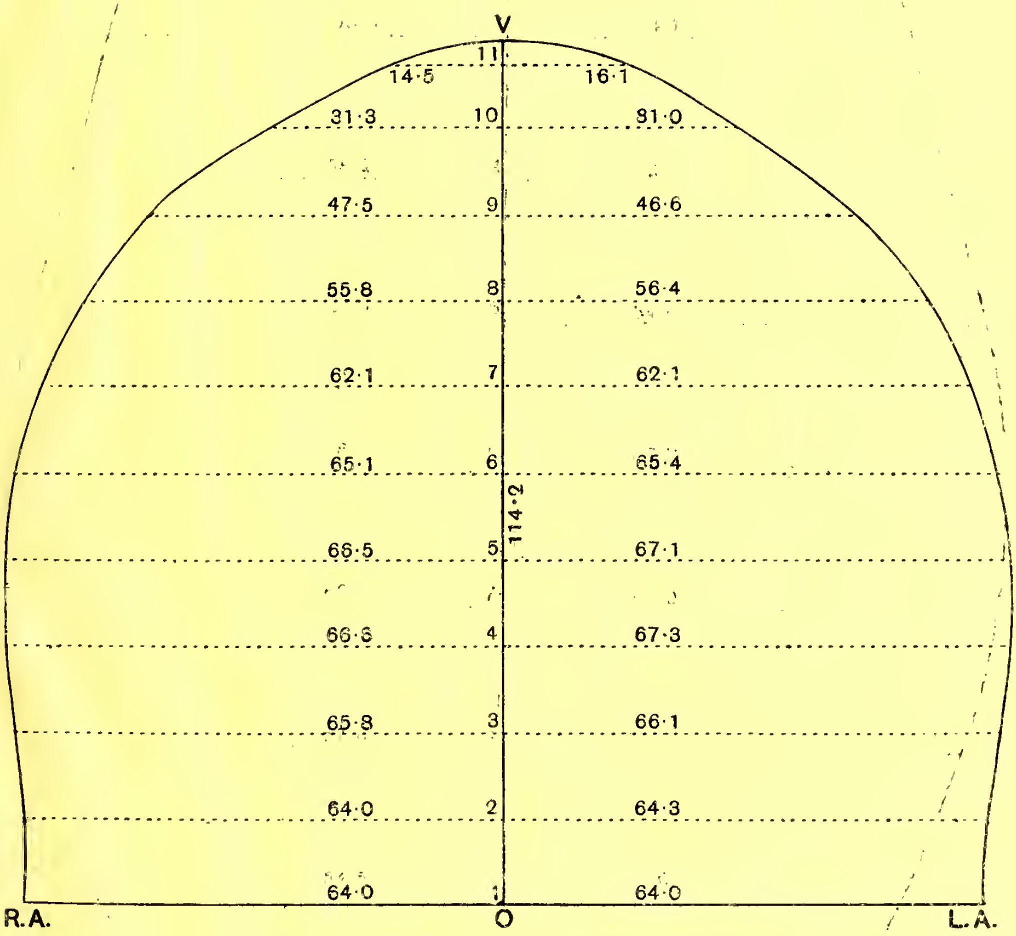


Fig. XXII. Eskimo Crania. Transverse Contour.

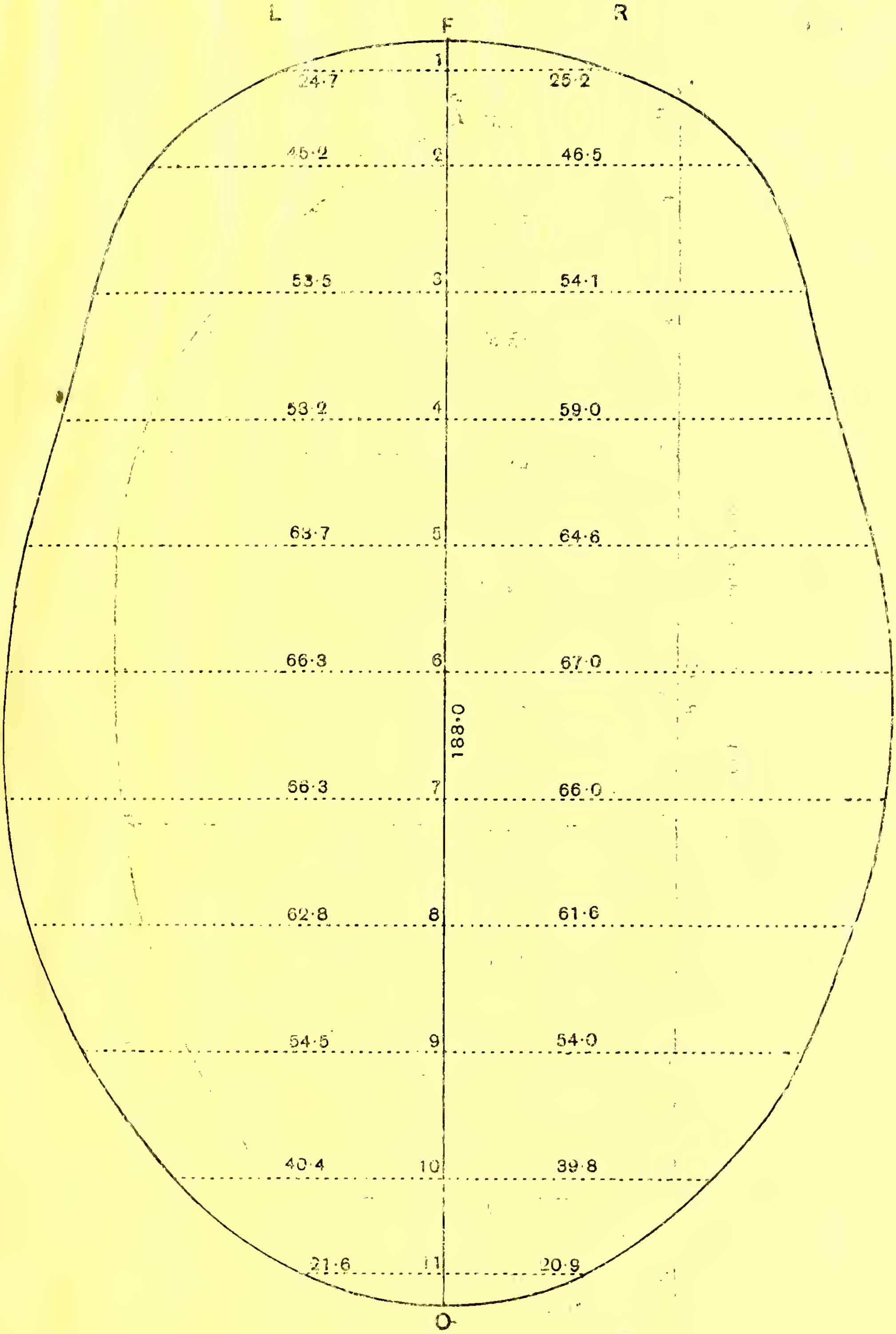


FIG. XXIII. Eskimo Crania. Horizontal Section.

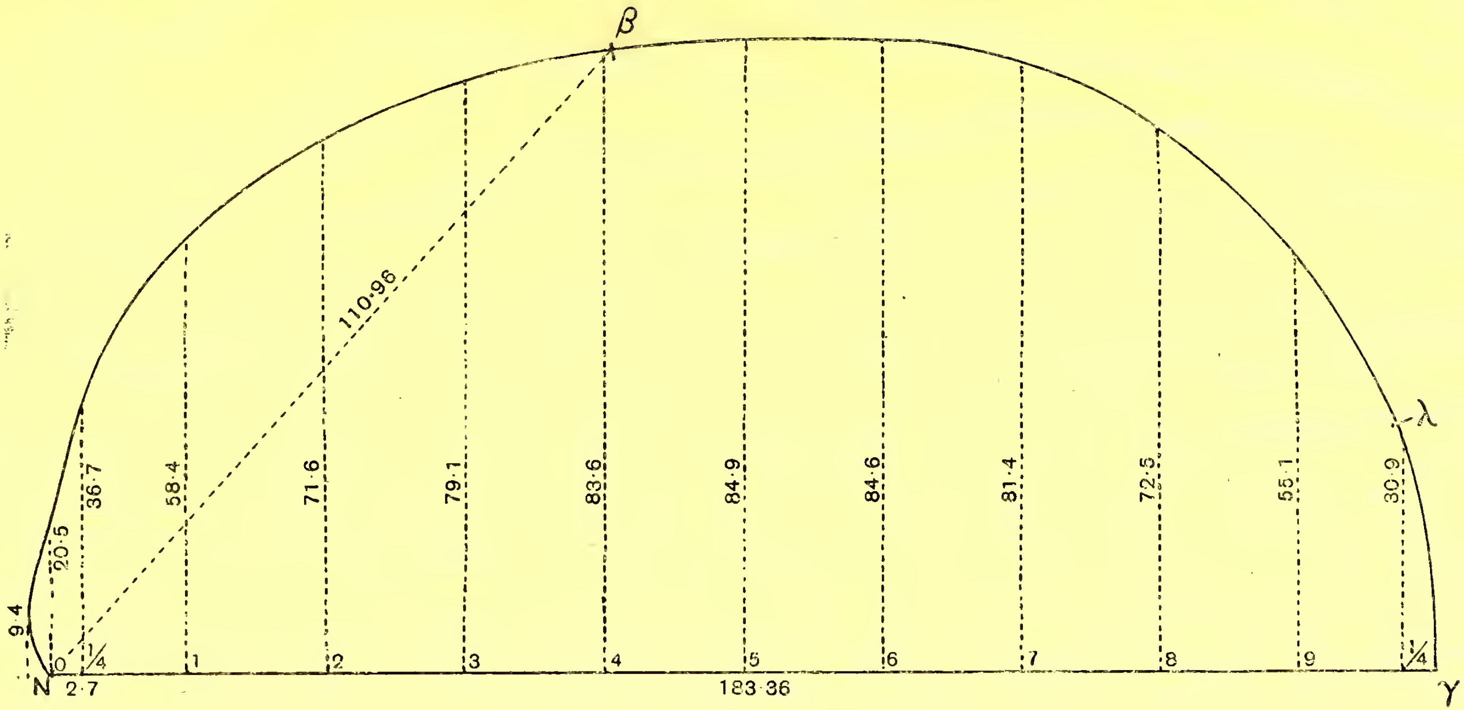


Fig. XXIV. Guanche Crania. Sagittal Contour.

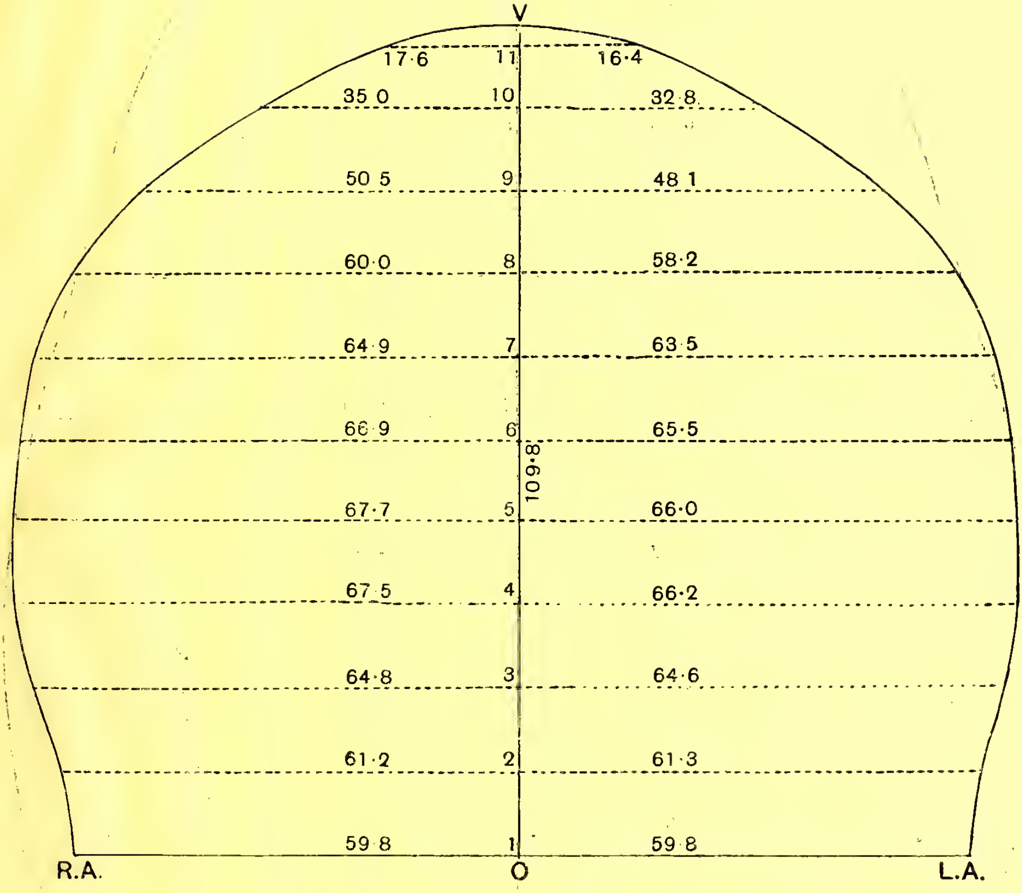


Fig. XXV. Guanche Crania. Transverse Contour.

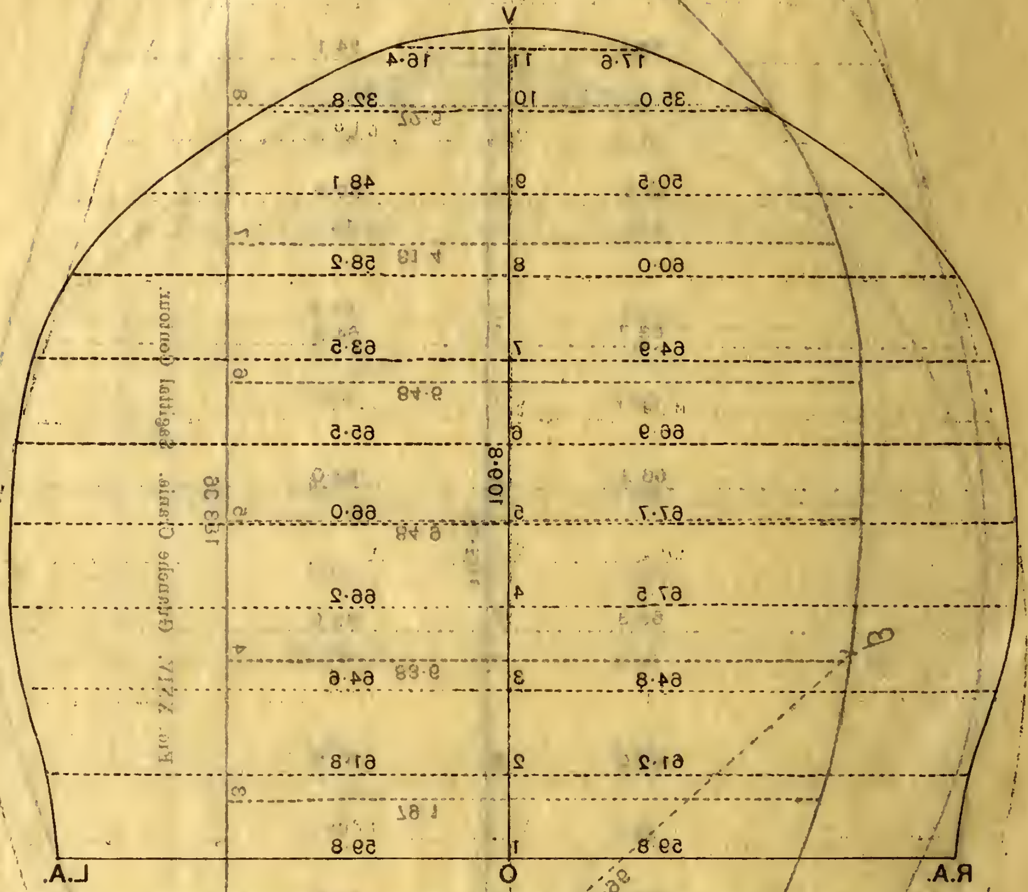


Fig. XXV. Guanche Crania. Transverse Contour.

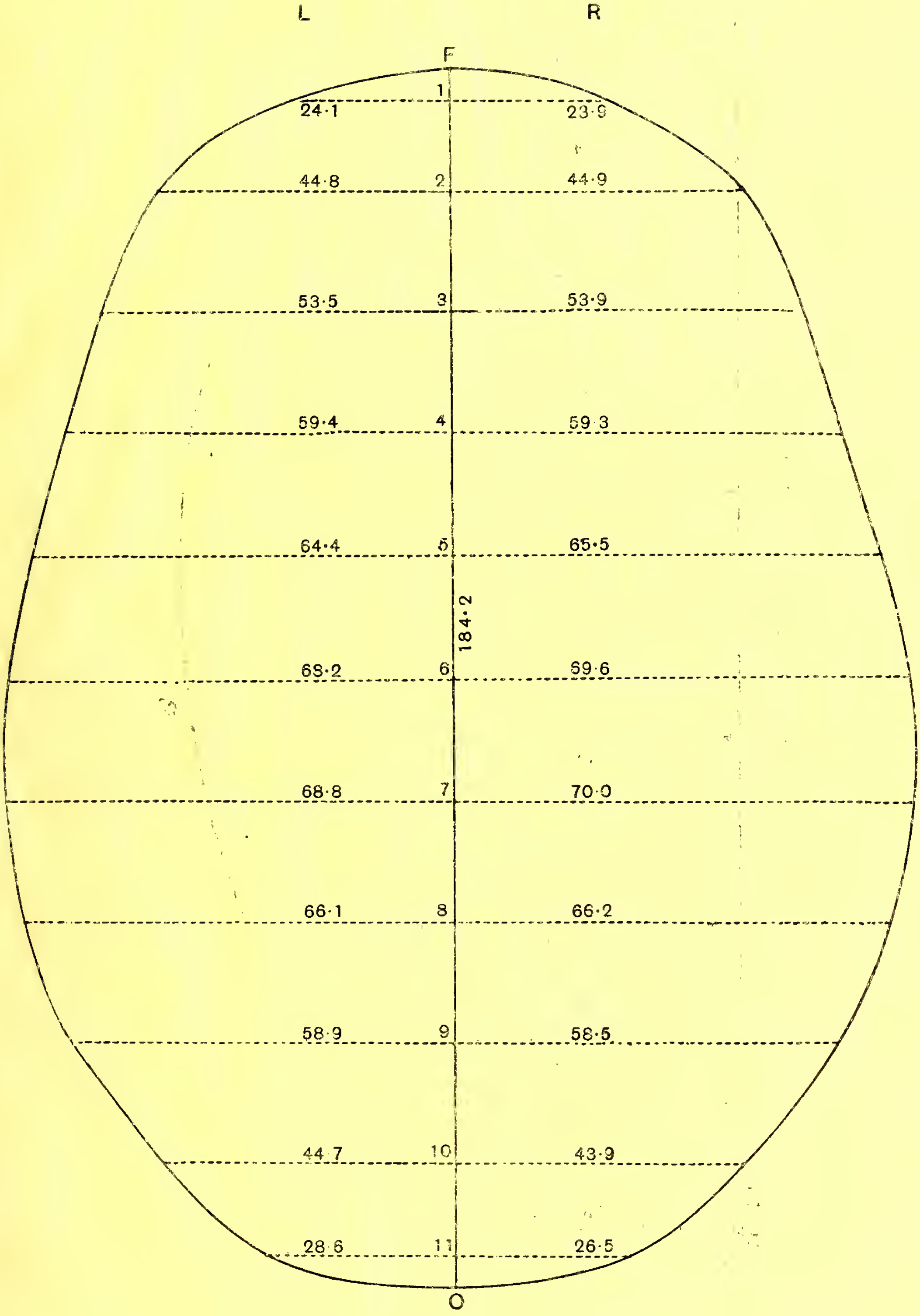


Fig. XXVI. Guanche Crania. Horizontal Contour.

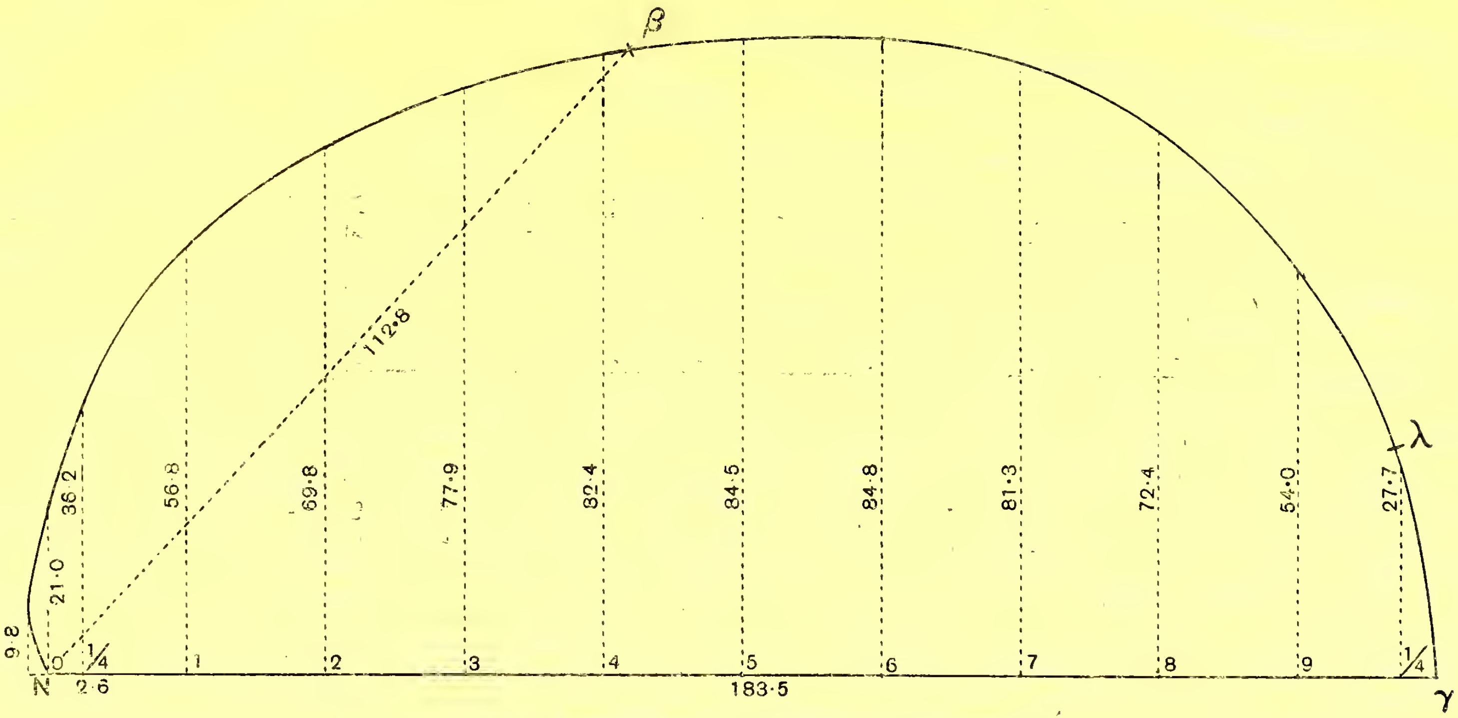


Fig. XXVII. English Crania, 17th Century. Sagittal Contour.

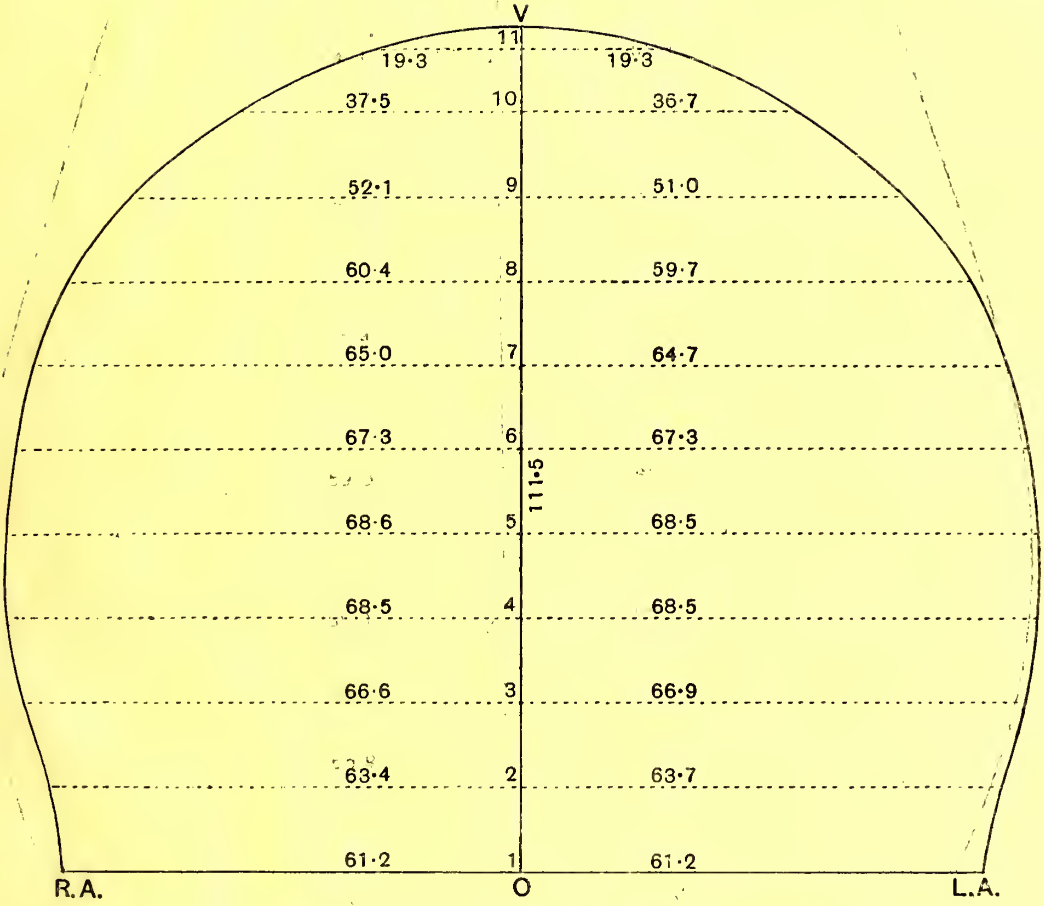


Fig. XXVIII. English Crania, 17th Century. Transverse Contour.

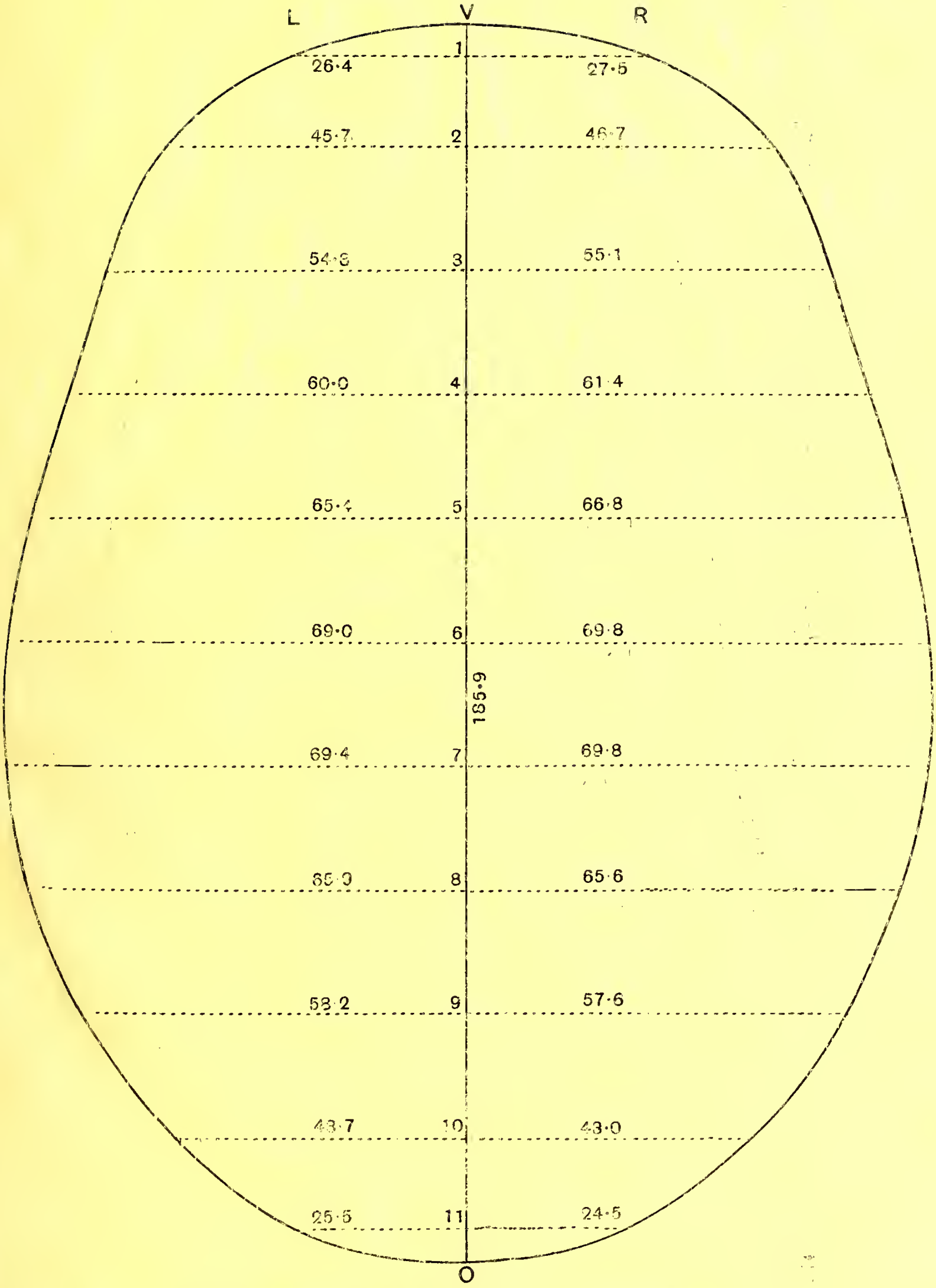


Fig. XXIX. English Crania, 17th Century. Horizontal Contour.

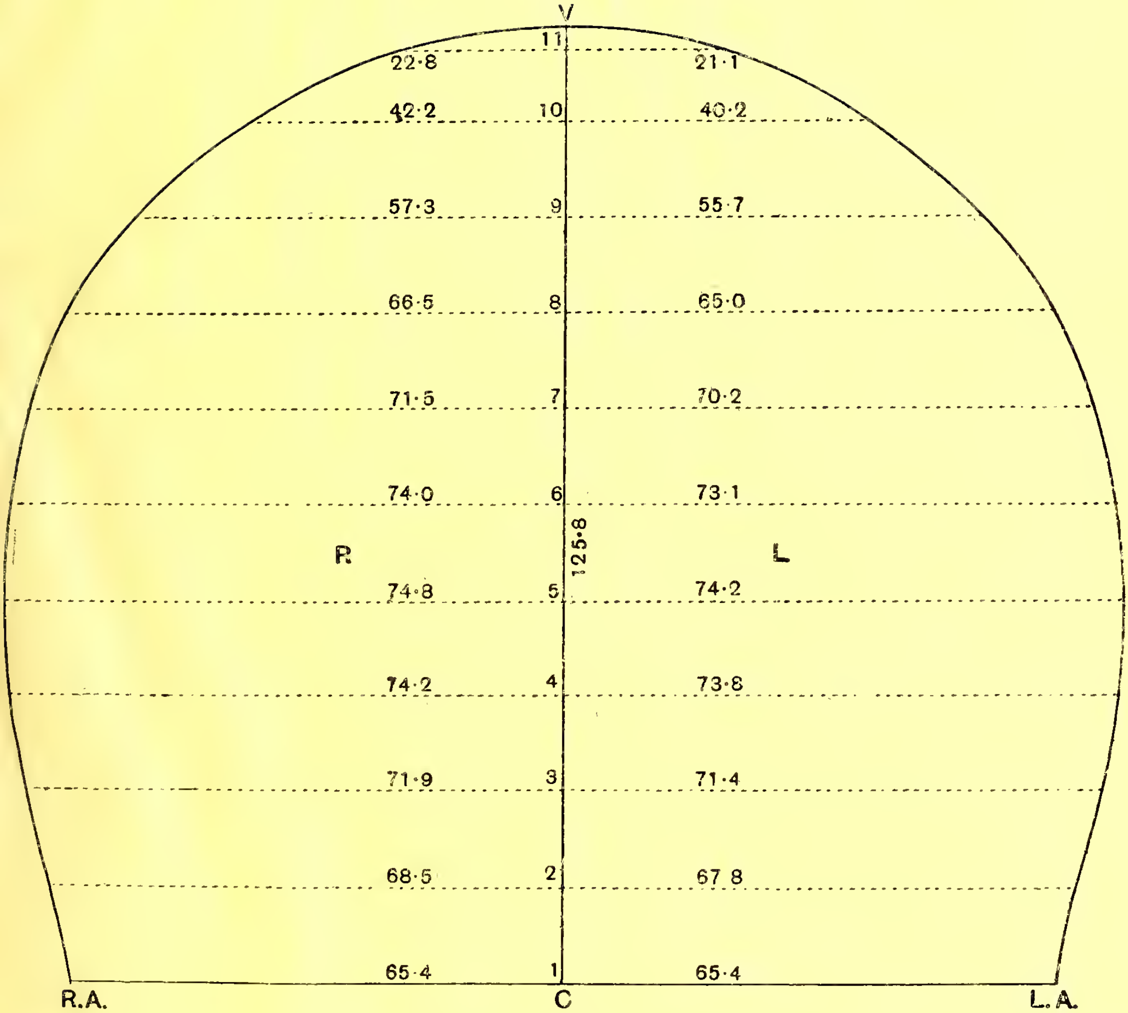


Fig. XXXI. Living English Head. Transverse Contour.

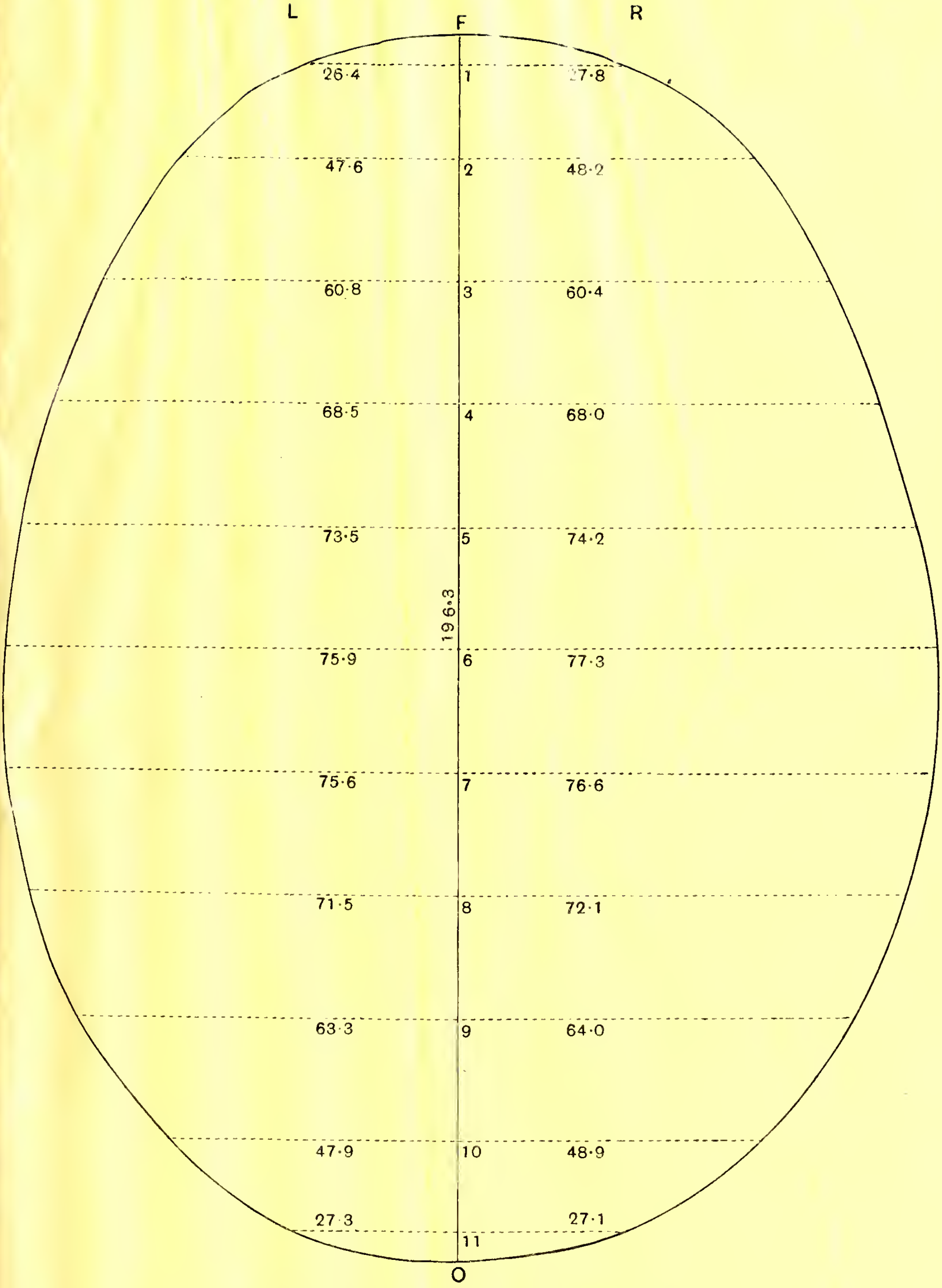


Fig. XXXII. Living English Head. Horizontal Contour.

Vol. VIII. Parts III and IV. January 1912

BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY
W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY
KARL PEARSON

CAMBRIDGE
AT THE UNIVERSITY PRESS

LONDON: FETTER LANE, E.C.
(C. F. CLAY, MANAGER)
AND H. K. LEWIS, GOWER STREET
EDINBURGH: 100, PRINCES STREET
NEW YORK: G. P. PUTNAM'S SONS
BERLIN: A. ASHER AND CO.
LEIPSIC: BROCKHAUS
BOMBAY AND CALCUTTA: MACMILLAN AND CO., LIMITED
Entered at the New York Post Office as Second Class Matter

Price Twenty Shillings net.

[*Issued February* 27, 1912]

UNIVERSITY OF LONDON, UNIVERSITY COLLEGE.

Drapers' Company Research Memoirs.

Biometric Series.

I. **Mathematical Contributions to the** Theory of Evolution.—XIII. On the Theory of Contingency and its Relation to Association and Normal Correlation. By Karl Pearson, F.R.S. Price 4*s. net.*

II. **Mathematical Contributions to the** Theory of Evolution.—XIV. On the Theory of Skew Correlation and Non-linear Regression. By Karl Pearson, F.R.S. Price 5*s. net.*

VI. **A Monograph on Albinism in Man.** By Karl Pearson, E. Nettleship, and C. H. Usher. With upwards of one hundred plates. *Text,* Part I and *Atlas,* Part I with 52 plates containing 185 illustrations of Albinism. Price 35*s. net.*

VII. **A Monograph on Albinism in Man.** By Karl Pearson, E. Nettleship, and C. H. Usher. *Text,* Part II and *Atlas,* Part II. *Nearly ready.*

Studies in National Deterioration.

III. **A Second Study of the Statistics of** Pulmonary Tuberculosis: Marital Infection. By the late E. G. Pope. Edited and revised by Karl Pearson, F.R.S. with an Appendix on Assortative Mating from Data reduced by Ethel M. Elderton. Price 3*s. net.*

V. **On the Inheritance of the Diatheses** of Phthisis and Insanity. A Statistical Study based upon the Family History of 1,500 Criminals. By Charles Goring, M.D., B.Sc. Price 3*s. net.*

VI. **A Third Study of Pulmonary Tuber**culosis. The Mortality of the Tuberculous and Sanatorium Treatment. By W. Palin Elderton, F.I.A. and S. J. Perry, A.I.A. Price 3*s. net.*

VII. **The Intensity of Natural Selection** in Man. By E. C. Snow, M.A. Price 3*s. net.*

Francis Galton Laboratory for National Eugenics.

Memoir Series.

IX. **The Treasury of Human Inheritance** (Pedigrees of physical, psychical and pathological Characters in Man). Part III. (Angioneurotic Oedema, Hermaphroditism, Deaf-Mutism, Insanity, Commercial Ability.) Price 6*s. net.*

X. **The Influence of Parental Alcoholism** on the Physique and Intelligence of the Offspring. By Ethel M. Elderton, assisted by Karl Pearson. Price 4*s.*

XI. **The Treasury of Human Inheritance** (Pedigrees of physical, psychical and pathological Characters in Man). Part IV. (Cleft Palate, Hare-Lip, Deaf-Mutism, and Congenital Cataract.) Price 10*s.*

XII. **The Treasury of Human Inheritance** (Pedigrees of physical, psychical, and pathological Characters in Man). Parts V and VI. (Haemophilia.) Price 15*s. net.*

XIII. **A Second Study of the Influence of** Parental Alcoholism on the Physique and Intelligence of the Offspring. By Karl Pearson, F.R.S., and Ethel M. Elderton. Price 4*s. net.*

XIV. **A Preliminary Study of Extreme** Alcoholism in Adults. By Amy Barrington and Karl Pearson, F.R.S., assisted by David Heron, M.A., D.Sc. Price 4*s. net.*

XV. **The Treasury of Human Inheritance** (Pedigrees of physical, psychical, and pathological Characters in Man). Parts VII. and VIII. (Dwarfism.) With 49 Plates of illustrations and 8 Plates of Pedigrees. Price 15*s. net.*

XVI. **A Second Study of Extreme Alco**holism in Adults. By David Heron, D.Sc. *Immediately.*

XVII. **The Treasury of Human Inheri**tance. Prefatory matter and complete name and subject Index to Vol. I. Parts I—VIII. *Just ready.* Price 2*s.* 6*d. net.*

Lecture Series.

II. **The Groundwork of Eugenics.** By Karl Pearson, F.R.S. Price 1*s. net.*

III. **The Relative Strength of Nurture** and Nature. By Ethel M. Elderton. Price 1*s. net.*

IV. **On the Marriage of First Cousins.** By Ethel M. Elderton. Price 1*s. net.*

V. **The Problem of Practical Eugenics.** By Karl Pearson, F.R.S. Price 1*s. net.*

VI. **Nature and Nurture, the Problem of** the Future. By Karl Pearson, F.R.S. Price 1*s. net.*

VII. **The Academic Aspect of the Sci**ence of National Eugenics. By Karl Pearson, F.R.S. Price 1*s. net.*

Questions of the Day and the Fray.

I. **The Influence of Parental Alcoholism** on the Physique and Ability of the Offspring. A Reply to the Cambridge Economists. By Karl Pearson, F.R.S. Price 1*s. net.*

II. **Mental Defect, Mal-Nutrition, and** the Teacher's Appreciation of Intelligence. A Reply to Criticisms of the Memoir on 'The Influence of Defective Physique and Unfavourable Home Environment on the Intelligence of School Children.' By David Heron, D.Sc. Price 1*s. net.*

III. **An Attempt to correct Some of the** Misstatements made by Sir Victor Horsley, F.R.S., F.R.C.S., and Mary D. Sturge, M.D. By Karl Pearson, F.R.S. Price 1*s. net.*

IV. **The Fight against Tuberculosis and** the Death-rate from Phthisis. By Karl Pearson, F.R.S. Price 1*s. net.*

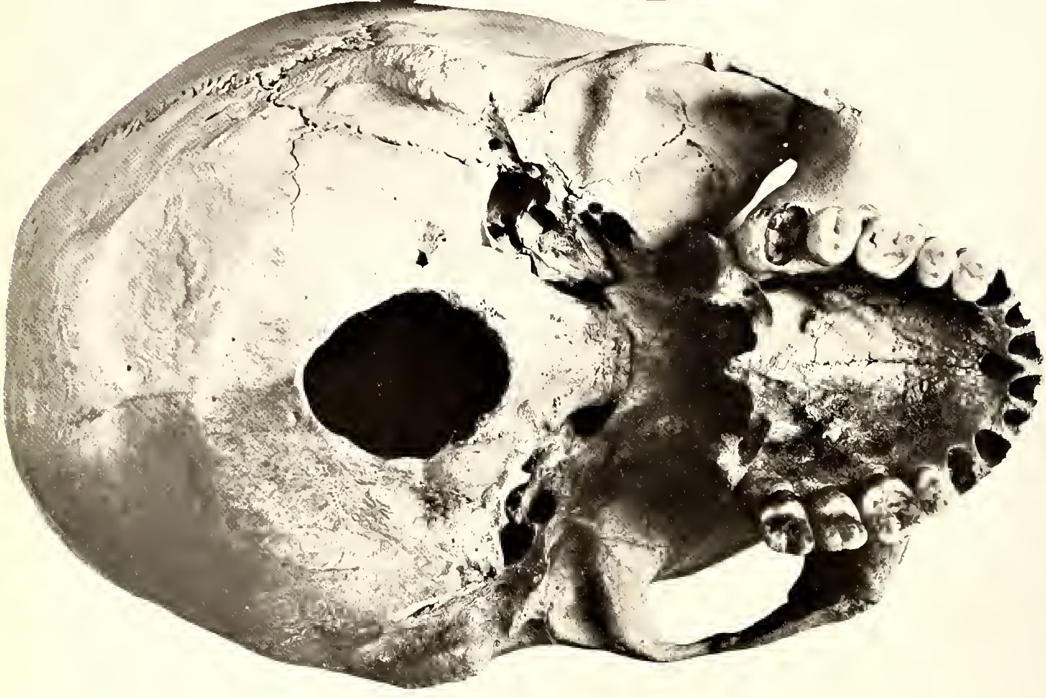
THE ABOVE MAY BE OBTAINED FROM DULAU & CO., 37, SOHO SQUARE, LONDON, W.

Plate I

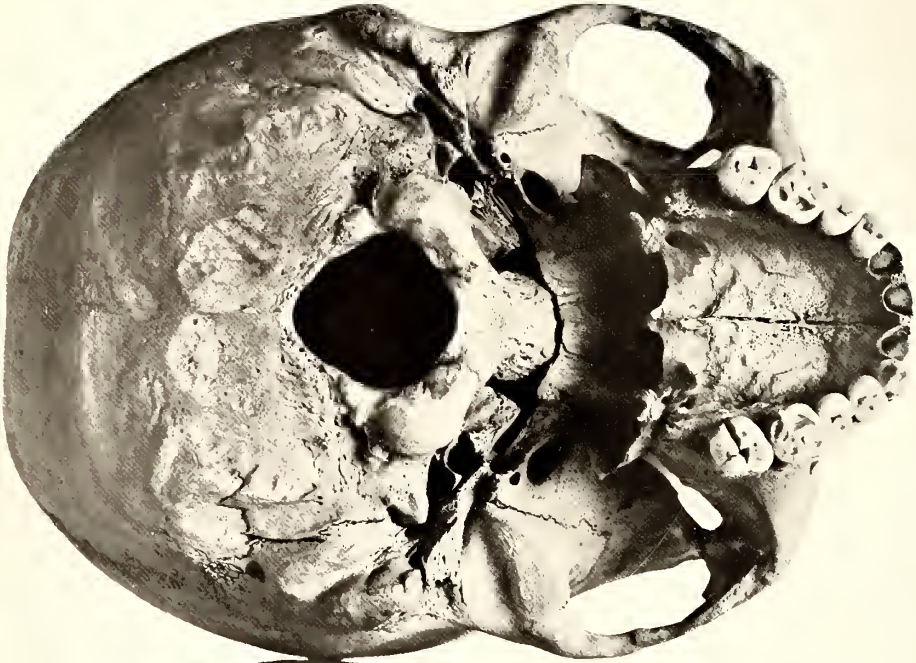
A. Complete synchondrosis on left, and on right to within 8 mm. Cranium E 167.

B. Left pars condyloidea and pars basilaris removed.

C. Same cranium with L. pars condyloidea and pars basilaris removed.



A. Unilateral incomplete synchondrosis on right. Cranium E 553.



B. Bilateral incomplete synchondrosis. Cranium E 576.

A. Bilateral slight synchondrosis. Cranium E 749.

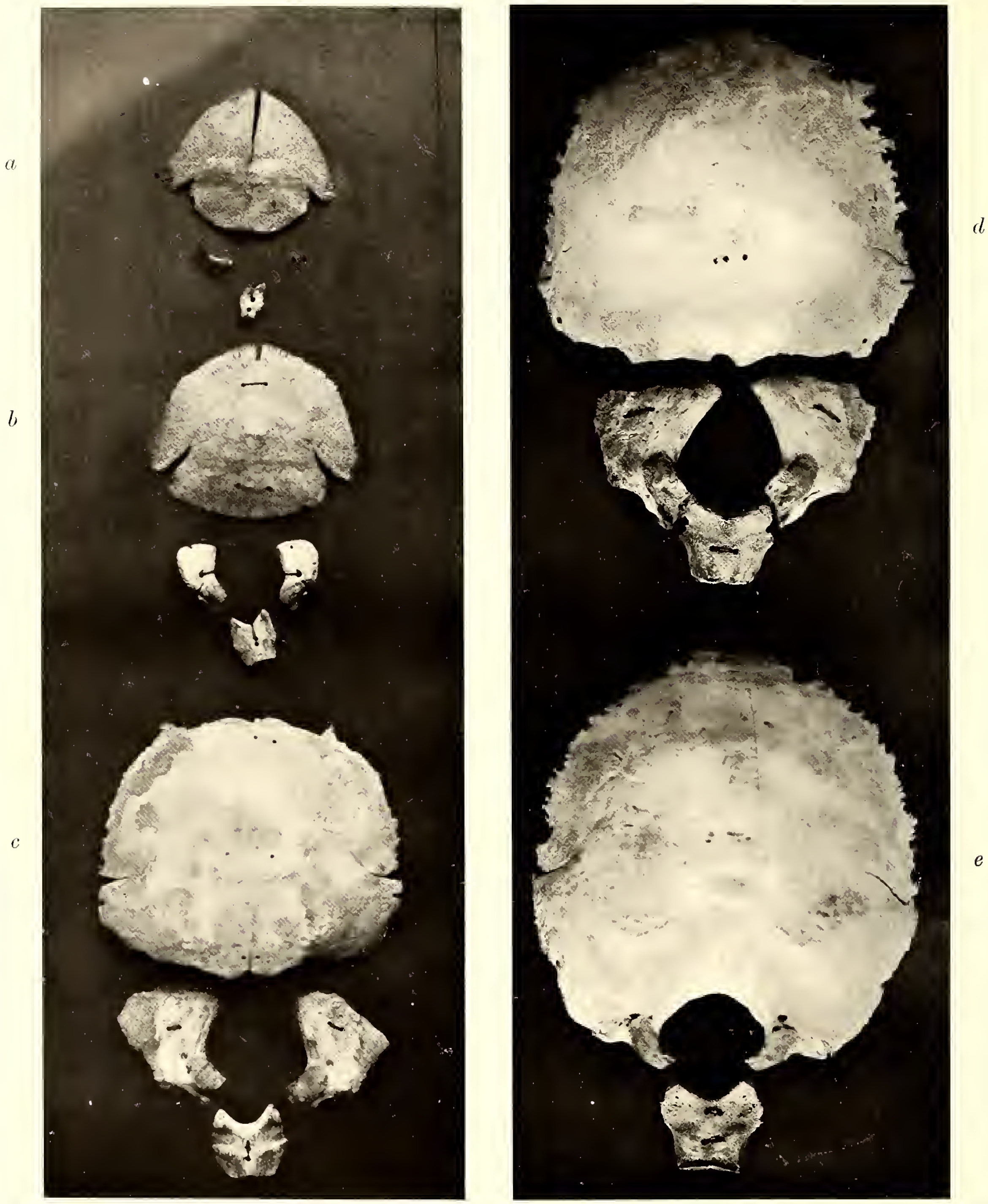
B. Bilateral incomplete synchondrosis. Cranium E 454.

A. Bilateral and complete synchondrosis from masto-occipital suture to foramen magnum. Child, Dr Derry, No. 14.

B. Synchondrosis on left side with ossicle at masto-occipital end. Cranium E 160.

A. Aged male skull, with slight trace of synchondrosis on right and opisthial notch. Cranium E 822.

B. Trace of synchondrosis on both sides; well-marked traces of *basiotique* of Albrecht and pressure facets on hinder margin of foramen magnum. Cranium E 801.



Development of the Human Occipital (Anatomical Museum, University College, London).

a. Foetus of about five months.
b. Infant at birth.
c. Child of one year.
d. Child of about three years.
e. Child of about five years.

VOLUME VIII JANUARY, 1912 Nos. 3 AND 4

OBSERVATIONS ON THE OCCIPITAL BONE IN A SERIES OF EGYPTIAN SKULLS, WITH ESPECIAL REFERENCE TO THE PERSISTENCE OF THE SYNCHONDROSIS CONDYLO-SQUAMOSA (ZAAIJER; SYNCHONDROSIS INTRAOCCIPITALIS POSTERIOR, B N A.)

BY H. DOROTHY SMITH, B.Sc. Crewdson-Benington Student
in Craniology.

WHILE taking a series of measurements at University College, under the supervision of Professor Pearson, on a collection of Egyptian crania of the third dynasty, the writer noted certain peculiarities in the occipital regions of several specimens. In particular a distinct fissure of varying length, extending inwards from the masto-occipital suture (the origin being located generally by the mastoid foramen) towards the foramen magnum, was well shown in several adult specimens. This was identified by Professor Thane with the persistent synchondrosis condylo-squamosa described by T. Zaaijer (*Anatomische Hefte,* herausgegeben von F. Merkel und R. Bonnet, 1. Abteilung, XII. Heft. 1894). An examination of the collection was then undertaken for comparative purposes, and in this the writer is much indebted to both Professor Pearson and Professor Thane for their kind assistance.

The first 1100 specimens examined included the crania of 62 children of whom the youngest was probably about 5 years old. The only instance of persistence of the synchondrosis condylo-squamosa in its whole extent from the masto-occipital suture to the margin of the foramen magnum was found in the cranium of a child of about 7 years of age—and this on the left side only, while on the right the synchondrosis persists to within 8 mm. of the margin of the foramen magnum. In Plate I. will be seen the condition of this cranium with the component parts in situ (A); in figure (C) will be seen the skull with pars basilaris and pars condyloidea removed and these parts are seen separately in Fig. (B).

The best examples among the adult specimens show an irregular fissure (Plates II. and III.), directed transversely inwards from the masto-occipital suture for about 25 mm. Of crania showing a fissure of 15 mm. or more in length, there are 12 adult and 6 young specimens; and, of these, 8 adult and 6 young specimens show the condition on both sides to a greater or less extent, while the remaining 4 adult (one a young adult) show it on the right side only.

Of the rest, 48 adult and 20 young specimens show a fissure of less than 15 mm. length—in some cases only a slight trace—and of these 29 adult and 11 young show the condition on both sides, and the remaining 19 adult and 9 young, on one side only.

Thus, in the 1100 crania examined, the conditions found were:—

Specimens Examined	Synchondrosis Obliterated	Fissure 15 mm. or more	Less than 15 mm.	Percentage showing persistent Synchondrosis Condylo-squamosa
1038 adult	979	12	47	5·7
62 young	36	6	20	41·9

The percentage of adult crania showing indications of the persistence of the synchondrosis condylo-squamosa agrees very closely with the figure (5·3) found by Zaaijer in the collection of adults examined by him, and similarly the condition has been found in a much higher percentage of young crania. The condition also appears to be of more frequent occurrence among the female crania, for of 322 adult female crania 30 show persistence of the synchondrosis condylo-squamosa (9·3 per cent.), while in 457 male crania the condition was found in only 20 specimens (4·3 per cent.).

It was thought worth while to examine the crania of other primates for comparison, and the writer was enabled by the courtesy of Professor Keith to examine the collection at the Royal College of Surgeons. This collection includes a higher percentage of young specimens, and also a number of younger individuals than any in the collection of human crania under observation, and, as was therefore to be expected, a great number of the young specimens show the condition with the synchondrosis extending from the masto-occipital suture to the margin of the foramen magnum, but few adults show more than traces of the developmental condition. Owing to the very great range in size of the crania of the other primates a standard of 15 mm. length of fissure has not been adopted here as in the case of the human crania, and the cases have been grouped according to whether the fissure reaches the margin of the foramen magnum or stops short of it.

Thus 13·3 per cent. of the adult anthropoid ape crania examined show some indication of the synchondrosis condylo-squamosa, while 21·8 per cent. of the young specimens show the whole length of the synchondrosis, and 54·5 per cent. show the condition in varying degrees.

Of the Lemuroidea 17 specimens (14 adult and 3 young) were examined, and a single adult showed a trace of the synchondrosis while two young showed the whole extent. The remaining specimen—a foetal cheiromys—showed the various components of the occipital entirely separate.

Specimens Examined		Synchondrosis Obliterated	Extending from Masto-occipital suture to Foramen Magnum	Shorter Portions
SIMIIDAE				
12 chimpanzees	7 adult	7	—	—
	5 young	1	3	1 (acrocephalic)
5 gorillas	3 adult ...	3	—	—
	2 young ...	2	—	—
11 orang-outans	5 adult	3	—	2
	6 young	1	4	1
6 gibbons	2 adult ...	2	—	—
	4 young ...	1	1	2
CERCOPITHECIDAE				
57 various	29 adult ...	27	—	2
	28 young ...	17	3	8
CEBIDAE				
19 various	10 adult ...	6	—	4
	9 young ...	2	1	6
HAPALIDAE				
5 various	4 adult ...	4	—	—
	1 young ...	1	—	—

Certain specimens among both the Egyptian crania, and the other primates, show the synchondrosis associated with an ossicle at the masto-occipital end (see our Plate IV. B), and the writer finds that the persistent portions of the synchondrosis are invariably at this end. In no case has the condition been found like that cited as typical by A. Rambaud and Ch. Renault (*Origine et Développement des Os*, p. 105 and Fig. 8, Plate VII.), where the ends of the fissure reaching the margin of the foramen magnum remain open after the other portions have been obliterated.

It will be seen in Plate I. A that the line of the synchondrosis condylo-squamosa terminates internally in a small angular projection which forms the lateral boundary of a distinct median bay in the hinder margin of the foramen magnum. This bay, which has a breadth of 9 mm. and a depth of 4 mm., may be termed the *opisthial notch*, as corresponding to the region of the opisthion. Of the 85 crania which show some degree of the persistence of the synchondrosis condylo-squamosa, this notch is more or less marked in 43, whereas of 100 skulls taken at random from the series, which have the synchondrosis completely obliterated, an appreciable notch is present only in 9. A faint indication of the opisthial notch is however frequently to be recognised in the adult skull.

In one somewhat aged male skull, shown in Plate V. A, and presenting only a slight trace of the synchondrosis condylo-squamosa at the end abutting on the masto-occipital suture of the right side, the opisthial notch has a depth of 11 mm. with a maximum breadth of 10 mm. In another adult male skull, with a trace of the synchondrosis on the right side only, the foramen magnum is prolonged backwards as a broad excavation which is not sharply defined laterally, but must be put into the same category. The antero-posterior diameter of the foramen magnum in this case measures 42·5 mm. and the greatest breadth is 30·5 mm.

The opisthial notch evidently results from a defective development of the lower division of the occipital squama, which is formed by the supra-occipital element of the bone, and especially of the part which is derived from the so-called ossicle of Kerckring (*manubrium squamae occipitis*, Virchow). The existence of such a notch in the adult bone is not mentioned in Ledouble's *Traité des Variations des Os du Crâne*, Paris, 1903; but Hamy records a case of a microcephalic foetal skull in which there was a small posterior encephalocele resulting from absence of the ossicle of Kerckring (*Bull. de la Soc. d'Anthropol. de Paris*, 1867, p. 511). Hamy refers to this case again in his paper "Recherches sur les fontanelles anomales du crâne humain," *Journal de l'Anatomie*, T. VII. 1870—71, pp. 591—601, and there uses the name *fontanelle cerebelleuse* for the membranous gap at the base of the occipital squama. This condition would appear to represent a persistence to a greater or less extent of Hannover's *spinoso-occipital membrane*. Ledouble states (*op. cit.* p. 53) that there is a similar defect of ossification in some skulls of normal foetuses in the *Museum d'histoire naturelle* of Paris, as well as in the skull of a hydrocephalic foetus in the Musée Dupuytren.

In only one instance among the Egyptian crania under observation did the writer find any indication of the division of the pars basilaris, which leads to the separation of an anterior segment, first observed by Etienne Geoffroy Saint-Hilaire and named by him *oto-sphenal*, and subsequently designated by Albrecht *basiotique*. This occurs in a young specimen, about 12 years of age (Plate V. B, Cranium E 801), which also shows a slight indication of the synchondrosis condylo-squamosa. Here the basilar portion has on each side a cleft directed transversely inwards from the petro-occipital fissure for a distance of 9 mm. on the right side and 4·5 mm. on the left. The intervening bridge of bone has a breadth on the surface of 15·5 mm. The division is more strongly marked on the endocranial aspect of the bone, where the fissures are only 12 mm. apart, and are connected by a shallow transverse furrow. An incomplete division of the basilar portion was found by Lucy 25 times in 496 skulls (5·04 per cent.). This skull also shows two well marked pressure facets at the hinder margin of the foramen magnum.

Owing to the large number of crania under consideration and difficulties of access it has not yet been possible to examine the whole of this Egyptian series very carefully, but the writer has made a more cursory examination of the remaining 629 crania in the collection for further examples of the persistence of the synchondrosis condylo-squamosa with the following results:—

Specimens Examined	Synchondrosis Obliterated	Fissure 15 mm. long or more	Less than 15 mm.	Percentage showing Synchondrosis Condylo-squamosa
608 adult	551	8	49	9·3
21 young	6	3	12	71·4

The cranium shown in Plate IV. A is that of a child of about 5 years of age which shows the whole extent of the synchondrosis condylo-squamosa from the masto-occipital suture to the foramen magnum on both sides. This is one of the Wood-Jones collection of Nubian specimens of Roman date obtained from cemetery 14. The condition of the occipital here was noticed by Dr Derry, and the specimen has been kindly lent for the purposes of this paper.

In addition to the Plates already mentioned is included one (Plate VI.) illustrating the development of the human occipital before and after birth, from a series in the Anatomical Museum of University College. The age of these specimens is estimated as follows. (*a*) Foetus of about five months. (*b*) At birth. (*c*) Child of about 1 year. (*d*) About three years. (*e*) About 5 years.

The condition of these specimens would indicate that the obliteration of the synchondrosis condylo-squamosa begins during the third or fourth year of life (Zaaijer, pp. 199—202), but in conclusion it would appear that there is a greater range of variation in the age at which the fusion takes place than is generally stated, and in certain cases the synostosis is not completed even in the adult.

A STUDY OF PYGMY CRANIA, BASED ON SKULLS FOUND IN EGYPT.

By H. DOROTHY SMITH, B.Sc. Crewdson-Benington Student
in Craniology.

Among the Egyptian crania of the Third Dynasty now being measured under the supervision of Professor Karl Pearson, at University College, London—it was found that nine adult specimens were remarkable for their small size. These crania present marked feminine characters, and have been sexed definitely as female in all cases except one (E 487) which, though exceedingly small, shows certain variations from the more usual form, and is more heavily developed than the others, being thus more open to question as to sex.

In Plates VII—XI will be seen this cranium in various aspects. Plate VII shows the profile, with some general projection of the glabella, beyond which the line rises and slopes somewhat markedly to the bregma. There is some slight post-coronal depression, beyond which there is a further slight rise towards the mid-sagittal area. Beyond this the line curves first gently downwards towards the lambda, where there is an aggregate of ossicles, and then descends almost vertically and finally curves in to the under surface of the skull. The orbits are less finely cut than is usual among these small crania, and the occipital region is somewhat more rugged, while the mastoid processes, though small, are somewhat clumsy, and the left process projects rather further than the right. Besides the ossicles of the lambda, there are two epipteric bones (16 and 14 mm. long) on the left, and one (10 mm. long) on the right, and small ossicles in the lambdoid suture. The sutures are somewhat simple. Both zygomatic arches, and the left orbit, and the nasal bones have been damaged and only three teeth remain in position. These are the first molar on the right side, and the first and second molars on the left side, which show considerable signs of wear, but there is no evidence of wisdom teeth. There is no sign of any ossification of the coronal, sagittal or lambdoid sutures, but the spheno-occipital synchondrosis is obliterated, and the cranium is evidently that of a fully grown individual.

Cranium E 506 (Plates XII—XVI) shows the more usual formation. Here the profile shows an 'infantile' forehead rising vertically from the nasion. The line then curves backwards, and runs almost horizontally, with a slight indication of post-coronal depression, to a point in the mid-sagittal area. Beyond this it curves down to the lambda, and shows some occipital projection before curving in towards the base of the skull. The orbits and zygomatic arches are extremely lightly developed, but the mastoid processes are of fairly large size. The cranium shows metopism with an accompanying breadth of forehead, but there is a tendency towards the obliteration of all sutures. There are some very small ossicles in the lambdoid suture. None of the teeth have been preserved, but the alveolar margin is in good condition, and shows the sockets of the full complement of permanent teeth, including the wisdom teeth. This is the only specimen to which it has been possible to fit a mandible, and the mandible in this case shows a somewhat low and sloping ramus, a shallow sigmoid notch, and slight mental prominence. The three lower molar teeth on each side have been preserved.

Plate XVII shows the above two crania in comparison with a typical adult male and female of normal size in the collection, the male being placed below the female in the plate.

The other crania of small size under consideration show a formation more closely resembling E 506; and, of these, E 21 shows considerable absorption of the hinder portions of the alveolar margin, but appears to have had the wisdom teeth in position. The forehead here is however not vertical, as in the case of E 506, but slopes backwards somewhat from the nasion.

E 420 shows no wisdom teeth, but the other molars in position are considerably ground down, and the sphenoid and occipital bones are united. This cranium is also metopic.

E 579 shows the three upper molar teeth in position on the left side, on the right the second and third molars are wanting, and their sockets are somewhat absorbed. This cranium is rather larger than the others under consideration, and shows somewhat marked supraorbital ridges, and postcoronal depression.

E 666. Here five teeth are present, viz. the two bicuspids and the three molars of the right side; and in association with the wisdom tooth is an accessory molar. The teeth are wanting on the left side.

E 862 shows no wisdom teeth, and the other teeth present show no great signs of wear, but the spheno-occipital synchondrosis is obliterated, and the cranium appears to be that of a young adult. There is an epipteric bone (26 mm. long) on the left side.

E 869 shows the sockets of both upper wisdom teeth. Slight supra-orbital ridges are present, and also a slight metopic crest, and postbregmatic eminence.

E 919 also shows the sockets of both upper wisdom teeth, and though remarkably small has somewhat heavier mastoid processes than is usual among these small specimens.

In addition to the above it has been possible through the kindness of Dr Derry to procure measurements and photographs of two small specimens of other periods.

Plates XVIII and XIX show a specimen of the Wood-Jones collection (*L* 2/63). This is a small female with a wisdom tooth erupted, but not yet quite in position in the mandible. This skull was taken from a cemetery of Christian period, on the Island of Hesa, to the south of the Asswan dam.

Plates XX and XXI show the skull of a small female (*L* 98/90 *B*) which has had the wisdom teeth in position. This was found by Dr Derry in a grave of Ptolemaic-Roman period at Dakka (70 kilometres south of Asswan on the left bank of the Nile), and, of this specimen, the pelvis has also fortunately been preserved. Plates XXII—XXIV show the pelvic bones in comparison with those of a female of normal size. It will be seen to be very much smaller than the latter, thus indicating a marked degree of correspondence between size of the skull and the probable size of the body.

A table of the series of measurements which it is customary to take in the Biometric Laboratory is given on p. 265, and in this table are included the measurements taken on the above crania, in the order of their notice above, and also, for purposes of comparison, as many of the corresponding measurements on Akka skulls, and Andamanese as were given by the late Sir W. H. Flower in his paper ("Description of Two Skeletons of Akkas, a Pygmy race from Central Africa," *Journal of the Anthropological Institute*, Vol. XVIII. 1888). The measurements of a typical female of normal size from the collection of third dynasty specimens are also given. It was hoped that the measurements taken on living Akkas by Prof. Elliot Smith ("Notes on African Pygmies," *Lancet*, Aug. 12, 1905), might also have been included, but, on investigation, it was found that the methods by which these were taken are hardly comparable, and certain of these measurements as printed appeared to be obviously in error. Unfortunately also the condition of the skulls lent by Dr Derry did not admit of the estimation of their capacity by the method of measurement with mustard seed, and E 919 was also too fragile for this.

It will be seen from the indices that the third dynasty specimens do not show prognathism, being for the most part orthognathous while two specimens are mesognathous. The specimen procured by Dr Derry at Dakka shows a very high degree of prognathism while the Hesa specimen is mesognathous. The Akka specimens have both been estimated as prognathous, but the Andamanese are mesognathous.

Similarly the majority of the third dynasty specimens fall into the mesaticephalic group, only one small, metopic, specimen, and the specimen of normal size being brachycephalic, while both the Dakka and Hesa specimens are brachycephalic. The male Akka is dolichocephalic, and the female is mesaticephalic, while the Andamanese are brachycephalic. With regard to the orbital index there is less

Pygmy Cranium, E 487. About Life Size. *Norma lateralis.*

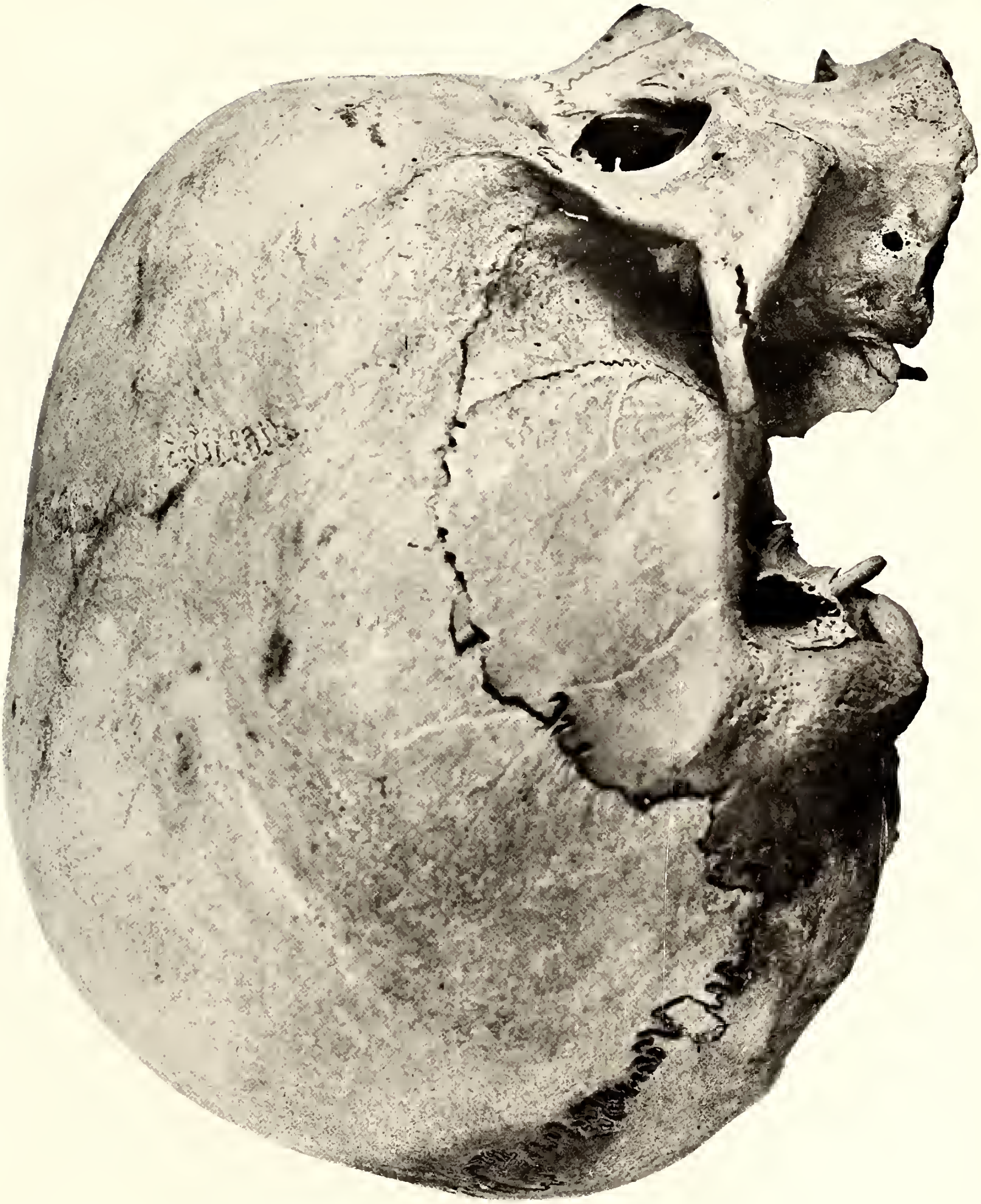
Pygmy Cranium, E 487. About Life Size. *Norma facialis.*

Pygmy Cranium, E 487. About Life Size. *Norma basalis.*

Pygmy Cranium, E 487. About Life Size. *Norma verticalis.*

Pygmy Cranium, E 487. About Life Size. *Norma occipitalis.*

Plate XII



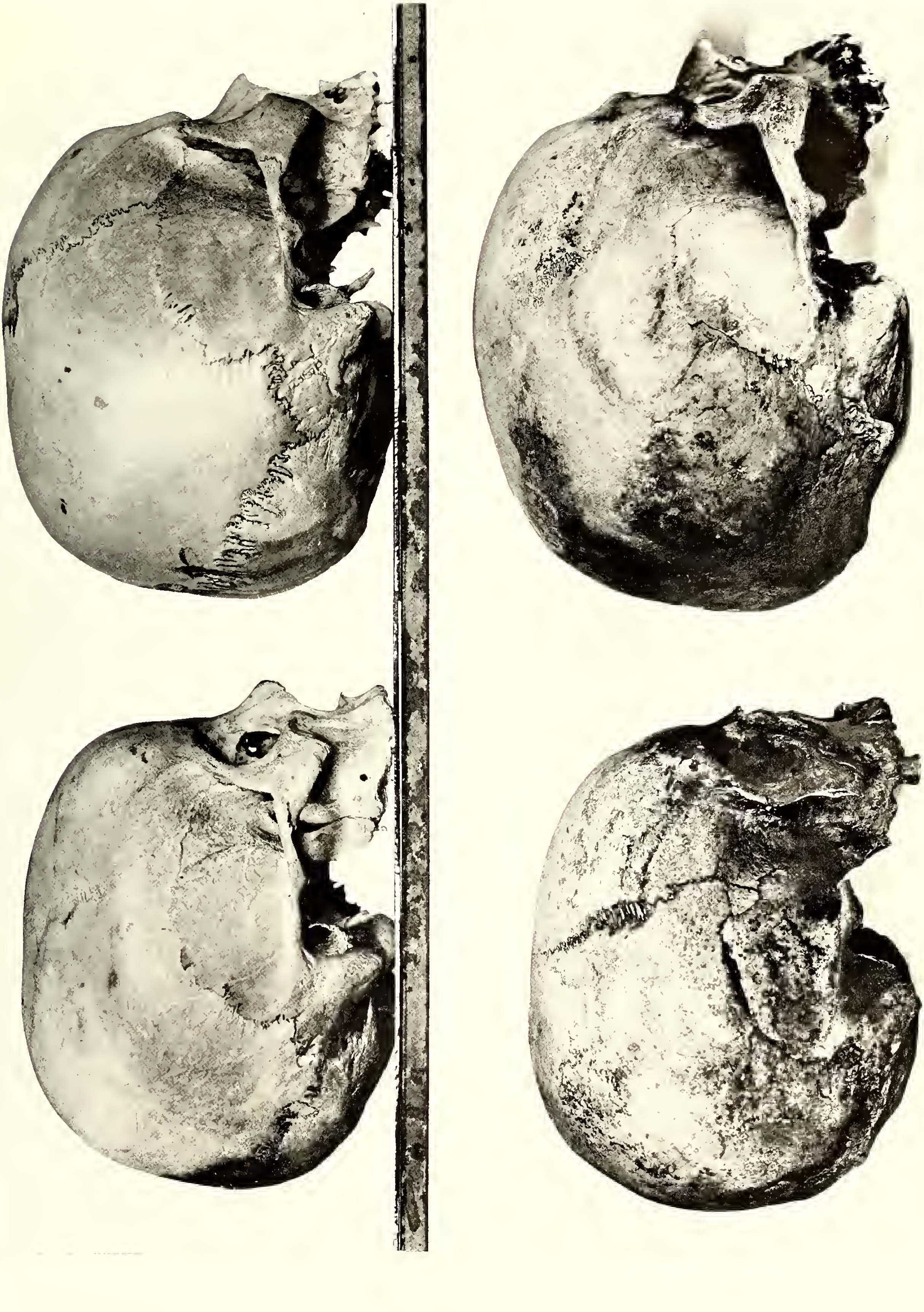
Pygmy Cranium, E 506. About Life Size. *Norma lateralis.*

Pygmy Cranium, E 506. About Life Size. *Norma frontalis.*

Pygmy Cranium, E 506. About Life Size. *Norma basalis.*

Pygmy Cranium, E 506. About Life Size. *Norma verticalis.*

Pygmy Cranium, E 506. About Life Size. *Norma occipitalis.*



Pygmy Crania, E 506 (above) and E 487 (below), placed longside normal female and male Egyptian Crania. About half Life Size.

Pygmy Cranium, L 2/63, from Hesa. About Life Size. *Norma lateralis.*

Pygmy Cranium, L 2/63, from Hesa. About Life Size. *Norma facialis.*

Pygmy Cranium, L 98/90 B, from Dakka. About Life Size. *Norma lateralis.*

Pygmy Cranium, L 98/90 B, from Dakka. About Life Size. *Norma facialis.*

Pelvis of Pygmy, L 98/90 B (below), from Dakka, compared with that of normal female (above). Bones apart.

Superior View of Pelvis of Pygmy, L 98/90 B (below), from Dakka, and of normal female (above).

Frontal View of Pelvis of Pygmy, L 98/90 B (below), from Dakka, and of normal female (above).

	E 487	E 506	E 21	E 420	E 579	E 666	E 862	E 869	E 919	E 921 Female of normal size	Specimen from Hesa	Specimen from Dakka	Akkas		Andamanese	
													♂	♀	♂	♀
Flowers' ophryo-occipital length ...	166	163·5	165	166	167	165	166	167·5	164	172	160	159	168	163	167·4	160·8
Length (glabella-occipital)	167	163	167	167	169	165	167	168	166	173	160	158·5	—	—	—	—
Length (German horizontal)	166	163	168·2	166·5	168·5	164·5	167	167·5	167	173·5	160	160	—	—	—	—
Greatest horizontal breadth	130	131	129	128	131	126	126·8	127	131	140·5	129	130	125	127	135·8	133·2
Least breadth of forehead	88·8	90	89·1	95	89	92·2	85	87	87·3	87	94	86·7	82	86	—	—
Height of skull (basi-bregmatic) ...	120	123	126	122	131	120	127·5	126·8	125	126	129	119	—	124	—	125·3
Auricular height	106·8	102	106	104	105	100	102	106	110	111	105	107	—	—	—	—
Length of base (basion to nasion) ...	93	92	98	86	100	86·2	92	96	90	95	89·5	92·5	92	92	94·6	90·3
Horizontal circumference	470	469	472·7	476	481	473	468	470	475	499	462	460	468	462	481	462
Sagittal circumference	336	340	333	355	337	344	333	340	348	364	342	328·5	—	—	—	—
Nasion to bregma	117	117	111	120·5	118	117	107	115	119	123	117	119·5	—	—	—	—
Bregma to lambda	99	118	119	125·5	121	125	118	118	116	116	123	109	—	—	—	—
Lambda to opisthion, with steel tape...	120	105	103·5	108	97·5	102	108	107	114	125	101·5	100	—	—	—	—
Lambda to opisthion, callipers ...	99·8	91	87·7	92·5	84·8	89	91·5	91·5	98	105	88·5	87·6	—	—	—	—
Cross circumference of skull... ...	288	282	286·5	283	284	276	277	287	299	312	286	293	—	—	—	—
Upper face height (nasion to alveolar point)	63·5	60	66·3	65	64·2	66·8	64·2	66·2	62·8	67·6	59·2	68	72 ophryon to alveolar point	72	—	—
Face breadth	*d*	85	90·8	96	89·2	91·5	82	85	88·2	89·6	89	90 ?	—	—	—	—
Zygomatic breadth	*d*	116	118·8	116·7	120·3	110·5	112	115·3	114	119·3	119·8	118 ?	—	—	—	—
Nasal height	48·2	44(45·3)	49·2	46·5	49	48	48	47	45	46·3	41·6	42	41	38	—	—
Nasal breadth	23	23	25	22·7	24·6	25	20·5	23	23	21	22	24·5	26	21	—	—
Breadth of Left orbit	*d*	36·5	39	36·5	38·8	*d*	36·3	41	37	38·2	38·2	38	35	35	—	—
Breadth of Right orbit...	37·5	37	39·2	37·5	40·2	*d*	36	39·3	37	38·8	37	37	—	—	—	—
Height of Left orbit	*d*	30	36	29	33·5	34·2	32	33·2	33	33	33·6	32	29	29	—	—
Height of Right orbit	30	29	35·7	29	32·3	34·2	32	32·3	31·5	30·3	30·5	31·5	—	—	—	—
Length of palate	44	43·3	47·8	46·3	44·8	44·5	40	49·6	49·5	45·2	48	53 ?	—	—	—	—
Breadth of palate	33·5	35	35·4	36	40	39·2	36	33·2	33·8	37·5	38·7	37·2	—	—	—	—
Palate length excluding alveolar point	40	40	46	43·5	43·2	42	39	46·2	45	42	45·7	48 ?	—	—	—	—
Profile angle	93°	87°·5	83°	82°	83°	79°	81°	82°	85°	87°	75°	78°	—	—	—	—
Profile length	84	84·5	92·9	88	93·2	85·2	83·7	92·8	88	91	90·9	104·5	100	96	—	—
Length of Foramen Magnum	33·8	33·5	35	33·5	36	32·8	39·3	33·8	32	33·8	31·2	26·5	—	28	—	—
Breadth of Foramen Magnum ...	26	31	30·5	31	29·8	30·3	31·8	29	27	28·8	26·2	22·3	—	—	—	—
Capacity...	1065	1200	1120	1154	1135	1130	1112·6	1072·5	*d*	1299·3	—	—	1102	1072	1244	1128
Length-breadth index	77·8	80·3	77·2	76·6	77·5	76·3	75·9	75·6	78·9	80·9	80·6	81·2	74·4	77·9	81·1	82·8
Gnathic index	90·3	91·8	94·8	102·3	93·2	98·8	90·9	96·6	97·7	95·8	101·5	112·8	108·7	104·3	101·1	101·1
Orbital index	80·0	78·3	92·3	79·4	86·8	*d*	88·1	80·9	89·1	86·3	87·9	84·2	82·9	82·9	90·4	91·4
Nasal index	47·7	52·2	50·8	48·8	50·2	52·0	42·7	49	51·1	45·3	52·9	58·3	63·4	55·3	mesorrhine	
Weight in grams	435·2	308·6	371·3	400·9	458·7	383·75	389·1	416·4	392·2	—	—	—	—	—	—	—

uniformity among the third dynasty crania. The orbits of four of the small specimens are microsemic, those of two small specimens, and the specimen of normal size, are mesosemic; and, of the remaining small specimens, two have megasemic orbits, while the last is defective. Both the Dakka and Hesa specimens have mesosemic, while the Akkas have both microsemic, and the Andamanese have megasemic orbits.

The nasal indices indicate no platyrrhiny in the third dynasty specimens under consideration. Two small specimens and the one of normal size are leptorrhine while the remainder are mesorrhine. The Hesa specimen is mesorrhine, and the Dakka specimen is markedly platyrrhine. Both Akkas also are platyrrhine while the Andamanese are stated to be mesorrhine. There is however in two of the small third dynasty specimens (E 21 and E 579) a marked rounding of the lower margin of the piriform aperture, which gives a somewhat negroid appearance, though for the most part the nasal bones are well formed, and somewhat prominent, and the root of the nose is not markedly broad.

The actual weight of the crania is also not without interest, and at the bottom of the table of measurements will be seen the weights of the small specimens of the third dynasty. In comparison with other crania, however, the weight of individual examples is of small value by reason of variations in preservation and defects; but the mean of the figures given above is 395·13, and this may be compared with the figure 491·15, which is the mean obtained from the weights of 36 adult female crania of normal size, of the same series, and showing the same general condition as regards preservation.

Thus, though closely resembling the Akkas in point of size, the characteristic features of the Negro races are not present in a marked degree in any of these small specimens, except in Dr Derry's specimen from Dakka. This also is the only specimen with which any other portion of the skeleton has been preserved, and, in this case there is marked correspondence between the size of the cranium and that of the pelvis. From what is known of the relation between size of cranium and stature, it is, however, not idle to suppose that the occurrence of adult individuals of small stature was fairly common in Egypt at the period of the third dynasty, although from the evidence afforded by these specimens it is not possible to assert that all such pygmies were of "Negrillo" race. The several forms of dwarfism were quite familiar to the Egyptians, and the possibility that these are cases of dwarfism of the "infantile type" is worthy of consideration. At the same time the fairly perfect character of the teeth rather favours ethnic dwarfism.

NOTES ON THE PIGMENTATION OF THE HUMAN IRIS.

By A. RUDOLF GALLOWAY, M.B., C.M., M.A., Aberdeen.

CONTENTS.

In the *Proceedings of the Royal Society* (B), Vol. 80, 1908, p. 85, C. C. Hurst has published a paper "On the Inheritance of Eye-Colour in Man." He therein states that there is distinct discontinuity between:

(1) "The eyes in which two kinds of pigments are present; the one yellow-brown in colour, deposited on the outer or anterior surface of the iris; the other, blue-black in colour, deposited on the inner or posterior surface of the iris." Such eyes he calls duplex.

(2) "The eyes in which the posterior pigment alone is present in the iris, the anterior pigment being absent." Such eyes he calls simplex.

At page 93 he states that "(1) simplex parents mated together give all simplex offspring; (2) duplex parents mated together give either (*a*) all duplex offspring, or (*b*) duplex and simplex offspring in the proportion of about 3 : 1; (3) duplex parents mated with simplex parents give either (*a*) all duplex offspring, or (*b*) duplex and simplex offspring in the proportion of about 1 : 1."

He adds "It is evident therefore that the simplex type, in heredity, behaves as a Mendelian recessive to the duplex type, which is dominant." "The unit characters concerned in the heredity of the duplex and simplex types of eyes are therefore presence (duplex) and absence (simplex) of anterior pigment in the iris, presence being dominant over absence, which is recessive."

From the above statements it follows:

1. That in studying the pigmentation of the human iris, the two surfaces—anterior and posterior—alone are of importance; the stroma which is about ten times as thick as the posterior layer of retinal pigment epithelium is negligible.

2. That two kinds of pigments may be present in the human iris, the one yellow-brown in colour located on the anterior surface, the other blue-black in colour located on the posterior surface.

3. That Mendelian laws regulate the inheritance of these types.

Since the publication of the paper cited, I have, as occasion allowed, in the out-patient department of the Eye Institution in Aberdeen, where the annual attendance is over 20,000, taken notes of eye-colours and pedigrees which might throw any light on this question. My first difficulty was in finding any eye in which I could state, after careful examination in daylight and by focal illumination with magnification, that there was no anterior pigmentation. This difficulty still continues, and any such case noted in this paper must be taken to mean that no anterior pigmentation was seen at the examination. Several times I have thought that I had discovered a pure blue, or pure grey, or pure blue and grey parentage with no anterior pigmentation, but on careful examination traces of pigment were found in one or other parent.

My next difficulty lay in the immense number of different types of pigmentation. Only very few (each covering many grades) can be given in this paper, and these refer solely to the pedigrees of the families mentioned. This multiplicity of types probably accounts for the following differences from Hurst's observations as noted at pp. 86, 87 of his paper.

1. "Self-coloured duplex" eyes are often "spotted."

2. "Ringed duplex" eyes are often "spotted."

3. The pigment ring of "ringed duplex" eyes is often outside the pupillary zone, occupying an irregular middle zone, and also sometimes, in spots, a peripheral zone of the iris.

4. "Self-coloured duplex" eyes occur with no denser pigment ring round the pupil.

5. Eyes with grades of anterior pigmentation so low that they can be mistaken for "simplices" do occur.

6. "The lighter shades of blue, and coarser greys" are not necessarily "due to the greater density of age" (Hurst, p. 88). In the B family mentioned in this paper, II. 2, aged 9 years, has as densely grey eyes as her mother.

With regard to the first deduction from Hurst's observations concerning the two surfaces of the iris, it is necessary to mention that the two layers of importance developmentally and anatomically are (1) the stroma, consisting mostly of connective tissue, about ten times as thick as (2) the posterior retinal pigment epithelium. The latter is epiblastic, and the former mesoblastic in origin.

For facility of description, the area of the iris may be divided into three zones:

1. Pupillary zone—from the pupil to the lesser circle.

2. Median zone—from the lesser circle to the beginning of the outer or folded part of the ciliary zone (where there is sometimes a greater circle).

3. Peripheral zone, consisting of the outer or folded portion of the ciliary zone, and the periphery of the iris as far as visible.

As the other deductions from Hurst's observations with regard to the two different kinds of pigments, and their Mendelian inheritance, could not be proved or disproved without examining microscopically an iris, showing both, I took occasion to do so in the case of G. C., aged 70, whose right eye I removed on March 30th, 1910, for sarcoma of the choroid in the lower and inner equatorial region. The tumour had not advanced far enough to affect the globe beyond the site of the growth, there being no pain or inflammation, and the sight being only partially obscured. The iris was normal and healthy, with free movement, and was coloured in exactly the same way as that of the left sound eye of the patient. The colour note of the eyes made on March 22nd, 1910, was:

"At a short distance distinctly blue; narrow yellow-brown pupillary zones; median and peripheral zones blue."

"The *right* eye has nine dark brown spots of the apparent size of ½ to 1½ mm. dotted over all three zones, one distinct dark one (up and in) and three more hazy ones occur on the yellow-brown pupillary ring. There is also an irregular peripheral ring, consisting of separate grey spots free of anterior pigment; the median and peripheral zones generally show reticulations well except where the dark brown pigment spots occur.

The *left* eye has seven similar dark brown pigment spots varying from ¼ to 2 mm. in apparent size."

The presence of a yellow-brown pigment ring, of dark brown pigment spots, and of blue median and peripheral zones, apparently mostly free from anterior pigment, promised valuable information. The eye, on removal, was taken to the Pathological Department of Aberdeen University, where Professor Dean prepared a large number of serial sections. The globe was divided into an upper and lower half, the lower containing the tumour. The upper half was used for the sections, which were cut antero-posteriorly from near the horizontal meridian.

Four figures will suffice to demonstrate the important appearances which may be followed all through the series of sections.

Fig. I. (× Zeiss A. unstained.)

Low power unilateral view of a section through the pupil, showing the dark brown posterior pigment epithelium, which increases in thickness near the pupil (behind the pupillary zone); in this region also, the *stroma* is seen to be thickly strewn with pigment cells, many rounded and full of pigment granules; towards the anterior surface these rounded pigment cells are less numerous, some appearing to have ruptured, while at the anterior surface, the pigment consists almost entirely of yellow granules, the cells apparently having distributed their pigment in a granular form, thus causing the pupillary ring. The anterior surface of this pupillary region is seen to be distinctly yellow in colour. In the median and peripheral regions of the iris, beyond the pupillary area, very few pigment cells are seen although in no part are they entirely absent; the anterior surface of these median and peripheral parts, which *to external examination before enucleation appeared blue and free from anterior pigment*, is yellowish and shows the presence of pigment granules, although more faintly than in the pupillary region. It will be noted also that near the pupillary margin, the dark brown posterior pigment epithelium passes suddenly and abruptly into the yellow colour of the anterior pupillary ring, also that the stroma of the pupillary margin has fewer pigment cells than the stroma behind the middle of the pupillary pigment ring. Towards the left limit of the field a dark brown anterior pigment spot is seen, as if some of the pigment cells had apparently passed through the stroma and located themselves on the anterior surface. In the stroma behind this spot, one or two disintegrated pigment cells appear with granular pigment around.

Fig. II. (× Zeiss A. unstained.)

Horizontal antero-posterior section through the pupillary margin.

At the mid-point of the section corresponding to the pupillary margin few stroma pigment cells are seen, while on each side in the parts corresponding to the stroma behind the pupillary pigment ring they are numerous. The yellow anterior surface is most distinct opposite the parts of the stroma which have most pigment cells.

Fig. III. (× Zeiss D. unstained.)

Horizontal antero-posterior section above the level of the pupil through the yellow-brown pupillary zone.

Note the dark pigment epithelium, and the stroma thickly strewn with pigment cells, which tend to disintegrate towards the yellow anterior surface, where pigment mostly in the granular form is seen.

Fig. IV. (× Zeiss D. logwood stain.)

Section through iris, showing the anterior pigment spot seen in Fig. I.

The spot is seen to be due to the presence of dark brown pigment cells of a similar nature to the stroma pigment cells, several of which, whole or disintegrated, appear in the field: the logwood staining shows the structure of the iris tissues.

Professor Dean suggests that these stroma pigment cells are possibly connective tissue cells that have the power of movement. These sections appear to show that they tend to pass from the pigment epithelium layer forward through the stroma, to distribute pigment either in the shape of pigment cells or granules to the anterior surface of the iris. Whether this view be correct or not the sections lend no corroboration to the idea that there are two kinds of pigment, a blue-black behind, and a yellow-brown in front. All the appearances are compatible with the belief that only one pigment is present, which when heaped up in the form of pigment cells, is dark brown in colour, and is yellow when thinly distributed especially in a granular form. Moreover the dark brown pigment cells of the anterior spot are exactly similar in colour to the retinal pigment epithelium of the posterior surface.

It appears probable from the presence of yellow anterior granular pigment in all sections of the median and peripheral zones which were blue and apparently free from pigment, that many so-called pure blue eyes may have this anterior coating of granular pigment, which is revealed only microscopically*. Hence, in addition to the errors concerning two kinds of pigment, etc., another error in the classification of some blue eyes as having no anterior pigment would arise.

Therefore any theory, Mendelian or otherwise, which depends on the supposition that there are two kinds of pigments, with different loci, and that there is no stroma in the human iris, seems based on inadequate examination of the human eye.

Hurst's results so far as they go give approximately the anticipated Mendelian proportions, but as I have endeavoured to indicate in this paper the amount of anterior pigment is a continuous quantity, and nothing short of microscopic post-mortem examination will determine to what extent an eye does or does not possess it. Hurst suggests that half the "clear" or pure blue eyes classified by ordinary observers really possess anterior pigment and that this is the reason why the data collected by Galton do not support the Mendelian theory. If, as I anticipate, many of Hurst's "simplex" irides would show some anterior pigment on microscopic examination, it is evident that he has been singularly fortunate in discovering that the limit of his personal method of examination was that which coincides numerically with Mendelian theory. This coincidence is the more

---

[* In the case of the eyes of "clinically complete albinos" both in man and in lower forms of life a post-mortem microscopic examination will frequently show some posterior pigmentation, and occasionally some slight anterior pigmentation as well. On p. 85 of his paper Hurst states that in the albino pigment is entirely absent from both surfaces of the iris. EDITOR.]

remarkable in that he quotes the investigations of Davenport as leading to Mendelism in human eye colour, and thus as confirming his own work. Now Davenport collected his data in precisely the same way as Galton did, i.e. by aid of schedules with verbal definitions of eye colour and categories practically identical with Galton's scheme, and therefore, according to Hurst, leading to a wrongful classification of 50 °/₀ of the "clear" or pure blue eyes, i.e. eyes without anterior pigment. Yet Davenport—precisely like Hurst—finds his Mendelian quarter. It is thus fairly obvious that if both these authors are right and that if their papers actually confirm each other it must be practically of no importance where the limit is drawn between a "simplex" and a "duplex" eye,—for two different methods of observation which ought to lead to widely different numerical results are appealed to by both observers as confirming each the other's work!

During my search for pure blue parentages, I made the following note on February 22nd, 1910, "Very many blue eyes have a slight yellowish staining, sometimes arranged as a pupillary ring, at other times as a faint wash here and there preferably following the lines of the reticulations. This staining is usually visible only in bright daylight with focal illumination, and not in gaslight." To this, I have now to add the observation that if, after deciding that a blue eye has no anterior pigment, such an eye be rotated upwards, so that an oblique view of the iris surface with foreshortened pupil is obtained, a yellow sheen of anterior pigmentation will often be seen.

These observations account for the fact that I have not, as yet, met with a family from a pure blue or grey, or blue and grey parentage of sufficient size to be of any value to record. Even although many such pure blue or grey-eyed families were obtained, however, I question whether this would indicate any Mendelian inheritance with regard to iris pigmentation: the nearest approach I have been able to make is represented by the following pedigrees, which show some interesting points in inheritance of iris colour. On looking over the notes made when the various individuals of these and other families were examined, I am impressed with the frequency with which presence of the yellow staining of blue and grey eyes is mentioned as occurring in the *upper* parts of the irides. In light brown irides also, the upper parts of the irides are often darker brown (*vide* F. family, II. 3). I would suggest as an explanation of this interesting fact that the upper lid prevents the pigment from being bleached as quickly as in other unprotected parts. This would agree with the microscopic appearances of the sections given in this paper, which show the gradual disintegration of the pigment cells towards the anterior part of the iris.

It is probable that the greatest incidence of light takes place at the most prominent or convex part of the iris surface, at a little distance from the slightly funnel-shaped pupil. Hence the greatest pigment activity is seen there in the sections, and hence the frequency of the ringed pupillary type of pigmentation in light eyes, and the tendency to have darker pupillary rings in dark eyes.

Fig. 1.

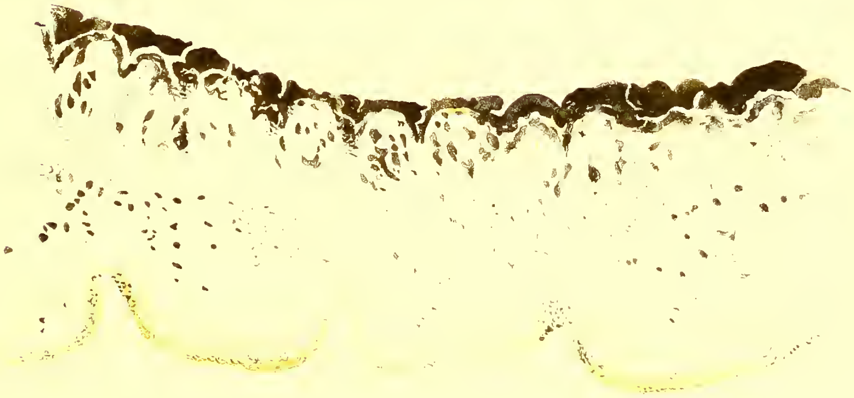


Fig. 2.

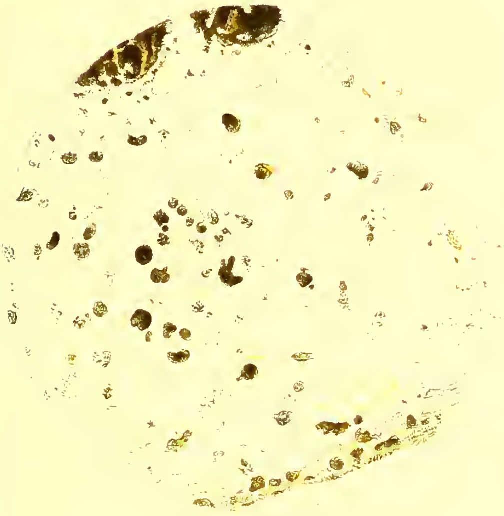


Fig. 3

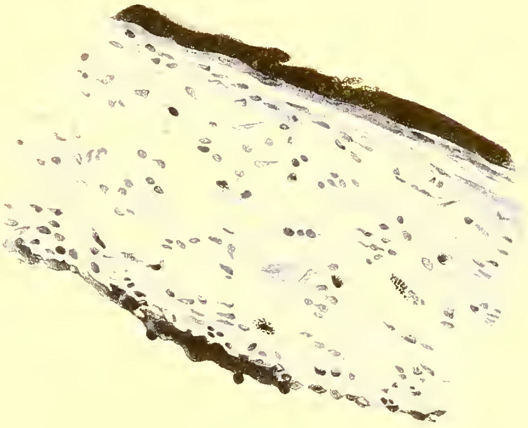


Fig. 4.

Wm Smith Jr del

E Wilson, Cambridge

SECTIONS OF THE EYE OF G. C. ILLUSTRATING A. RUDOLF GALLOWAY'S MEMOIR.

REMARKS ON THE PEDIGREES OF IRIS PIGMENTATION.

A. Family:

This pedigree was constructed prior to April 1910, mainly on account of the large family of sixteen—from two blue-eyed parents II. 3, and II. 4. At that time, following Hurst's classification of simplex and duplex irides, I noted as having no anterior pigment those eyes which appeared to me to have none with focal illumination in good daylight. I have since found that many of these show the yellowish sheen of anterior pigmentation when rotated strongly upwards so that a foreshortened view of the pupil is obtained. This test was not applied either to the parents or children of this interesting family. It will be noted, however, that the mother II. 4 had a faint buff tint on the blue of the upper parts of each iris.

It would serve no useful purpose, therefore, to look for the manifestation of Mendelian phenomena in this family, although the proportion of 14 blues without visible anterior pigment to 2 with tells against this mode of inheritance. I feel that the classification of eyes into "duplex" and "simplex" according to the pigmentation of the iris, is an impossible one, and must lead to inevitable confusion.

It is interesting to note the inheritance, in the daughter III. 9, of the mother's brown spot of anterior pigment, and in the daughter III. 15, as a pupillary ring, of the mother's buff tinting*.

In the eyes of I. 3, faint buff tinting was at first noted, and on a second examination deleted. This yellowish sheen would probably appear on upward rotation in good light in this case also. Three of her sisters, I. 4, 5, 6, had anterior pigmentation well marked.

The mating of III. 3, a pure blue, with III. 4, a uniform rich dark self-brown, is interesting, and has resulted in no iris that is, or can ever become, self-brown. Three of the children have well marked pupillary rings when the parents had none, and IV. 6 is almost an albino. In the mating of II. 5 with II. 6, unfortunately the husband could not be seen, but again from dark brown eyes and almost pure blues there have resulted three with pupillary rings.

F. Family:

The chief points of interest in this pedigree—the mating of two blues which would be classified as pure blues with no anterior pigment by any non-professional examiner—are:

1. The occurrence of a son, II. 3, with brown irides with a complete coating of anterior pigment.

---

* For examination and report on III. 15 I am indebted to Major F. M. Mangin, R.A.M.C., Specialist in Ophthalmology, Aldershot Command.

2. The buff or yellowish tinting being confined to the upper parts of the irides in two members (I. 2 and II. 1), and pigment being rather more dense in the same regions in II. 3.

3. The appearance of pupillary circles in II. 4, while they are absent in the parents.

B. Family:

This pedigree is interesting:

1. From the mating of blue, almost free from visible anterior pigment, with very rare and peculiar dense grey, almost white, irides free from visible anterior pigment.

2. From the occurrence of the same peculiar white or light grey irides in three of the children, viz. II. 1, 2, 3.

3. From the appearance of pigment circles in four of the five children, the parents being free from them.

4. From the buff staining occurring in the upper parts of the irides of I. 1.

5. From the blue peripheral zones of II. 2. This appearance may partly be due to the fact that in this region the iris is thinner than elsewhere. In a clear yellow yellow-ammer—a *clinically* complete albino with pink eyes—which I have alive at present, this same grey colour occurs as a pupillary ring, where the iris is thickest, the median and peripheral zones being translucent and showing the red fundus reflex through them*.

6. From the absence of photophobia and other signs of albinism.

C. Family:

1. The eyes of I. 1, to any ordinary observer, are blue with no visible anterior pigment.

2. Mated with a blue with no visible anterior pigment, there appears the heterochromic iris of II. 1, showing a large deposit of anterior pigment.

3. Neither of the parents is ringed, but two of the children are, viz. II. 2 and 3.

D. Family:

1. The red pupillary ring of I. 1 corresponds, as is frequently the case, with the original reddish colour of the hair.

2. Red hair is more subject to change of colour, and is more closely connected with albinism than other colours.

3. The two children are verging on albinism and show variation in refraction, II. 1 being hypermetropic, and II. 2 myopic.

---

* This bird (*Emberiza citrinella* ♀) is now dead, and Dr Usher, who kindly examined the eyes, reports them to be entirely free of pigment, except in the posterior pigment epithelium of the irides, which is very lightly pigmented: see "Canary Breeding," *Biometrika*, Vol. VII. Plate II.

4. The pupillary rings of I. 1 are repeated in both the children.

5. Yellow staining on the blue irides of I. 2 occurs only in the upper and inner regions.

E. Family:

1. The blue eyes and reddish hair of I. 2 are probably connected with the appearance of an incomplete albino II. 2.

2. The examination of this patient II. 2, as detailed, combined with the microscopic examination of G. C.'s eye, has convinced me that it is quite impossible to say when any living person's blue eye is free from anterior iris pigment, and has led me to doubt that this ever truly occurs.

CONCLUSIONS.

1. The human iris cannot be held to consist merely of an anterior and a posterior surface, each, in certain eyes, exhibiting its own kind of pigment.

2. The stroma may contain pigment cells more especially in its deeper parts and granular pigment towards the anterior surface.

3. Pure blue, or so-called "simplex" eyes, which to any outward examination appear free from anterior pigmentation, may show stroma pigment cells, and anterior granular pigmentation when examined microscopically.

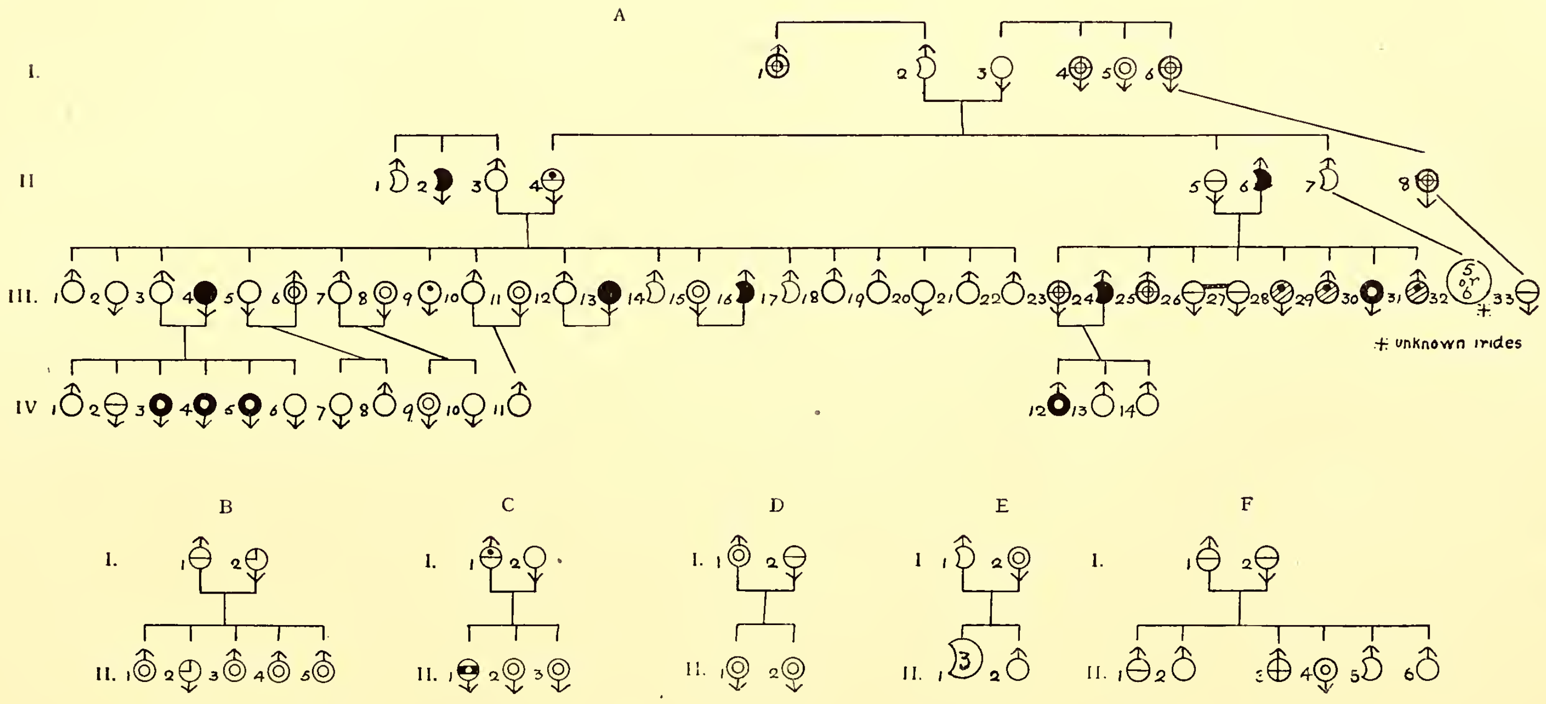
4. Hence the classification of "duplex" and "simplex" eyes is erroneous, and there is no discontinuity or Mendelian inheritance between them.

PEDIGREES OF IRIS PIGMENTATION.

All eyes that have been seen have been examined by myself, except III. 15, Fig. A.

The symbols used have been devised to indicate the main types of iris pigmentation occurring in the following pedigrees, but, as already mentioned, these varieties are so numerous that several grades of colouration must be included under some of the symbols.

The aim of these pedigrees is to indicate how complex is the problem of heredity of eye colour, when we pass from a mere rough appreciation of the extent of pigmentation, as provided by the ordinary verbal descriptions, to an analysis of its distribution throughout the iris.



○ Blue irides.

Ɔ Blue eyes, not examined.

⊖ Slightly buff tinted, blue irides.

◎ Blue or grey irides with various coloured circles.

⊖ Slightly buff tinted blue irides with one or more brown anterior pigment spots.

⊙ Blue irides with one or more brown anterior pigment spots.

● Dark self-brown irides.

◗ Brown eyes, not examined.

◉ Brown irides, with darker-brown pupillary circles.

▨ Brown pigment spotting on lighter brown irides.

⊕ Brown or greenish brown irides with pupillary circles, some also with pigment spots.

⦶ Greenish blue irides with yellow rayed circles mostly in outer pupillary and middle zones.

⊝ Heterochromic blue and brown.

◷ Grey irides.

⊕ Light brown irides.

APPENDIX, PEDIGREES.

Fig. A. (April, 1910.) I. 1, male, aet. 77. Poorhouse. *Hair*, black. *Eyes*, Right—brown with darker pupillary circles, three or four $\frac{1}{4}$ mm. darker brown spots in middle and peripheral zones; Left—greenish-brown with darker brown pupillary circles, four or five $\frac{1}{4}$ mm. brown spots in middle and peripheral zones. I. 2, male, seaman, aet. 55 at death. *Hair*, dark. *Eyes*, pale blue. I. 3, female, aet. 80. *Hair*, light brown. *Eyes*, pale blue, no pigment seen. I. 4, female, aet. 76. *Hair*, brown. *Eyes*, greenish-brown, large yellow-brown rayed pupillary circles, occupying pupillary and middle zones. Peripheral zone, bluish-green. I. 5, female, aet. 72. *Hair*, brown. *Eyes*, pale blue with small buff pupillary circles. I. 6, female, aet. 67. *Hair*, black. *Eyes*, green-brown. Brown pupillary circles, middle and peripheral zones, green. Right has one small ($< \frac{1}{4}$ mm.) brown tick inner peripheral zone. Left has one small ($< \frac{1}{4}$ mm.) brown tick outer middle zone. II. 8, female, aet. 34. *Hair*, dark brown. *Eyes*, green-brown. Brown pupillary circles: middle and peripheral zones, green. Right has 3 or 4 small brown dots inner middle zone. Left ditto outer peripheral. III. 33, female, aet. 7 (daughter of II. 8). *Hair*, light brown. *Eyes*, blue; buff mottling arranged in irregular pupillary and peripheral circles. II. 1, male, dead. Blue eyes. II. 2, female, dead. Brown eyes. II. 3, male, aet. 58. *Hair*, blond. *Eyes*, pale blue, no anterior pigment; reticulations well marked. II. 4, female, aet. 57. *Hair*, brown. *Eyes*, blue, each with faint buff tint especially upper part of left iris, which has also one $1\frac{1}{2}$ mm. brown pigment spot in lower pupillary zone. II. 5, female, aet. 55. *Hair*, brown. *Eyes*, pale blue with faint buff mottling on pupillary and peripheral zones. II. 6, male, aet. 51, trawler. *Hair* and *eyes*, dark brown. II. 7, male. *Eyes*, blue. III. 1, male, single, aet. 40, coachman. *Hair*, light brown—lighter on left side when young. *Eyes*, blue, no anterior pigment. III. 2, female, aet. 38. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 3, male, aet. 36. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 4, female, aet. 34. *Hair*, used to be auburn now brown. *Eyes*, rich dark self-brown, uniform colour. III. 5, female, aet. 34. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 6, male, aet. 32. *Hair*, dark brown. *Eyes*, greenish-blue with yellow rayed circles mostly in outer pupillary and middle zones (one in each eye). III. 7, male, aet. 32. *Hair*, black. *Moustache*, light brown. *Eyes*, pale blue, no anterior pigment. III. 8, female, aet. 29. *Hair*, dark brown. *Eyes*, pale blue with pupillary circles of buff. III. 9, female, single, aet. 30. *Hair*, dark brown. *Eyes*, blue—right has one $\frac{1}{2}$ mm. brown spot upper pupillary zone. III. 10, male, aet. 28. *Hair*, brown. *Eyes*, blue with no anterior pigment. III. 11, female, aet. ?. *Hair*, brown. *Eyes*, blue with buff pupillary circles. III. 12, male, aet. 27. *Hair*, light brown. *Eyes*, blue, no anterior pigment. III. 13, female. *Hair*, black. *Eyes*, dark self-brown. III. 14, male, single, aet. 25, in Vancouver. *Hair*, brown. *Eyes*, blue. III. 15, female. *Hair*, light brown. *Eyes*, pale blue with buff pupillary circles. III. 16, male, soldier, Aldershot. *Hair* and *eyes*, brown. III. 17, male, died, aet. 9 mos. *Eyes*, blue. III. 18, male, aet. $21\frac{1}{12}$. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 19, male, aet. 19. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 20, female, aet. 17. *Hair*, light brown. *Eyes*, blue, no anterior pigment. III. 21, male, aet. $11\frac{8}{12}$. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 22, male, aet. 10. *Hair*, brown. *Eyes*, blue, no anterior pigment. III. 23, female, aet. 32. *Hair*, black. *Eyes*, greenish-brown, large pupillary circles of yellow-brown, and dots of same elsewhere, both zones. III. 24, male, died, aet. 31. *Hair*, dark brown. *Eyes*, dark brown. III. 25, male, aet. 30. *Hair*, dark brown, used to be light brown. *Eyes*, greenish-brown—well-marked pupillary circles of reddish-brown with rays meeting similar spots in peripheral zones—middle and peripheral zones dull green. III. 26, female, aet. 25. *Hair*, light brown. *Eyes*, blue. Right has two or three minute ticks of yellow in peripheral zone down and out. Left very similar. Twin with III. 27. III. 27, female, aet. 25. Twin with III. 26. *Hair*, brown. *Eyes*, blue, both with very faint spots of buff staining. III. 28, female, aet. 23. *Hair*, dark brown. *Eyes*, self light brown. Right has one 1 mm. dark brown spot in peripheral zone down and out. Left ditto in pupillary zone upwards. III. 29, male, aet 20. *Hair*, once black has now a slight reddish tint. *Eyes*, light brown, uniform yellow-brown pigment layer, no rings. Left has one ½ mm. brown spot peripheral zone up and in. III. 30, female, aet. 16. *Hair*, brown. *Eyes*, light brown with darker pupillary circles. III. 31, male, aet. 14. *Hair*, dark brown. *Eyes*, light self-brown with one or two darker spots. IV. 1, male, aet. 12. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 2, female, aet. 10. *Hair*, light brown. *Eyes*, pale blue, very faint mottling of buff in sinuous lines. IV. 3, female, aet. 8. *Hair*, brown. *Eyes*, light brown with narrow pupillary circles of yellow-brown. IV. 4, female, aet. 5. *Hair*, light brown. *Eyes*, light brown, rather browner round pupils. IV. 5, female, aet. 3. *Hair*, light brown. *Eyes*, light brown, rather browner round pupils. IV. 6, female, aet. 2. *Hair*, light flaxen. Eyebrows and lashes the same—almost an albino. *Eyes*, pale blue, no anterior pigment. IV. 7, female, aet. 4½. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 8, male, aet. 9 mos. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 9, female, aet. 4. *Hair*, brown. *Eyes*, pale blue with indistinct buff pupillary circles and other buff dots here and there. IV. 10, female, aet. 2. *Hair*, brown. *Eyes*, blue, no anterior pigment. IV. 11, male, aet. 2. *Hair*, light brown. *Eyes*, blue, no anterior pigment. IV. 12, male, aet. 6. *Hair*, brown. *Eyes*, brown with darker pupillary circles. IV. 13, male, aet. 5. *Hair*, brown. *Eyes*, blue, no anterior pigment, a greyish white ring in peripheral zone. IV. 14, male, aet. 2. *Hair*, light brown. *Eyes*, blue, no anterior pigment.

Fig. B. (March, 1910.) I. 1, male, aet. 33. *Hair*, brown. *Eyes*, blue, very faint stain of buff on upper part of each iris. I. 2, female, aet. 31. *Hair*, brown, used to be lighter. *Eyes*, very pale grey, almost white, no visible anterior pigment. By magnification the greyish-white colour of the iris is seen to be due to closely packed sinuous fibrous lines. II. 1, male, aet. 11. *Hair*, brown. *Eyes*, pale grey like the mother's, but with pupillary circles of yellow-brown, and one or two other ticks of the same colour here and there. II. 2, female, aet. 9. *Hair*, light brown. *Eyes*, dense pale grey all over except peripheral zones of blue, no visible anterior pigment. The grey irides very like the mother's, and quite as grey, verging on white. II. 3, male, aet. 8. *Hair*, brown. *Eyes*, the same pale grey, almost white irides but with median circles of yellowish-brown. II. 4, male, aet. 6. *Hair*, light brown. *Eyes*, pale blue, with faint pupillary circles of buff. II. 5, male, aet. 2. *Hair*, light flaxen. *Eyes*, blue, pupillary circles of *grey*, and other lines and spots of same, no visible anterior pigment.

Fig. C. (Feb. 1911.) I. 1, male, aet. 43. *Hair*, medium brown. *Eyes*, blue with rather darker pupillary circles. Touched with one or two minute ticks of yellow-brown; one brown pigment spot in *Right* periphery down and out of ·25 mm. size, a smaller one up and out in median zone. I. 2, female, aet. 45. *Hair*, dark brown. *Eyes*, blue, no visible anterior pigment. II. 1, female, aet. 13. *Hair*, medium brown, rather reddish at tips. *Eyes*, blue. *Right* has a transverse heterochromic band formed by two distinctly red-brown patches of anterior pigment, one at each side of pupil, the outer one 4½ × 5½ mm., widest at periphery, the inner one 3 × 4 mm., widest also at periphery. II. 2, female, aet. 11. *Hair*, red. *Eyes*, light blue, median irregular circles of reddish-yellow anterior pigment. II. 3, female, aet. 9. *Hair*, pale brown with reddish-golden tints. *Eyes*, pale blue with faintest possible staining of pale yellow in pupillary zones.

Fig. D. (March, 1911.) I. 1, male, aet. 37. *Hair*, used to be very fair, gradually got darker, now dark brown, almost black. Eyebrows and moustache *light* reddish brown, they have not altered. *Eyes*, bluish-grey with pale green tint in median and peripheral zones. *Red* complete pupillary rings, with rays stretching across median and peripheral zones. *Refraction*, $\frac{6}{6}$ with manifest hypermetropia of ·5 D.: probable total hypermetropia of 1·5 D. I. 2, female, aet. 38. *Hair*, medium brown, used to be very *fair*. *Eyes*, blue, yellow staining in patches, one in each eye, at upper and inner parts only. Greyish hazy spots form irregular peripheral circles.

Refraction, hypermetropic astigmatism in each eye of about +2 D. cylinder, axis down and in. II. 1, female, aet. 8. *Hair*, yellowish flaxen. *Eyes*, pale blue, narrow yellow pupillary rings. *Refraction*, hypermetropia of 1 D. II. 2, female, aet. 5. *Hair*, pale yellowish-brown, a shade darker than II. 1. *Eyes*, pale blue, lighter than her sister's—faintest possible yellowish staining in irregular pupillary circles. *Refraction*, myopia −6 D.

Fig. E. (April, 1911.) I. 1, male. Unexamined. *Hair*, light brown, darker than that of I. 2. *Eyes*, blue, darker than those of I. 2. I. 2, female, aet. 35. *Hair*, light reddish-yellow, used to be lighter. *Eyes*, blue-grey with irregular pupillary rings of yellow-brown, and one or two small separate ticks of brown in median and peripheral zones of each eye. *Refraction*, +1 D. II. 1. Two older and one younger children (than no. 2) with considerably darker hair and eyes. *Eyes*, blue. II. 2, male, aet. 10. Incomplete albino. Brought by his mother (I. 2) on account of sensitive eyes—photophobic in bright light—and difficulty with lessons. *Hair*, almost white, white eyebrows, lashes a shade darker—pale yellowish. *Eyes*, light blue, no pigment visible anteriorly, and would certainly pass all ordinary tests for absence of anterior iris pigment ; but when the eyes are turned strongly upwards so that a foreshortened view of the pupils is obtained, a faint yellowish sheen appears over both irides, and this is more visible round the pupils than elsewhere. Both eyes have peripheral grey dots in irregular circles, no nystagmus, nor pink pupils. *Fundi*, distinctly albinotic, the choroidal interspaces being much paler than the choroidal vessels. *Refraction*, +2 D.

Fig. F. (April, 1910.) I. 1, male, aet. 58. *Hair*, very light flaxen—"white" in youth. *Eyes*, pale blue, very faint buff mottling here and there. He has four brothers and two sisters with blue eyes and one sister with dark eyes. I. 2, female, aet. 57. *Hair*, light brown. *Eyes*, slight buff or yellowish staining in upper part of each iris, seen only by focal illumination in bright daylight. II. 1, male, aet. 37. *Hair*, light brown. *Eyes*, blue, faint touch of buff upper parts of each iris. II. 2, male, aet. 35. *Hair*, dark brown. *Eyes*, pale blue, no visible anterior pigment. II. 3, male, aet. 33. *Hair*, brown. *Eyes*, light brown, left rather darker—diffuse brown anterior pigment—most marked at upper part of each iris, especially *left*. II. 4, female, aet. 31. *Hair*, light brown. *Eyes*, blue with pupillary circles of pale yellow-brown. II. 5, male, aet. 28. *Hair*, brown. *Eyes*, blue. Abroad, not examined. II. 6, male, aet. 20. *Hair*, brown. *Eyes*, blue, no visible anterior pigment.

THE INCREASE IN THE NUMBER OF ERYTHROCYTES WITH ALTITUDE.

By Captain HUGH W. ACTON, I.M.S., and Major W. F. HARVEY, I.M.S.,
Pasteur Institute, Kasauli.

THE subject has previously been investigated by numerous observers and their work has invariably shown that there is a definite increase in the number of erythrocytes in the peripheral blood with a change of altitude. Most of the investigations have however, been conducted either on animals alone, or on a small number of human beings. We on our part have carried out these investigations on biometric lines in order to determine the undermentioned points:—

(1) The normal variations in the number of erythrocytes in apparently healthy individuals.

(2) The effect that a high altitude has on this number.

(3) The effect, if any, that a change of altitude has on anaemic individuals.

The opportunities that were afforded to us to investigate these points were unique in many ways, as we had access to a considerable number of more or less healthy individuals (Natives of India) and furthermore they remained on the spot some 18—20 days whilst undergoing anti-rabic treatment (Högyes dilution method).

This treatment may possibly affect the leucocyte content of the blood but there is no evidence to show that it has any influence on the erythrocyte content, and therefore should not lead to any alteration in our findings on that account.

Kasauli is a hill station in the Punjab in India situated in the Sub-Himalayan range, Lat. 30° 58′ 4″, Long. 77° 2′ 3″ and 6335 feet (about 1920 metres) above sea-level. The patients examined numbered 127 all told; they were taken in the order they came for treatment and were all adult men. Cases severely bitten and those suffering from inflammatory conditions, as the result of the bite, were excluded. We may therefore affirm that the men represented a varied sample of the population. Our patients came from all parts of Burmah, Assam, Central

India and Upper India. If any selection was brought into play, it would be to this extent that more Indians of the lower classes were examined than those of the upper. Amongst our cases some had recently suffered from malarial fever and others probably were suffering from helminthic diseases, if we take eosinophilia to be a characteristic of the latter. Thirty-five were judged to be suffering from anaemia as indicated by the pallor of the mucous membranes; this point was always noted before any blood counts were taken; unfortunately it was not possible at the time to make a haemoglobin estimation as well. Out of the total of 127 individuals 16 came from an altitude of 3000—8000 feet and showed as might have been expected a higher count than the majority who came from the plains of India.

Before giving the results of our analysis, it is necessary to give a brief account of the conditions that may possibly influence an examination of erythrocytes, as well as some of the tests that were applied by us, in order to show how these faults could be excluded.

I. Expansions of glass with diminished atmospheric pressure. As Bürker* has conclusively shown that there is no appreciable effect produced by the expansion of the glass in a Thoma-Zeiss instrument as the result of the decreased atmospheric pressure which occurs at altitudes, this point does not concern us.

II. Errors of technique. When commencing this investigation a number of trial counts were first made on ourselves and the laboratory attendants in order to acquire a greater technical skill and the following points were carefully attended to in order to avoid obtaining false results:—

(*a*) The diluted blood was thoroughly mixed both at the time it was drawn and before blowing it out.

(*b*) The glass cover slip was applied immediately before the corpuscles had time to settle down. Even a few seconds' delay has been shown to materially affect the count*.

(*c*) An exact measurement of the drop was made so that when the cover-slip was applied the fluid extended right to the periphery and none ran over into the trough. No slide was examined unless Newtonian rings were seen.

The blood was diluted 1—200 times with Toisson's fluid and 50 squares of a Thoma-Zeiss slide were counted, which meant that some 350—500 erythrocytes were counted in each individual case.

III. Variability of observations:—

(*a*) To test the degree of error of one's own observations, ten counts were carried out on two consecutive days on the blood of one of our laboratory attendants (see Fig. 1 and Table I).

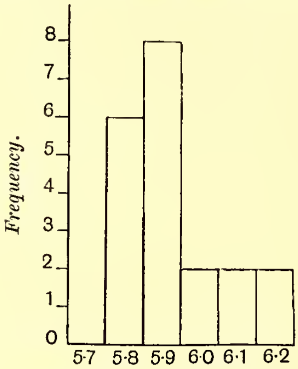
---

* R. Bürker, "Die Wirkungen des Höhenklimas auf das Blut," *Münchener medizinische Wochenschrift*, Jahrgang LII., p. 249, 1905, Feb. 7th.

This table shows that the mean or average of these twenty observations was 5,930,000, with a probable error of ± 19,000 and a standard deviation of 127,000.

The probable error of the mean is small compared with the mean, showing that the error of technique was also small. The counts were registered only to the nearest 100,000.

FIG. 1. Results of 20 Observations on one Individual.



Erythrocytes in Millions.

(*b*) If a theoretical expression of the probable distribution of erythrocytes per unit of volume can be obtained this would form, when compared with the actual state of affairs, a valuable test of observational and technical accuracy. We have utilized for our purpose a computation given in an article written under the name of "Student" in *Biometrika* (Vol. v. p. 351). The computation is expressed in the formula

$$e^{-m}\left\{1 + m + \frac{m^2}{2\,!} + \ldots \frac{m^r}{r\,!} + \ldots\right\}^*,$$

this being a very good approximation to the point binomial which would still more closely describe the distribution.

Table I gives the results of a count of 400 squares of a Thoma-Zeiss haemocytometer. The blood in this case was for convenience diluted 500 times.

TABLE I.

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Erythrocytes per square
30	72	105	104	52	27	8	1	1	Number of squares, observed
33	82	103	86	52	26	11	3	2	Number of squares, calculated

It seems evident from the above table that the observed and calculated figures do not differ greatly. And when we apply the necessary statistical tests (*vide*

---

* m = mean of frequency distribution.

table for testing the goodness of fit of theory to observations, Elderton, *Biometrika* Vol. I. p. 155) we find that $\lambda^2 = 7{\cdot}987$ and $P = {\cdot}43$, which means that the two distributions cannot be considered significantly different (Fig. 2 shows graphically the two distributions). From the foregoing we may assume that our obtained results conform more or less to theoretical requirements.

We will now leave the subject of errors of technique and random sampling and proceed to consider the observations themselves. If an analysis is made of the total first counts (Table II and Fig. 3), it will be seen that the distribution of our cases is a long drawn out one starting at 2,700,000 erythrocytes per c.mm. and

FIG. 2. **Firm Line.** Actual Observations.

Broken Line. Calculated Values.

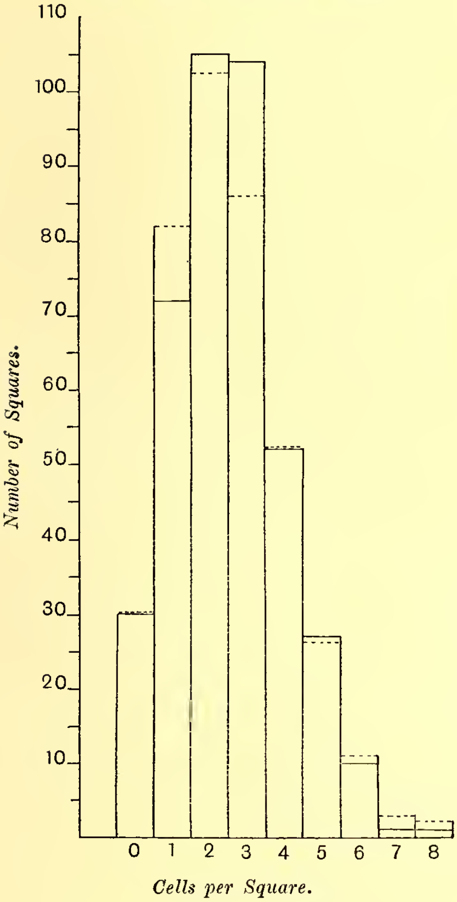


FIG. 3. Total First Counts.

extending to 8,500,000 per c.mm. The mean of the distribution is 5,300,000 erythrocytes per c.mm. with a probable error of ± 67,000 and a standard deviation of 1,127,000. These counts include all individuals, whether suffering from anaemia, enlarged spleens or helminthic diseases and also persons coming from high altitudes as well as those in whom none of these characters are present. They were in fact a random sample of the Pasteur Institute population. In every case, in order to avoid being influenced by the count itself, these points (anaemia, etc.) were carefully noted beforehand and recorded.

TABLE II.

Showing the Frequency of the Total First Counts.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
27	1	*59*	2
28	0	*60*	5
29	1	*61*	4
30	2	*62*	1
31	0	*63*	4
32	2	*64*	3
33	1	*65*	2
34	2	*66*	2
35	1	*67*	3
36	0	*68*	1
37	1	*69*	2
38	3	*70*	1
39	3	*71*	1
40	2	*72*	0
41	4	*73*	1
42	2	*74*	0
43	1	*75*	1
44	0	*76*	1
45	2	*77*	1
46	6	*78*	0
47	0	*79*	0
48	4	*80*	0
49	3	*81*	0
50	6	*82*	0
51	6	*83*	0
52	8	*84*	1
53	4	*85*	1
54	12		
55	3	Total	130
56	2		
57	6	Mean	53 ± ·67
58	5		
		S. D.	11·3

Now if we eliminate from the total the persons suffering from anaemia and enlarged spleens and those who have come from high altitudes, we are left with the distribution shown in Table III and Fig. 4. The total number now consists of only 75 individuals.

The mean is now increased to 5,620,000 erythrocytes per c.mm. and the increase is chiefly due to the removal of a rather large number of anaemic persons contained in our original total. The probable error is now only ± 43,000 and the standard deviation only 556,000, that is to say the exclusion of the above individuals causes a considerable decrease in the range and variability of our distribution. *The above figures so contracted probably represent fairly accurately the mean and variability for an apparently healthy adult Indian male population living in the plains of India.* The effect of altitude on the number of erythrocytes is well shown in Table IV and Fig. 5 (*a*) and (*b*). These exhibit the frequency distribution of 100 first counts and the corresponding second counts. It was not necessary in this investigation to consider the exclusion of abnormal individuals (anaemics, etc). The second count was made at an interval of 18 days after the first, that is to say after an 18 days' residence at Kasauli. The counts were limited to cases free from any development of malarial fever during their course of anti-rabic treatment. The means of the first and second counts are respectively 5,328,000 ± 69,600 and 6,492,000 ± 66,500. The difference between the means is 1,164,000, and the probable error of the difference = ± 65,000, from which we may conclude that these two means are significantly different from one another, and that the second count represents an increase on the first. We might have applied our tests to the distributions themselves instead of to their means.

TABLE III.

Showing the Frequency Distribution of the Total First Counts with the omission of Persons suffering from Anaemia, or Enlarged Spleens, and of Persons from High Altitudes.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
42	1	61	4
43	0	62	1
44	0	63	3
45	0	64	1
46	1	65	0
47	0	66	0
48	1	67	2
49	2	68	0
50	5	69	2
51	6	70	0
52	7	71	0
53	4	72	0
54	11	73	1
55	3		
56	2	Total	75
57	6		
58	5	Mean	56 ± ·43
59	2		
60	5	S. D.	5·56

It is of further interest to determine the degree of relationship between the first and second counts and from this obtain an abbreviated expression giving the mean value of a second count as determined from the first for a given altitude. This we believe should have besides a purely scientific interest, one which is of therapeutic value; for it may enable us to determine how far the increase of erythrocytes is likely to occur in anaemic individuals, and furthermore whether the consequent improvement in their condition is or is not counterbalanced by the additional requirements and strain of high altitudes.

FIG. 4. Total First Counts with the omission of Persons suffering from Anaemia, or Enlarged Spleens, and Persons from High Altitudes.

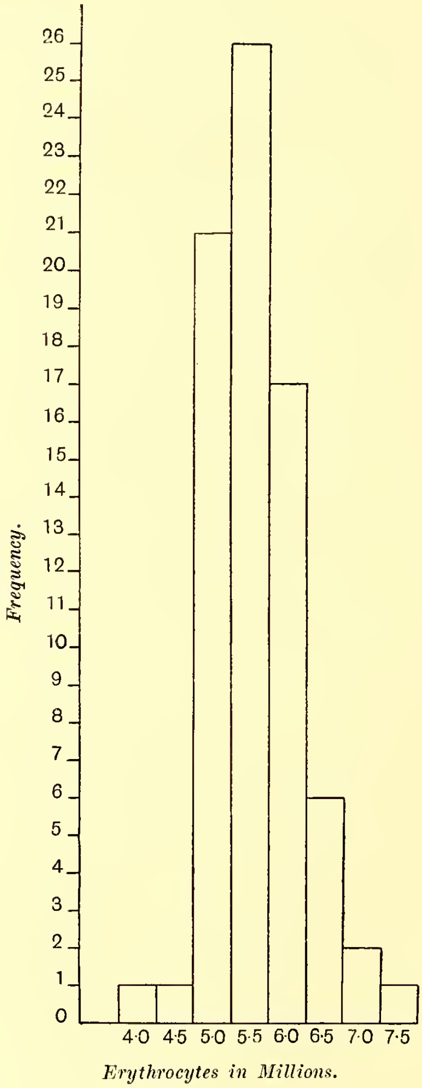


Fig. 5 (*a*). 100 First Counts.

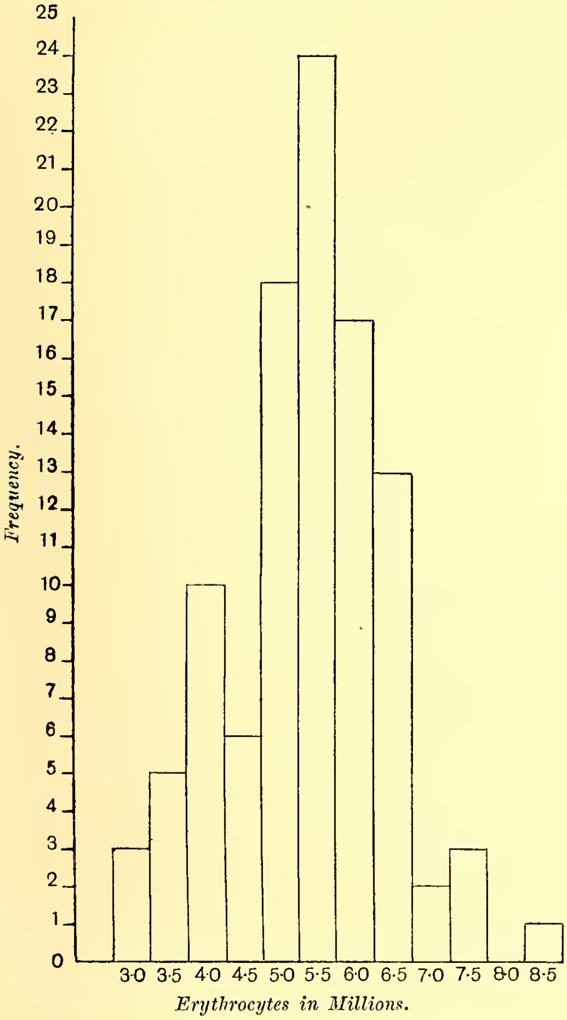


Fig. 5 (*b*). 100 Corresponding Second Counts.

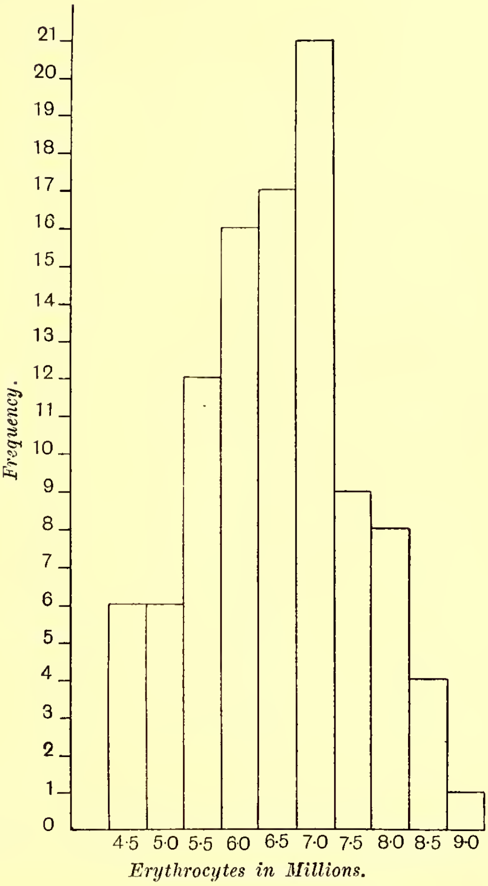


Table IV is a correlation table of the relationship we are considering. In this table the frequency distribution has been somewhat compressed; instead of a unit of 10,000, a unit of 50,000 has been taken. The correlation coefficient works out at $r = 0{\cdot}65 \pm 0{\cdot}04$ which as we might have anticipated is a high one. The relationship between the first and second values obtained from this table is given by the equation

$$C_2 = \bar{C}_2 + r \frac{\sigma_1}{\sigma_2} (C_1 - \bar{C}_1).$$

Where C_2 = the second count, $\bar{C}_2$ = the mean of the second count, which in our correlation table is 6,500,000: σ_1, σ_2 = the standard deviations of the first and second distribution respectively. $\bar{C}_1$ = mean of the first counts, i.e. 5,325,000. An example of the use of this formula will make matters clear.

TABLE IV.

Erythrocytes in 100,000's. First Count.

Erythrocytes in 100,000's. Second Count.	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	71—75	76—80	81—85	Totals
46—50	1	5	1	—	—	—	—	—	—	—	—	—	7
51—55	—	3	3	3	4	—	—	—	—	—	—	—	13
56—60	—	2	—	4	6	1	—	—	—	—	—	—	13
61—65	—	2	—	1	6	7	1	—	—	—	—	—	17
66—70	—	—	2	3	4	5	6	4	—	—	—	—	24
71—75	—	1	1	—	1	2	3	1	1	—	—	—	10
76—80	—	—	1	—	2	3	2	1	1	—	—	—	10
81—85	—	—	—	1	1	—	—	—	1	1	—	—	4
86—90	—	—	—	1	—	1	—	—	—	—	—	—	2
	1	6	9	5	13	24	19	12	7	1	2	1	100

$r = 0{\cdot}65 \pm 0{\cdot}04$.

For a first count of 5,000,000 erythrocytes, what should be the expectation for a given second count taken 18 days later, at an altitude of 6000 feet?

$$C_2 = 6{,}500{,}000 + 0{\cdot}65 \times \frac{1{\cdot}95}{2{\cdot}16}(5{,}000{,}000—5{,}325{,}000),$$

$$= 6{,}325{,}000 \text{ erythrocytes per c.mm.}$$

The following table (Table V) gives the calculated and observed values.

TABLE V.

First Count	Mean of Observed Second Counts	Calculated Mean Second Count
2,800,000	4,800,000	5,027,000
3,300,000	5,220,000	5,322,000
3,800,000	5,800,000	5,617,000
4,300,000	5,900,000	5,912,000
4,800,000	6,220,000	6.207,000
5,300,000	6,500,000	6,502,000
5,800,000	7,090,000	7,092,000
6,300,000	7,160,000	7,387,000
6,800,000	7,800,000	7,682,000
7,300,000	8,050,000	7,977,000
7,800,000	8,300,000	8,282,000

And $\lambda^2 = {\cdot}311$.

$P = {\cdot}998$.

Table VI shows the frequency distribution of 52 observations made on permanent Kasauli residents. They are few in number and widely varying. An average under such circumstances describes but poorly the character of the distribution. Still we may draw this conclusion from our inspection of the figures that the increase which we noted in the erythrocytes of individuals remaining at an elevation of 6000 feet for only 18 days would seem to be one of a permanent character, as judged from our observations on residents.

TABLE VI.

Showing the Frequency Distribution of Number of Erythrocytes in 52 Kasauli Residents.

Number of Erythrocytes in 100,000's	Frequency	Number of Erythrocytes in 100,000's	Frequency
52	2	*70*	4
53	2	*71*	1
54	1	*72*	0
55	1	*73*	0
56	1	*74*	0
57	4	*75*	1
58	4	*76*	1
59	3	*77*	1
60	3	*78*	1
61	3	*79*	1
62	1	*80*	1
63	0	*81*	1
64	2	*82*	0
65	4	*83*	0
66	3	*84*	0
67	0	*85*	1
68	1		
69	4		
		Total	52

During the stay at Kasauli 9 out of the 127 cases developed typical malarial fever and in nearly all of these cases there was definite evidence of blood destruction, especially when the blood count was taken a few days after the fever. As the point is an interesting one in relation to malarial fever, we give the figures in Table VII.

TABLE VII.

First Count	Date of development after arrival, in days	Parasites	Second Count	Loss or gain on First Count
6,640,000	1	B. T.*	6,688,000	+ 48,000
3,952,000	1	B. T.	5,939,000	+1,987,000
5,440,000	8	B. T.	5,200,000	− 240,000
5,172,000	6	B. T.	5,104,000	− 68,000
5,232,000	16	B. T.	2,784,000	−2,448,000
3,984,000	5	M. T.†	4,960,000	+ 976,000
5,952,000	17	B. T.	3,216,000	−2,736,000
5,024,000	10	B. T.	4,160,000	− 864,000
5,392,000	6	Nil	5,280,000	− 112,000

* B. T. = Benign Tertian. † M. T. = Malignant Tertian.

We note that the defect due to destruction in malarial fevers would seem to be rapidly recovered from. Those cases which developed fever early after their arrival had at least recovered their original number of erythrocytes by the end of the anti-rabic treatment.

We extracted from our original distribution all anaemias as well as cases of malarial fever. This was done in order to determine the variability of the number of erythrocytes in apparently healthy individuals. Table VIII gives the first and second counts of these extracted anaemias. Most of these, although not all, have reached at the time of their second count the total which we found on the average to be necessary for apparently healthy individuals living at a height of 6000 feet above sea-level. Those who failed to do so, would, we should expect, not benefit markedly by residence at a high altitude, unless carefully treated and kept absolutely at rest in bed. This is in accordance with clinical experience.

TABLE VIII.

Anaemias.

First Counts	Second Counts	Total Gain
2,920,000	4,040,000	1,120,000
3,024,000	3,835,000	811,000
3,040,000	4,241,000	1,201,000
3,168,000	4,672,000	1,504,000
3,168,000	4,188,000	1,020,000
3,376,000	4,632,000	1,256,000
3,440,000	4,688,000	1,248,000
3,536,000	7,328,000	3,792,000
3,696,000	5,104,000	1,508,000
3,792,000	6,496,000	2,704,000
3,840,000	5,208,000	1,368,000
3,840,000	5,080,000	1,240,000
3,928,000	4,696,000	768,000
3,952,000	6,208,000	2,256,000
4,048,000	7,280,000	3,232,000
4,112,000	5,152,000	1,040,000
4,128,000	5,632,000	1,504,000
4,272,000	5,478,000	1,206,000
4,608,000	6,096,000	1,488,000
4,752,000	5,760,000	1,008,000
4,848,000	5,568,000	720,000
5,216,000	6,768,000	1,552,000
5,360,000	5,376,000	16,000
5,424,000	8,704,000	3,280,000
5,792,000	6,308,000	516,000

Concluding Remarks.

1. The mean of our observations is higher than that usually given for an erythrocyte count in a healthy individual, namely, 5,600,000 as against 5,000,000.

2. The deviations from the mean may be considerable and yet consonant with apparent health. The range of our observations extended from 4,200,000 to 7,300,000 for a random sample of a varied population.

3. The effect of altitude is to cause a rapid increase in the erythrocyte content of the blood—quite apparent within a few weeks.

4. Permanent residents at a high altitude exhibit a higher erythrocyte content than those dwelling at lower altitudes.

5. Anaemic individuals do respond to the stimulus of altitude with production of additional erythrocytes but in very different degrees and may not do so in a degree sufficient to meet the requirements of a high altitude.

A STUDY OF THE NEGRO SKULL WITH SPECIAL REFERENCE TO THE CONGO AND GABOON CRANIA.

BY THE LATE R. CREWDSON BENINGTON, M.D.

Prepared for Press by KARL PEARSON, F.R.S.

(1) *Introductory.* Attention has already been drawn in *Biometrika* (Vol. VIII. p. 123) to the fact that the late Dr Crewdson Benington during his two years of work in the Biometric Laboratory had spent many months measuring the Congo crania at the Royal College of Surgeons and the crania brought by Du Chaillu from the Gaboon in 1864 and 1880, which latter series are both in the British Museum (Natural History). At the time of Dr Benington's death there was a great deal done, but each of these series was far from completed, and the delay in publication of this paper has arisen from the need of finishing the actual measurements, and reducing statistically not only Dr Benington's series, but other series for comparative purposes. The measurements on the Congo crania at the Royal College were completed by Miss E. Y. Thomson of the staff of the Biometric Laboratory; the Gaboon crania at the British Museum were re-examined especially with regard to their anomalies, by Miss H. Dorothy Smith, Crewdson-Benington Student in craniology. Figs. (i)—(iii) in the text are due to her and the majority of the observations in the "Remarks" on the sheets of individual measurements are either due to or have been revised by her. The whole work of numerical reduction has been carried out by Miss Julia Bell of the Biometric Laboratory. In all anatomical points we have consulted Professor G. D. Thane, who with his wonted kindness has given us aid in a great variety of difficulties. The conclusions which Dr Benington would have drawn from his material of course fail us, for not even the mean characters had been calculated; he had devoted the whole of his reduction work on these skulls to deducing the type contours discussed in the last issue of this Journal. I have made as limited observations on the material as seemed consistent with the general scheme of craniometric work in my Laboratory, leaving the comparative measurements to speak largely for themselves. My own impression after examining the numbers on which many of

the means obtained by both Dr Benington and Dr Shrubsall are based is that they are far too few to base at present any *very* dogmatic statements of racial differentiation upon them. Dr Shrubsall most courteously placed at our disposal a number of unpublished measurements of negro crania, but the frequency of each racial group seemed too small to render them of much service from the standpoint of statistical investigation. Not till at least 50, better 100 crania of each sex and local race are available will it be possible to draw valid conclusions biometrically as to the finer racial differences among the African negroid races. Meanwhile a comparison of the statistical constants* recorded in this paper may be of value as suggesting *possible* but *not proven* differences.

No one appreciates better than the present writer the real difficulty of getting long series of crania, and further how often such long series run the risk of being heterogeneous. But the deduction of local or racial differences by craniometric argument from series of 10 to 20 crania is a perfectly idle task. It cannot be too often asserted that the publication of measurements on small series of crania is of value only because, if they have been made on a standardised system, they can later be pooled with other material. That is the immense advantage that would arise from an international standardised catalogue. The anatomist may from inspection and by appreciation be able to draw conclusions as to racial peculiarities from small cranial series—the biometrician does not feel called upon either to dispute the fact, or to accept such conclusions. He is working by other methods; but he does feel justified in asserting that such conclusions do not receive adequate confirmation by appeal to the slight craniometric differences discoverable in small series of allied human races. Too often we find craniometric distinctions made between two small series, where no evidence is given that two random samples of the same local race might not have exhibited as large differences. Furthermore even, when probable errors are provided, we often find in small, apparently random, samples of the same race, differences quite beyond those of random sampling. The probable explanation of this is that we are meeting in such cases of small numbers with a marked influence of heredity in clan or even with family peculiarities. A cranial series ought to be sufficiently large to insure that the members of a single family or blood group will not markedly influence the results; this is wholly impossible with any cranial series under 50, and the minimum dealt with ought to be 100 adult crania of one sex. The craniologist may often have to content himself with fifty or fewer, but if he does, it will be only wise, as long as he is arguing from craniometry and not from anatomical appreciation, to replace dogmatic statement by cautious suggestion.

(2) *Material used.* The following series have been dealt with, the first three being measured and reduced, the second three reduced only. The seventh series is one which was reduced by Cicely D. Fawcett and published in *Biometrika*, Vol. I, p. 426.

---

* The statistical constants of Dr Shrubsall's published material have all been independently recalculated in the Laboratory.

A. *Congo Crania* in the Royal College of Surgeons, measured by Dr Benington at the suggestion of Professor Keith. These crania, belonging to the Batetela tribe, provide 50 males and 27 females and 5 non-adult, although it has not been possible to obtain a complete series of measurements on all of them. There are no mandibles. Professor Keith has kindly given us the following account of their origin :

> The Collection of Batetela crania was brought home by Mr E. Torday in 1909 when he had completed his travels and explorations in the eastern central parts of the Congo Free State. He gives the following account of them.
>
> "The Batetela skulls were collected by the natives of one or two villages near Makunji. They may be considered as mostly fresh ; some were not quite dry when I received them. I was assured that all had died from sleeping sickness and they were given to me to find a remedy for that disease. Sleeping sickness did not appear in this part until thirty to forty years ago. The district in which they were gathered does not cover more than three or four square miles."
>
> The Batetela occupy a large area between 9° and 6° S. and 24°—26° E. The Sungu Batetela are about 24° 20′ E. and 4° 50′ S. Their village is Mokunji or Mukunji, quite close to the Labefu River, the most eastern tributary of the Sankuru.
>
> Some of the crania show marks of having been cut or notched in the supra-orbital region. Parts of the bone may have been removed for medicinal purposes.

B. *Crania from the Gaboon, Group* I, brought by Du Chaillu from Fernand Vaz in 1864. This and the following series appear never to have been adequately measured, still less properly discussed. There are 50 male and 44 female crania in this series. This and the following series are in the British Museum (Natural History), South Kensington.

C. *Crania from the Gaboon, Group* II, brought by Du Chaillu from Fernand Vaz in 1880. There are only 18 male and 19 female in this series, and the numbers being so small it has not been considered worth while calculating more than the means.

D. *Zulu Crania.* Twenty male crania measured by Dr Shrubsall. The measurements are published in his paper on "A Study of A-Bantu Skulls and Crania," in *Journal of the Anthropological Institute,* Vol. I, p. 55 *et seq.*, London, 1899. They belong to the Eastern group of Southern Bantus, inhabiting country south of Zambesi.

E. *Angoni Crania.* Thirty-eight male crania measured by Dr Shrubsall. The measurements are given in the memoir just cited. The crania were sent by Sir H. H. Johnston to the British Museum (*Natural History*), who wrote about them as follows (Shrubsall, p. 77): "The skulls that I sent you were obtained by me in M'ponda's town at the south end of Lake Nyasa. I was given to understand that they were the skulls of Angoni hostages, who had been slaughtered by M'ponda. The heads were taken by me off the stakes of M'ponda's stockade. If, as I imagine, they are the heads of Angoni, they would be the heads of a slightly mixed negro race mainly belonging to the Anyanja stock with a slight Zulu intermixture. It is possible, however, that one or more of them may be Yaos mixed with Arab blood." Sir H. H. Johnston further indicates that the Angoni were very unlikely themselves to be at all homogeneous; for he tells us that a horde of Zulus early in the 19th century trekked northwards to avoid the tyranny of Chalea, some founded a Zulu kingdom in Fipa, some went northwards even as far as the Victoria Nyanza, "while others working eastwards became the dominant caste among the Wahehe and Wanzindo. After the disruption of the Zulu kingdom in Fipa country the bands formed the present Angoni and Magwang wara." (*loc. cit.* p. 77.)

With such a history there would be almost certainly a mixture of the migrating Zulus and the original population, and it may be seriously doubted whether any weight at all can be given to such differences as are significant between Shrubsall's Zulus and Angoni. All we should be justified in saying would be that the differences may be due to the presence of crania belonging to a mixed race, or to crania belonging to a mixture of races, but no real light can be thrown on what elements go to build up the heterogeneity of the Angoni series.

F. *Kaffir Crania.* Thirty-eight Kaffir male crania measured by Dr Shrubsall (*loc. cit.* p. 55). These Kaffirs appear to consist chiefly of Arna-mpondo, who inhabit Pondoland along the bank of lower Umzi-mvubu, Aba-temlu from between Umtata and Kei rivers, and Ama-xosa, who formerly were between Kei and Fish rivers, but were driven into the Transkei. Again we must feel on very unsafe ground in drawing any very dogmatic conclusion with regard to Kaffir cranial characters from this mixed and sparse material.

G. *Northern Negro Crania.* This series consists of 39 male crania, the measurements of which were extracted from the German Anthropological Catalogue. They belonged to negroes from the North of Africa (see *Biometrika*, Vol. I. p. 426).

G$^{\text{bis}}$. Broca took measurements of 77 modern negro skulls in the collection at Paris, and copies of these measurements were sent to K. Pearson by M. Manouvrier. The skulls were chiefly from North and West coasts of Africa. The following table is taken from Fawcett's paper (*loc. cit.* p. 428):

Character	Male		Female	
	Number	Mean	Number	Mean
Capacity (C)*	*54*	1430 (? 1347)	*23*	1256 (? 1175)
Maximum Length (L) ...	*54*	185·04	*23*	174·52
Maximum Breadth (B) ...	*54*	135·20	*23*	130·52
Height (H)	*54*	134·77	*23*	126·91
100 B/L	*54*	73·28	*23*	74·85
100 H/L	*54*	72·83	*23*	72·75

---

* Broca's measures of capacity are exaggerated. See footnote to Fawcett's paper *loc. cit.* p. 428. The means calculated from the series G and G$^{\text{bis}}$ can only be of service when we are comparing negroes in general with other races.

The series involved are too heterogeneous to throw any light on intra-negroid differences. The values found for G and G^{bis}, however, lie well inside the differences found by Shrubsall for small series of local negro races, and it would therefore be impossible to assert on the basis of these results *alone* that the negroes of Northern Africa differ sensibly from those of the South.

(3) *Measurements.* The measurements taken by Dr Crewdson Benington are in the main those of previous biometric craniologists. They were made in the manner described by Fawcett (*loc. cit.* p. 416) and Macdonell, *Biometrika* (Vol. III. p. 200). The measurements are as follows: C = capacity. Taken with mustard seed, packing and measuring glass,—not in the present manner of the Biometric Laboratory with the balance and weighing (see *Biometrika*, Vol. III. p. 203). The latter method was not possible as the skulls were measured away from the Laboratory. Dr Benington, however, practised on the "cranes étalons" and against other workers in the Laboratory until his average difference was under 10 cm.^{3} F = Flower's ophryo-occipital length; L' = glabellar horizontal length; L = maximum length from glabella to occiput. B = maximum horizontal breadth; B' = least breadth of forehead from one temporal crest to the other; H = basio-bregmatic height*; OH = auricular height as measured on the craniophor, that is the height perpendicular to the horizontal plane, above the line joining the *upper* margins of the auricular orifices. LB = basi-nasal length, from basion to nasion. P = profile angle measured with the goniometer, the skull being on craniophor. Q = cross or tranverse arc from upper rim of one auricular passage to that of the other over the bregma. Q_1 = the same measurement taken not over the bregma, but perpendicular to the horizontal plane. S = sagittal arc from nasion to opisthion; S_1 = part of S from nasion to bregma; S_2 = part of S from bregma to lambda; S_3 = part of S from lambda to opisthion; S_3' = chord of arc S_3. U = horizontal circumference, measured directly above the superciliary ridges and round the most projecting part of the occiput.

$G'H$ = upper face height, nasion to alveolar point; GB = face breadth, from the lower end of one zygomatico-maxillary suture to that of the other; J = zygomatic breadth, from the outermost point of one zygomatic arch to that of other; NH = nasal height, from nasion to the lowest edge of pyriform aperture†; NB = nasal breadth, greatest breadth of nasal aperture, wherever it may be; O_1 = breadth of orbit, R and L, the greatest breadth from side to side (see *Biometrika*, Vol. III. p. 201); O_2 = height of orbit, R and L, taken perpendicular to O_1; G_1 = length of palate, from the point of the *spina nasalis posterior* to an

---

* H is usually measured in the Biometric Laboratory from the basion to the pencil line on the skull vertically above the auricular axis, but for comparative purposes Dr Benington followed Dr Macdonell and measured to the bregma and thus his measurements are comparable with those of Shrubsall and Broca. In the case of the Congo crania he had the craniophor at the Royal College, and I think his H is the pencil line measurement, and that this accounts for the great difference between the Congo and Gaboon cranial heights.

† *Theoretically* we say from nasion to the point in the median plane which is determined by a tangent to the lower edges of the pyriform aperture. In practice, however, this gives a value sensibly the same as the above.

imaginary surface tangential to the inner alveolar surfaces of the mesial incisors; G_1' = length from base of spine. The reader should consult *Biometrika*, Vol. I. p. 429 and Vol. III. p. 201, for obscurities in different systems of measuring the palate length*:—G_2 = breadth of palate between the inner alveolar walls at the second molars; B_2 = breadth of palate, from end to end of palato-maxillary suture; GL = profile or basi-alveolar length, from basion to alveolar point. fml = length of foramen magnum; fmb = greatest breadth of foramen magnum. In order to determine the flattening of the bridge of the nose, Dr Benington at my suggestion started to measure the transverse distance from dacryon to dacryon with the callipers and also the corresponding minimum arc with the tape. The ratio of the excess of the mesodacryal arc over the mesodacryal chord to the mesodacryal chord, i.e. $DA - DC$ to DC forms a measure of the flattening of the bridge of the nose. Possibly a better measure than this ratio would be the ratio of the subtense of this arc to the chord and a means of determining this is discussed later. These two ratios may be called the Mesodacryal or more shortly the Dacryal Indices. It might be anticipated that they would show very considerable differences between European and negro races.

Pt = Bi-pteric breadth, measured from the hinder extremity of one spheno-parietal suture to the other. St = Bi-stephanic breadth.

The indices calculated are: the cephalic indices $100\,B/L$, $100\,H/L$ and $100\,B/H$; the upper facial index $100\,G'H/GB$; the nasal index $100\,NB/NH$; the right and left orbital indices $100\,O_2/O_1$; the palate index $100\,G_2/G_1$; the occipital index† $Oc.I = 100\dfrac{S_3}{S_3'}\sqrt{\dfrac{S_3}{24(S_3 - S_3')}}$; the foraminal index $100\dfrac{fmb}{fml}$; and the dacryal indices, α and β, as defined above.

On the mandibles, where available, were measured: W_1 = condylar width, or greatest width from outside one condyle to outside of second; W_2 = greatest width at angles from outside one angle to outside of other; h_1 = greatest height, from lowest median projection to top of alveolar margin between mesial incisors; and f = distance between foramina mentalia.

---

* The following comparative results are of interest:

Length of spina nasalis posterior.

Naqada Skulls	Male 5·05,	Female 4·00.	Fawcett
English Skulls, Whitechapel ...	Male 3·61,	Female 3·60.	Macdonell
Negroes, Congo	Male 3·09,	Female 2·93.	Benington
Negroes, Gaboon, 1864 ...	Male 3·73,	Female 3·37.	Benington

It will be noticed that the spine is sensibly longer in the Egyptian than in the English or Negro series. I presume Shrubsall has measured G_1, but in many cases craniologists do not precisely define what they understand by palate length.

† This index, which I now term the Occipital Index, was suggested by me to Dr Macdonell and misnamed by me at that date as the Cerebellar Index.

TABLE I. *Males.*

Character	CONGO SERIES				GABOON, 1864 SERIES			
	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean	Standard Deviation	Coefficient of Variation
C	47	1343·91 ± 12·45	126·57 ± 8·81	9·42 ± ·66	49	1380·51 ± 10·38	107·69 ± 7·34	7·80 ± ·53
F	50	176·34 ± ·62	6·55 ± ·44	3·71 ± ·25	50	177·62 ± ·52	5·46 ± ·37	3·08 ± ·21
L'	47	177·78 ± ·65	6·80 ± ·46	3·82 ± ·26	—	—	—	—
L	50	177·87 ± ·69	7·05 ± ·49	3·96 ± ·28	50	179·48 ± ·53	5·61 ± ·38	3·12 ± ·21
B	50	138·52 ± ·48	5·00 ± ·34	3·61 ± ·24	50	135·48 ± ·32	3·38 ± ·23	2·49 ± ·17
B'	50	97·46 ± ·37	3·85 ± ·26	3·95 ± ·27	50	96·46 ± ·30	3·19 ± ·22	3·31 ± ·22
H	48	133·81 ± ·40	4·13 ± ·28	3·09 ± ·21	50	135·42 ± ·48	5·02 ± ·34	3·60 ± ·24
OH	47	113·85 ± ·40	4·05 ± ·28	3·56 ± ·25	—	—	—	—
LB	48	98·19 ± ·45	4·58 ± ·32	4·66 ± ·32	48	100·29 ± ·37	3·77 ± ·19	3·76 ± ·26
P	47	82°·76 ± ·37	3°·74 ± ·26	4·52 ± ·32	—	—	—	—
Q	47	310·74 ± 1·01	10·29 ± ·72	3·31 ± ·23	50	310·54 ± ·86	9·02 ± ·61	2·90 ± ·20
Q_1	47	309·28 ± 1·02	10·40 ± ·72	3·36 ± ·23	—	—	—	—
S	50	361·80 ± 1·02	10·73 ± ·72	2·97 ± ·20	50	364·26 ± 1·19	12·51 ± ·84	3·43 ± ·23
S_1	50	125·46 ± ·52	5·47 ± ·37	4·36 ± ·29	50	126·32 ± ·62	6·53 ± ·44	5·17 ± ·35
S_2	50	128·30 ± ·71	7·42 ± ·50	5·78 ± ·39	50	128·58 ± 1·12	11·75 ± ·79	9·14 ± ·62
S_3	49	107·67 ± ·60	6·20 ± ·42	5·76 ± ·39	50	109·14 ± ·94	9·83 ± ·66	9·00 ± ·61
S_3'	50	94·98 ± ·45	4·69 ± ·32	4·94 ± ·33	50	95·92 ± ·65	6·77 ± ·46	7·06 ± ·48
U	50	504·34 ± 1·64	17·24 ± 1·16	3·42 ± ·23	50	501·18 ± 1·09	11·44 ± ·77	2·28 ± ·15
$G'H$	49	63·41 ± ·40	4·14 ± ·28	6·53 ± ·45	50	66·44 ± ·57	6·01 ± ·41	9·04 ± ·61
GB	46	94·76 ± ·55	5·57 ± ·39	5·88 ± ·41	48	95·60 ± ·42	4·32 ± ·30	4·52 ± ·31
J	33	126·48 ± ·81	6·90 ± ·57	5·46 ± ·45	45	129·04 ± ·49	4·85 ± ·35	3·76 ± ·27
NH	49	47·22 ± ·27	2·83 ± ·19	5·99 ± ·41	50	48·16 ± ·28	2·98 ± ·20	6·18 ± ·42
NB	50	26·04 ± ·20	2·05 ± ·14	7·87 ± ·53	50	26·56 ± ·21	2·20 ± ·15	8·29 ± ·56
O_1R	46	41·14 ± ·16	1·64 ± ·12	3·99 ± ·28	50	41·46 ± ·14	1·46 ± ·10	3·52 ± ·24
O_1L	47	40·40 ± ·14	1·44 ± ·10	3·56 ± ·25				
O_2R	46	34·57 ± ·19	1·95 ± ·14	5·64 ± ·40	50	35·04 ± ·20	2·09 ± ·14	5·96 ± ·40
O_2L	47	34·62 ± ·22	2·19 ± ·15	6·33 ± ·44				
G_1'	33	47·24 ± ·45	3·83 ± ·32	8·11 ± ·68	45	48·38 ± ·32	3·23 ± ·23	6·67 ± ·48
G_1	30	50·33 ± ·55	4·49 ± ·39	8·92 ± ·78	35	52·11 ± ·41	3·56 ± ·29	6·82 ± ·55
G_2	46	38·87 ± ·39	3·95 ± ·28	10·16 ± ·72	44	39·50 ± ·30	2·93 ± ·21	7·41 ± ·54
B_2	45	27·64 ± ·42	4·13 ± ·29	14·94 ± 1·09	44	30·43 ± ·26	2·60 ± ·19	8·54 ± ·62
GL	47	96·47 ± ·54	5·46 ± ·38	5·66 ± ·40	48	100·50 ± ·45	4·67 ± ·32	4·64 ± ·32
Fml	48	35·94 ± ·23	2·39 ± ·16	6·65 ± ·46	50	35·86 ± ·27	2·80 ± ·19	7·81 ± ·53
Fmb	48	29·98 ± ·22	2·30 ± ·16	7·67 ± ·53	50	29·96 ± ·27	2·80 ± ·19	9·34 ± ·64
DC	50	24·90 ± ·25	2·67 ± ·18	10·72 ± ·73	50	24·50 ± ·23	2·38 ± ·16	9·70 ± ·66
DA	50	34·52 ± ·44	4·65 ± ·31	13·47 ± ·92	—	—	—	—
PT	50	108·66 ± ·49	5·17 ± ·35	4·76 ± ·32	—	—	—	—
ST	50	115·74 ± ·52	5·50 ± ·37	4·75 ± ·32	50	113·66 ± ·51	5·33 ± ·36	4·69 ± ·32
100 B/L	50	77·99 ± ·25	2·58 ± ·17	3·31 ± ·22	50	75·48 ± ·26	2·77 ± ·19	3·66 ± ·25
100 H/L	48	75·36 ± ·28	2·88 ± ·20	3·82 ± ·26	50	75·50 ± ·23	2·43 ± ·16	3·21 ± ·22
100 B/H	48	103·58 ± ·44	4·55 ± ·31	4·39 ± ·30	50	100·07 ± ·41	4·27 ± ·29	4·26 ± ·29
100 $G'H/GB$	46	66·88 ± ·48	4·82 ± ·34	7·21 ± ·51	48	69·73 ± ·54	5·57 ± ·38	7·99 ± ·55
100 NB/NH	49	55·23 ± ·47	4·91 ± ·33	8·89 ± ·61	50	55·31 ± ·47	4·89 ± ·33	8·84 ± ·60
100 O_2/O_1L	47	85·71 ± ·46	4·72 ± ·33	5·51 ± ·38	50	84·56 ± ·52	5·41 ± ·36	6·40 ± ·43
100 O_2/O_1R	45	84·14 ± ·51	5·12 ± ·36	6·09 ± ·43				
100 G_2/G_1	30	76·80 ± ·77	6·27 ± ·55	8·16 ± ·72	33	76·08 ± ·76	6·50 ± ·54	8·55 ± ·72
Oc. I	48	68·83 ± ·77	7·91 ± ·54	11·49 ± ·80	49	68·30 ± ·65	6·78 ± ·46	9·92 ± ·68
100 Fmb/Fml	48	83·50 ± ·58	5·96 ± ·41	7·14 ± ·49	50	83·86 ± ·79	8·24 ± ·56	9·82 ± ·67
100 DI, *a*	50	39·85 ± ·61	6·32 ± ·43	15·85 ± 1·11	50	45·41 ± ·58	6·04 ± ·41	13·30 ± ·91
W_1	—	—	—	—	27	112·63 ± ·70	5·42 ± ·50	4·81 ± ·44
W_2	—	—	—	—	30	94·27 ± ·77	6·23 ± ·54	6·61 ± ·58
h_1	—	—	—	—	29	32·69 ± ·51	4·11 ± ·36	12·57 ± 1·13
f	—	—	—	—	30	45·37 ± ·29	2·36 ± ·21	5·19 ± ·45

TABLE I. *Males (continued).*

Character	Gaboon, 1880 Series		Zulu		Angoni		Kaffir		Northern Negro	
	No.	Mean	No.	Mean	No.	Mean	No.	Mean	No.	Mean
C	16	1447·44	—	—	—	—	—	—	39	1347·7
F	18	180·50	—	—	—	—	—	—	—	—
L'	—	—	—	—	—	—	—	—	—	—
L	18	182·33	20	184·15	25	184·34	38	190·62	39	182·85
B	18	138·22	19	137·05	25	134·58	36	137·36	39	133·15
B'	18	98·39	20	99·83	23	98·02	38	100·86	32	95·91
H	18	136·67	20	138·13	24	136·98	38	137·37	39	135·31
OH	—	—	—	—	—	—	—	—	27	115·04
LB'	18	101·89	20	101·85	25	102·46	38	105·05	27	102·96
P	—	—	—	—	—	—	—	—	11	82°·00
Q	15	311·73	17	307·24	23	305·17	37	307·16	27	306·85
Q_1	—	—	—	—	—	—	—	—	—	—
S	16	370·56	19	372·37	24	368·92	37	380·78	32	367·72
S_1	16	129·38	19	128·32	24	129·63	36	129·51	—	—
S_2	16	126·75	19	132·37	24	130·42	37	132·12	—	—
S_3	16	114·13	19	111·68	24	108·88	36	119·31	—	—
S_3'	16	100·25	—	—	—	—	—	—	—	—
U	15	509·53	17	520·35	23	511·43	37	526·68	32	508·47
$G'H$ ...	15	67·27	19	69·47	25	70·32	36	69·14	31	68·00
GB	17	98·18	19	98·47	17	95·59	32	96·16	27	96·63
J	17	132·53	15	133·07	—	—	33	134·14	32	129·47
NH	18	47·94	20	47·20	25	48·46	36	48·64	27	49·52
NB	18	27·06	20	27·30	25	28·02	36	27·33	34	27·29
O_1R ...	18	42·06	20	38·73	20	38·03	37	39·80	27	39·56
O_1L										
O_2R	18	35·17	20	33·20	22	33·32	37	34·00	27	34·19
O_2L										
G_1'	14	50·07	—	—	—	—	—	—	—	—
G_1	7	57·29	17	52·00	25	51·94	36	52·13	26	49·42
G_2	12	40·67	18	37·94	25	35·80	36	37·21	26	39·73
B_2	13	31·38	—	—	—	—	—	—	—	—
GL	15	102·73	18	101·94	24	103·52	36	105·14	27	105·33
Fml ...	16	35·63	20	37·30	24	36·38	38	37·50	—	—
Fmb ...	16	30·00	20	30·15	25	29·56	38	30·03	—	—
DC	17	25·53	20	24·30	24	24·25	37	24·14	—	—
DA	—	—	—	—	—	—	—	—	—	—
PT	—	—	20	107·35	25	108·50	37	109·49	—	—
ST	16	115·63	19	115·00	25	114·32	37	110·78	—	—
100 B/L ...	18	75·86	19	74·27	25	73·05	36	72·13	39	72·87
100 H/L ...	18	74·96	20	75 04	24	74·25	38	72·13	39	74·15
100 B/H ...	18	101·22	19	98·89	24	98·21	36	99·97	39	98·28
100 $G'H/GB$	14	68·94	18	72·15	17	74·06	32	72·18	27	70·37
100 NB/NH	18	56·49	20	58·05	25	58·03	36	56·33	27	55·43
100 O_2/O_1L	18	83·73	20	85·82	20	88·09	37	85·51	27	87·20
100 O_2/O_1R										
100 G_2/G_1 ...	7	70·64	17	73·07	25	69·10	36	71·56	25	81·52
$Oc. I$ ...	15	66·27	—	—	—	—	—	—	—	—
100 Fmb/Fml	16	84·31	20	80·8	24	81·3	38	80·1	—	—
100 DI, a ...	—	—	—	—	—	—	—	—	—	—
W_1	7	120·86	—	—	—	—	19	118·53	16	117·00
W_2	7	99·00	—	—	—	—	19	96·05	16	97·15
h_1	7	31·71	—	—	—	—	—	—	—	—
f	7	46·29	—	—	—	—	—	—	—	—

TABLE II. *Females.*

Character	CONGO SERIES				GABOON, 1864 SERIES				GABOON, 1880 SERIES	
	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean	Standard Deviation	Coefficient of Variation	No.	Mean
C	21	1205·88 ± 15·85	107·68 ± 11·21	8·93 ± ·94	43	1231·70 ± 13·03	126·63 ± 9·20	10·28 ± ·76	17	1240·41
F	27	170·83 ± ·77	5·93 ± ·54	3·47 ± ·32	44	170·57 ± ·63	6·16 ± ·44	3·61 ± ·26	19	170·05
L′	26	171·23 ± ·70	5·26 ± ·49	3·07 ± ·29	—	—	—	—	—	—
L	27	171·37 ± ·79	6·06 ± ·56	3·54 ± ·33	44	171·57 ± ·61	6·01 ± ·43	3·51 ± ·25	19	170·95
B	27	130·91 ± ·72	5·53 ± ·51	4·63 ± ·43	44	130·57 ± ·47	4·67 ± ·34	3·58 ± ·26	19	130·47
B′	27	92·56 ± ·49	3·74 ± ·34	4·04 ± ·37	43	92·84 ± ·41	4·00 ± ·29	4·30 ± ·31	19	93·05
H	26	127·50 ± ·60	4·53 ± ·42	3·55 ± ·33	43	130·05 ± ·60	5·80 ± ·42	4·46 ± ·32	18	131·50
OH	26	109·00 ± ·58	4·35 ± ·41	3·99 ± ·37	—	—	—	—	—	—
LB	27	93·31 ± ·44	3·42 ± ·31	3·68 ± ·34	43	95·84 ± ·42	4·05 ± ·29	4·22 ± ·31	18	97·39
P	25	81°·08 ± ·46	3°·38 ± ·32	4·17 ± ·40	—	—	—	—	—	—
Q	26	298·19 ± 1·39	10·53 ± ·98	3·53 ± · 33	44	296·27 ± 1·10	10·79 ± ·78	3·64 ± ·26	19	298·16
Q_1	26	297·31 ± 1·38	10·44 ± ·98	3·51 ± · 33	—	—	—	—	—	—
S	27	349·81 ± 1·50	11·56 ± 1·06	3·30 ± · 30	44	350·39 ± 1·39	13·67 ± ·98	3·90 ± ·28	19	348·21
S_1	27	120·91 ± ·86	6·61 ± ·61	5·47 ± · 50	44	121·59 ± ·68	6·72 ± ·48	5·52 ± ·40	19	121·11
S_2	27	119·80 ± 1·13	8·70 ± ·80	7·26 ± ·67	44	122·77 ± 1·06	10·44 ± ·75	8·50 ± ·62	19	120·74
S_3	27	109·43 ± 1·01	7·78 ± ·71	7·11 ± ·66	44	105·93 ± 1·04	10·18 ± ·73	9·61 ± ·70	19	106·47
S_3'	27	95·81 ± ·74	5·67 ± ·52	5·92 ± ·55	44	93·05 ± ·77	7·56 ± ·54	8·13 ± ·59	19	93·79
U	27	489·30 ± 1·79	13·80 ± 1·27	2·82 ± ·26	44	481·16 ± 1·41	13·86 ± 1·00	2·88 ± 21	19	480·68
G′H	26	59·27 ± ·64	4·83 ± ·45	8·15 ± ·77	38	62·74 ± ·53	4·89 ± ·38	7·79 ± ·61	19	62·47
GB	27	92·20 ± ·57	4·36 ± ·40	4·73 ± ·44	41	90·85 ± ·64	6·07 ± ·45	6·68 ± ·50	17	90·65
J	18	122·14 ± ·68	4·25 ± ·48	3·48 ± ·39	40	120·50 ± ·63	5·92 ± ·45	4·91 ± ·37	15	120·33
NH	27	45·02 ± ·38	2·90 ± ·27	6·44 ± ·59	43	45·14 ± ·32	3·08 ± ·22	6·82 ± ·50	18	45·33
NB	27	25·04 ± ·23	1·74 ± ·16	6·95 ± ·64	43	24·74 ± ·20	1·98 ± ·14	8·00 ± ·59	18	25·67
O_1R	26	41·21 ± ·18	1·36 ± ·13	3·30 ± ·31	43	39·56 ± ·18	1·72 ± ·13	4·34 ± ·32	19	40·11
O_1L	26	40·50 ± ·20	1·50 ± ·14	3·70 ± ·35						
O_2R	27	33·57 ± ·23	1·81 ± ·17	5·39 ± ·50	43	33·67 ± ·19	1·84 ± ·13	5·47 ± ·40	19	34·11
O_2L	27	33·35 ± ·24	1·83 ± ·17	5·49 ± ·51						
G_1'	21	48·26 ± ·39	2·63 ± ·27	5·45 ± · 57	36	46·72 ± ·44	3·92 ± ·31	8·40 ± ·67	13	46·69
G_1	21	50·19 ± ·43	2·90 ± ·30	5·78 ± ·60	33	50·09 ± ·47	4·02 ± ·33	8·02 ± ·67	11	51·73
G_2	21	35·93 ± ·36	2·43 ± ·25	6·76 ± ·71	35	37·40 ± ·38	3·33 ± ·27	8·90 ± ·72	12	37·33
B_2	24	30·92 ± ·28	2·01 ± ·20	6·50 ± ·64	36	28·94 ± ·26	2·33 ± ·19	8·06 ± ·64	13	29·69
GL	26	93·90 ± ·50	3·75 ± ·35	3·99 ± ·37	38	98·45 ± ·54	4·91 ± 38	4·98 ± ·39	18	98·17
Fml	27	35·33 ± ·39	2·99 ± ·27	8·46 ± ·78	41	33·98 ± ·28	2·61 ± ·19	7·68 ± ·58	18	33·72
Fmb	27	28·80 ± ·27	2·09 ± ·19	7·26 ± ·67	42	28·14 ± ·27	2·55 ± ·19	9·07 ± ·67	18	28·50
DC	27	24·72 ± ·21	1·58 ± ·15	6·39 ± ·59	43	23·09 ± ·30	2·87 ± ·21	12·43 ± ·92	19	22·68
DA	27	40·22 ± ·44	3·38 ± ·31	8·40 ± ·77	—	—	—	—	—	—
PT	27	106·24 ± ·63	4·83 ± ·44	4·55 ± ·42	—	—	—	—	—	—
ST	27	112·24 ± ·51	3·96 ± ·36	3·53 ± ·32	42	108·48 ± ·61	5·87 ± ·43	5·41 ± ·40	19	105·32
100 *B/L*	27	76·46 ± ·32	2·50 ± ·23	3·27 ± ·30	44	76·15 ± ·34	3·38 ± ·24	4·44 ± ·32	19	76·43
100 *H/L*	26	74·58 ± ·33	2·51 ± ·23	3·37 ± ·32	43	75·67 ± ·32	3·07 ± ·22	4·06 ± ·30	18	76·78
100 *B/H*	26	102·58 ± ·55	4·19 ± ·39	4·08 ± ·38	43	100·72 ± ·54	5·28 ± ·38	5·24 ± ·38	18	99·68
100 *G′H/GB*	26	64·04 ± ·67	5·05 ± ·47	7·89 ± ·74	38	69·28 ± ·54	4·96 ± ·38	7·15 ± ·56	17	68·20
100 *NB/NH*	27	55·81 ± ·62	4·78 ± ·44	8·56 ± ·79	43	54·93 ± ·43	4·20 ± ·31	7·66 ± ·56	18	56·90
100 O_2/O_1L	26	82·54 ± ·50	3·81 ± ·36	4·62 ± ·43	43	85·23 ± ·48	4·65 ± ·34	5·45 ± ·40	19	85·17
100 O_2/O_1R	26	81·23 ± ·47	3·59 ± ·34	4·42 ± ·41						
100 G_2/G_1	17	70·79 ± ·80	4·90 ± ·57	6·92 ± ·80	30	74·80 ± ·75	6·13 ± ·53	8·20 ± ·72	9	71·98
Oc. I	27	67·04 ± ·59	4·58 ± ·42	6·83 ± ·63	44	69·23 ± ·87	8·53 ± ·61	12·32 ± ·90	19	69·45
100 *Fmb/Fml*	27	81·80 ± ·75	5·79 ± ·53	17·07 ± ·65	41	81·16 ± ·74	6·99 ± ·52	8·61 ± ·65	18	84·67
100 *DI*	27	41·48 ± ·71	5·71 ± ·51	13·75 ± 1·24	38	46·71 ± ·82	7·49 ± ·58	16·03 ± 1·27	—	—
W_1	—	—	—	—	18	110·00 ± ·68	4·25 ± ·48	3·86 ± ·43	9	106·33
W_2	—	—	—	—	21	90·05 ± 1·05	7·10 ± ·74	7·89 ± ·83	9	90 11
h_1	—	—	—	—	20	30·50 ± ·57	3·78 ± ·40	12·38 ± 1·34	9	28·11
f	—	—	—	—	22	44·27 ± ·34	2·38 ± ·24	5·37 ± ·55	9	47·44

(4) *Biometric Constants for Negro Crania.* Tables I and II give the means, standard deviations and coefficients of variation (accompanied where justifiable by their probable errors) of the Congo and Gaboon 1864 series for males and females respectively. The values of the Gaboon 1880 series, for Fawcett's modern negroes and for Shrubsall's Zulu, Angoni and Kaffir series are given for comparison. Let us first compare the Gaboon and Congo crania with regard to a few of the measured characters. First: Capacity. Taking the males there is a greater difference between the 1880 Gaboon crania and the 1864 Gaboon than between the latter and the Congo. The difference of the last two is 37 cm.^{3} and the probable error of that difference about 16 cm.^{3}, or the difference is less than 2·5 times its probable error. It is difficult to lay stress on this. Now turn to the two Gaboon series. The probable error of the capacity of the 1864 males is 10·38, of the 1880 about 18·16*; the difference of their capacities is 67 and the probable error of that difference 21 cm.^{3} Hence the difference is more than three times the probable error and likely to be significant. On the other hand the modern negroes from North Africa are not in capacity significantly different from either the Congo or Gaboon 1864 series! Shall we not conclude that the Gaboon 1880 series has only larger capacity, because it consists of solely 16 crania, and that these have been probably subject to some unknown form of selection? If we turn to the female crania we find this view is entirely confirmed. There is no significant difference between either Gaboon series and the Congo. Shrubsall has given us no capacities, but, I think, we may say provisionally that for the negro skull the capacity is about 1350 for males and 1230 for females, a result very close to Galton's rule of adding one twelfth to the female to get the male equivalent. Clearly the capacity of the negro skull is for males about 140 cm^3. and for females about 100 cm^3. less than that of modern Europeans †. These are significant differences. The capacity is probably slightly less than that of the ancient Egyptians as judged by Naqada or Theban crania, it is sensibly less than that of a short and primitive race like the Aino, and appears only equalled by that of the modern Copt or Hindoo ‡.

Diameters: If we take the chief diameters of the head we have:

	Congo		Gaboon, 1864		Gaboon, 1880	
	♂	♀	♂	♀	♂	♀
Length (*L*) ...	177·9	171·4	179·5	171·5	182·3	171·0
Breadth (*B*) ...	138·5	130·9	135·5	130·6	138·2	130·5
Height (*H*) ...	133·8	127·5	135·4	130·1	136·7	131·5

---

* Assuming the S.D. of the 1880 series would be approximately the same as that of the 1864 series.

† English ♂ 1477, ♀ 1300; Bavarian ♂ 1503, ♀ 1335; Würtemberger ♂ 1494, ♀ 1337; French ♂ 1494. Data reduced by Macdonell, Fawcett and Lee.

‡ For ancient and modern Egyptians; see *Biometrika*, Vol. I. p. 426; the Hindoo skull is judged from material in the Biometric Laboratory not yet published.

The striking similarity in value of the three diameters for the three female series is only marred by the value of the Congo height i.e. 127·5. The males are in less good agreement, but here again the differences where significant are sometimes such that the Congo is nearer to one Gaboon series than the two Gaboon series are to each other. I do not think any differentiation of these series can be made on the basis of differences so wholly unsystematic as those of this table.

Compared with Europeans the negro has both a shorter and a narrower skull; it is shorter but broader than the ancient Egyptians, being in length and breadth very like the modern Copt (♂: $L = 177{\cdot}0$, ♂$B = 136{\cdot}8$, ♀: $L = 171{\cdot}5$, ♀$B = 131{\cdot}1$). On the other hand the negro skull is higher than most European skulls, thus we have:

	Height (H)		$100\ H/L$	
	♂	♀	♂	♀
Aino	139·5	135	75·6	76·7
Copt	137	130	77·5	76·1
Gaboon, 1880	137	132	75·0	76·8
Gaboon, 1864	135	130	75·5	75·7
Northern Negro ...	135	—	74·2	—
Congo	134	128	75·4	74·6
Naqada	135	129·5	73·3	73·2
Bavarian	134	128	74·2	73·9
English {Whitechapel ...	132	125	70·0	69·1
Moorfields ...	130	124	68·4	67·2
Würtemberger	131	126	73·0	73·2
French	131	—	72·6	—

In the feature therefore of height of head the negro excels the European, and resembles both in height and cephalic height index a primitive race like the Aino, or even the modern Copt ($H = 137$ and $100\ H/L = 77{\cdot}5$ for males!). This excessive height of the negro skull is maintained in all Shrubsall's series, Zulu 138, Angoni 137, Kaffir 137, although their great length reduces the height index to 75, 74 and 72 respectively. Our North African negroes give 135 and 74·2,—all these for males,—while the mixed series G^{bis} gives 135 and 72·8.

It is worth investigating whether this greater height of the negro skull is above or below the auricular passages. Unfortunately the auricular height has not been determined for Shrubsall's series or our Gaboon series, and is probably 5 mm. too great in Ranke's Bavarian series, which were I expect measured from the *centre line* of the auricular orifices (see *Biometrika*, Vol. VIII. p. 137). We have the results given in the table on the following page.

It will be seen that the Congo crania agree very closely with the North African negroes and that both do not exceed in auricular height the European group. The exaggerated height of the negro skull is therefore associated with the portion of the skull between the basion and the auricular axis. This is well brought out by taking the index formed of the ratio of auricular to total height, when it is seen that the Europeans have a greater development above the auricular

	Auricular Height		Index 100 OH/H	
	♂	♀	♂	♀
Aino	119	115	85·5	85·1
Copt	116	109	84·6	83·9
Gaboon, 1880	—	—	—	—
Gaboon, 1864	—	—	—	—
Congo	114	109	85·1	85·5
Northern Negro ...	115	—	85·0	—
Naqada	115·5	113	85·5	87·4
Bavarian	115·5 ?*	109 ?*	86·5 (?)*	85·3 (?)*
English Whitechapel ...	115	109	86·8	87·6
Moorfields ...	114	109	87·7	88·5
Würtemberger	115	112	87·8	89·2
French	113	—	86·4	—

axis, while the negroes most closely resemble the primitive Aino, and the Copt alone stands lower than they do in the relative proportion of the upper height. It must not, however, be overlooked that while European crania are in breadth and length notably superior to the negro of the Congo-Gaboon type, and quite significantly superior to the Zulu-Kaffir groups, they are not superior in auricular height. The negro is just as high from auricular passage to the vertex; the largeness of the index 100 OH/H is not due to a development of the upper portion of the European's skull; it appears due to a lessening of the basal part of his skull.

Skull-Base: To test this we may compare the basi-nasal length LB of the skull in negro and other races. We have

	Basi-nasal length (LB)		Ratio L/LB	
	♂	♀	♂	♀
Aino	105	100	176·2	176·7
Kaffir	105	—	181·5	—
Northern Negro ...	105	—	177·6	—
Zulu	103	—	180·8	—
Angoni	102	—	179·9	—
Gaboon, 1880	102	97	178·9	175·5
Gaboon, 1864	100	96	179·0	179·0
Copt	101	96	175·4	178·6
French	100	—	180·5	—
Bavarian	100	96	180·0	181·1
English Moorfields ...	98·5	96	192·0	191·2
Whitechapel ...	101·5	95	186·1	189·2
Naqada	99	95	186·4	187·1
Würtemberger	98·5	94	182·0	184·5
Congo	98	93	181·1	183·7

---

* OH from Ranke's measurements is 120·75 for males and 114·17 for females, giving for the indices 90·2 and 89·2. I have deducted 5 mm. from his values, believing that he has measured from centre of auricular passages, and in accordance with my observation on actual sections, see *Biometrika*, Vol. VIII. p. 137.

It is of interest to see that for the base of the skull the Congo crania appear sensibly differentiated from the other negro crania. Although the Kaffir and Zulu crania are long and give a ratio of maximum length to skull basis equal to that of the French or Bavarian, we see that on the whole the European development has been in the direction of a reduced skull basis, and much increased cranial length.

Arcs: If we now turn to the tape measurements we should expect little difference between European and negro in the transverse arc (Q)*, but marked differences in the sagittal arc (S) and the horizontal circumference (U). The following table shows results:

Cranial Arcs.

	MALE				FEMALE			
	$Q+S+U$	Q	S	U	$Q+S+U$	Q	S	U
Aino	1224	328	373	523	1180	317	361	502
Bavarian	1219	330	365	524	1173	319	353	501
Kaffir	1215	307	381	527	—	—	—	—
English, Moorfields...	1211	305	379	527	1172	293	366	513
English, Whitechapel	1209	308	377	524	1161	294	363	504
Würtemberger ...	1209	324	367	518	1174	315	358	501
Zulu	1199	307	372	520	—	—	—	—
French	1196	312	366	518	—	—	—	—
Gaboon, 1880 ...	1192	312	371	509	1127	298	348	481
Naqada	1188	304	373	511	1154	296	364	494
Angoni	1185	305	369	511	—	—	—	—
Northern Negroes ...	1183	307	368	508	—	—	—	—
Copts...	1179	311	366	502	1133	296	352	485
Congo	1177	311	362	504	1137	298	350	489
Gaboon, 1864 ...	1175	310	364	501	1127	296	350	481

It will be seen that the Germans (Bavarians and Würtemberger) head the list for the transverse arc (Q), but while the negroes in this arc excel the English and are close to the French, yet with the exception of the Kaffirs and Zulus in the sagittal and horizontal circumferences they are behind the Europeans. The Congo and the longer Gaboon series of 1864 are at the bottom of the list, and are strikingly alike and also similar as in other characters to the modern Copts. The Angonis and the Northern negroes are very close together and fall between the Naqada and the Copts. It is the Kaffir and Zulu who stand out with a markedly bigger skull, and it is the more to be regretted that we have not data

---

* Because the negro cranium is not deficient in vertical height, but markedly so in breadth and length.

for the capacities of these races*; they may like the Aino have a very considerable capacity, or the bone of the cranium may be thicker than that of Europeans.

Another extremely interesting point is a comparison of the development of the various sections of the sagittal arc. We can unfortunately only compare the Moorfields English with the Congo and Gaboon series, where alone these measurements have been made. We have :

	Male				Female			
	English	Congo	Gaboon, 1864	Gaboon, 1880	English	Congo	Gaboon, 1864	Gaboon, 1880
Nasion to Bregma...	129·5	125·5	126·5	129·5	125·5	121·0	121·5	121·0
Bregma to Lambda...	128·5	128·5	128·5	127·0	123·5	120·0	123·0	120·5
Lambda to Opisthion	120·5	107·5	109·0	114·0	117·0	109·5	106·0	106·5
Chord of latter ...	98·0	95·0	96·0	100·0	96·0	96·0	93·0	94·0
Occipital Index	58·5	68·8	68·3	66·3	59·2	67·0	69·2	69·4

Now the peculiarity of these results is that the English have only slightly more arcual length between nasion and bregma or bregma and lambda than the three negro groups, but the arc from lambda to opisthion is much larger in the English skull; that is to say the occipital part is much more markedly developed. The measure of this is well seen in the occipital index, the ratio of the radius of curvature of the occipital arc to the chord from lambda to opisthion, a *low* occipital index meaning a well-developed occiput. Clearly the occiput is far more developed in the English than in the negro crania—the difference in the occipital indices being very significant.

In the upper face height $G'H$, there is little to differentiate Negro and European, for though the Germans (Würtemberger and Bavarians) stand at the top, the English and French (68 to 70) are almost identical with Zulus, Kaffirs and Angonis. Only the Congo and Gaboon crania come at the bottom, following closely the Naqada†.

---

* The known capacities (in cm^3.) are as follows:

	♂	♀		♂	♀
Bavarian	1504	1336	Naqada	1381	1288
Würtemberger ...	1494	1337	Gaboon, 1880 ...	1447	1240
English, Whitechapel	1477	1300	Gaboon, 1864 ...	1381	1232
English, Moorfields ...	1474	1365	Copts	1356	1214
French	1473	—	Northern Negro ...	1348	—
Aino	1462	1308	Congo	1344	1206

These results place the European 100 cms. above the negro who is comparable with the prehistoric and modern Egyptians. But as I have said we cannot generalise until the Kaffir and Zulu capacities are found.

† The same remarks apply to the bi-zygomatic breadth, J, which appears in no way as an essentially negroid character.

A more striking comparison may be made between the upper face indices which run as follows:

Upper Face Index, 100 *G′H/GB.*

	♂	♀		♂	♀
Würtemberger ...	78·1	74·1	Angoni	72·2	—
English, Whitechapel	76·5	77·9	Copts	72·1	68·5
Bavarian	74·5	74·4	Naqada	70·6	70·4
French	74·2	—	Gaboon, 1864	69·7	69·3
Kaffir	74·1	—	Gaboon, 1880	68·9	68·2
English, Moorfields ...	72·8	73·6	Aino	68·4	67·7
Theban Mummies ...	72·3	72·3	Northern Negro ...	68·0	—
Zulu	72·2	—	Congo	66·0	64·0

It appears from this list that no modern European race occurs in the second half, but that Zulu and Kaffir appear in the first half*. It is clear that on the whole Europeans have a relatively longer upper face than the negro races.

Another marked feature of the face, its upper prognathism, is measured by the profile angle *P*. Unluckily we have but few records of this for the negro races, but they suffice to show how markedly it differentiates their crania from the modern European, and even from the prehistoric as well as modern Egyptian. We have:

Profile Angle, P.

	♂	♀		♂	♀
Bavarian	89°·1	88°·8	Naqada	84°·4	84°·5
English, Whitechapel	86°·1	87°·1	Copts	84°·2	?
French	86°·5	—	Long Barrow	83°·0	?
Würtemberger ...	86°·4	84°·9	Congo	82°·8	81°·1
Theban Mummies ...	85°·5	85°·6	Aino	82°·0	81°·0
English, Moorfields ...	84°·5	84°·5	Northern Negroes ..	82°·0	—

Thus the prognathous character of the negro's profile is only equalled or excelled by that of the primitive Aino.

It would seem unusual to pass over the discussion of the cephalic indices and yet it is doubtful how far they are really indicative of important racial differences. Thus the Gaboon 1864 males and the Zulus differ but slightly in the height-length index, but the value in one case is produced by the head being unusually short and in the other case by its being abnormally high, relatively, to the majority of human races.

Now it would puzzle any one to take much of profit out of this list as it stands! It represents no clear order in the scale of civilisation nor in that of

---

* The position of the Kaffirs and Zulus may be due to personal equation in determining the alveolar point. Dr Shrubsall appears to get larger values than we do for *G′H*.

Cephalic Indices.

	MALE		FEMALE	
	$100B/L$	$100H/L$	$100B/L$	$100H/L$
Bavarians	83·2	74·2 (+9·0)	83·1	73·9 (+9·2)
Würtemberger ...	82·5	73·0 (+9·5)	82·8	73·2 (+9·6)
French	79·8	72·6 (+7·2)	—	— —
Congo	78·0	75·4 (+2·6)	76·5	74·6 (−1·9)
Copts	77·3	77·5 (−0·2)	76·6	76·1 (+0·5)
Aino	76·5	75·6 (−0·1)	77·6	76·7 (+0·9)
Gaboon, 1880 ...	75·9	75·0 (+0·9)	76·4	76·8 (−0·4)
Gaboon, 1864 ...	75·5	75·5 (0·0)	76·2	75·7 (+0·5)
English, Moorfields ...	75·5	68·4 (+7·1)	75·1	67·2 (+7·9)
Theban Mummies ...	75·1	74·7 (+0·4)	76·5	74·7 (+1·8)
English, Whitechapel	74·3	70·1 (+4·2)	74·7	69·1 (+5·6)
Zulus	74·3	75·0 (−0·7)	—	— —
Angonis	73·1	74·3 (−1·2)	—	— —
Naqadas	73·0	73·3 (−0·3)	74·2	73·2 (+1·0)
Northern Negroes ...	72·9	74·2 (−1·3)	—	— —
Kaffirs...	72·0	72·0 (0·0)	—	— —

racial relationship. But quite a different light is thrown on the whole matter, if we take as our character the index $100(B-H)/L$, i.e. the difference of the two cephalic indices as shown by the numbers in brackets; we then have the following arrangement:

Würtemberg Women	+9·6
Würtemberg Men	+9·5
Bavarian Women	+9·2
Bavarian Men	+9·0
English (Moorfields) Women ...	+7·9
French Men	+7·2
English (Moorfields) Men ...	+7·1
English (Whitechapel) Women	+5·6
English (Whitechapel) Men ...	+4·2
Congo Men	+2·6
Long Barrow Women... ...	+2·3
Long Barrow Men	+2·2
Theban Egyptian Women ...	+1·8
Naqada Women	+1·0
Aino Women	+0·9
Gaboon Men, 1880	+0·9
Coptic Women...	+0·5
Gaboon Women, 1864 ...	+0·5
Theban Egyptian Men ...	+0·4
Gaboon Men, 1864	+0·0
Kaffir Men	+0·0
Aino Men	−0·1
Coptic Men	−0·2
Naqada Men	−0·3
Gaboon Women, 1880 ...	−0·4
Zulu Men	−0·7
Angoni Men	−1·2
Northern Negro Men ...	−1·3
Congo Women	−1·9

It is at once seen that this list is far more significant than the previous one; the modern European races stand at one end of the scale, the negroes at the other and the prehistoric and primitive races in the centre. The women as a rule stand above the men; probably, not because the numerator $B-H$ is for women relatively greater, but because L owing to the development of the superciliary ridges is relatively greater in man. On this account when this index has been calculated for a larger number of races, it may be desirable to keep the two sexes in separate lists. That there is something very anomalous in the position of the Congo males is clear; not only is this one of the two exceptions to the rule that the female stands above the male in the list, but the highest negro male series is associated with a female series at the very bottom of the list. It should however be noted that the other case of interchanged sex order occurs in the 1864 Gaboon series, a series very closely allied to the Congo crania. The affinity already noted between the prehistoric, historic and modern Egyptians and the Gaboon Congo crania is also emphasised in this series. Modern advanced races of man tend to a considerable excess of cranial breadth over height, primitive and prehistoric races like the Naqada and Aino to an equality of these diameters; the negroid races tend to fall below equality and exhibit an excess of cranial height over breadth. The index $100\,(B-H)/L$ may be used to measure this characteristic, and if it has not yet been given a name—and I am not aware that it has—might be called the *acroplatic* index.

Of the measures of the mandible we can say but little, because so few comparative measures exist. Macdonell gives measures of the Whitechapel mandibles, but was unable to sex them satisfactorily, so that they are at present almost useless for comparative purposes*. Shrubsall gives condylar and angle widths for 19 Kaffir mandibles. Benington provides four characters for the two Gaboon series; there were no mandibles with the Congo crania. All we can do is to put against these negro mandible measurements those of Fawcett on the prehistoric Egyptians and Koganei on the Aino. We have the following results:

	W_1		W_2		h		f	
	♂	♀	♂	♀	♂	♀	♂	♀
Aino	102·0	95·5	—	—	34·0	32·5	—	—
Naqada	110·5	106·5	93·5	87·5	33·0	31·5	44·5	43·0
Gaboon, 1864... ...	112·5	110·0	94·0	90·0	32·5	30·5	44·5	45·5
Gaboon, 1880... ...	121·0	106·0	99·0	90·0	31·5	28·0	47·5	46·5
Northern Negro ...	117·0	—	97·0	—	—	—	—	—
Kaffir	118·5	—	96·0	—	—	—	—	—
English, Whitechapel	113·0		95·5		31·0		43·5	

This table shows that in every respect (W_1, W_2 and f) the negro mandible is broader than that of either Aino or prehistoric Egyptian, but that he has a less depth (h) of mandible, i.e., crudely described, less chin.

Another interesting index in the case of the negro is the alveolar index, or the ratio of the profile length to the base of the skull; it is, like the profile angle, a partial measure of the prognathism.

---

* See *Biometrika*, Vol. VIII. p. 213.

Alveolar (*or Gnathic*) *Index*, 100 GL/LB[*].

	♂	♀		♂	♀
English (Whitechapel)	94·4	94·8	Aino	99·5	99·8
Scottish[†]	94·9	94·9	Kaffir	100·1	—
Theban Mummies ...	95·2	96·5	Zulu	100·1	—
Naqada	95·4	95·9	Gaboon, 1864... ...	100·2	102·7
English (Moorfields)...	96·4	96·1	Gaboon, 1880... ...	100·8	100·8
Copts	96·6	98·9	Angoni	101·0	—
Congo	98·2	100·6	Northern Negro ...	102·3	—
Long Barrow... ...	99·4	99·9			

It is regretable that we have not the measurements of any but British crania among the European races. The table brings out, however, several points: the female sex tends to have a higher alveolar index than the male. The Egyptians as represented by prehistoric, historic and modern (Coptic) crania are very sensibly differentiated from the bulk of the negro; the one exception is the Congo male series, the anomalous character of which is again emphasised by the marked difference of male and female indices.

There are few determinations of the *foraminal index*. Macdonell gives it as 84·2 for English males (Moorfields) and 84·5 for English females. Schuster's values are 77·6 for Long Barrow males and 87·5 for females, the numbers being only 11 and 6 crania respectively[‡]. The Congo crania give: males 83·5, females 81·8; Gaboon, 1864, give males 83·9, females 81·1; Gaboon, 1880, give: males 84·3, females 84·7; Zulu males 80·8, Angoni males 81·3, Kaffir males 80·1. These numbers tend to show that the foramen magnum is somewhat more oval in the southern negroes than in the Congo-Gaboon series, the latter of which approaches close to the English value. But more data are requisite before the matter can be discussed adequately. As far as the material reaches, it is not inconsistent with the Congo-Gaboon crania standing closer to the European than the Zulu-Kaffir groups do. So far as the limited evidence goes, the form of the foramen is not a markedly differential racial characteristic.

Passing to the *palate*, we should, judging from results on the mandible and the negro prognathism, expect to find G_1 and G_2 both greater than in European crania and thus possibly not much differentiation in the palate index. But as the Biometrical School has several times indicated, it is very difficult to be certain that two different measurers of the palate take exactly the same measurements, and there are serious pitfalls likely to arise from personal equation and a lack of standardisation. The following table gives the best results available:

---

[*] Determined from means of GL and LB.

[†] See *Biometrika*, Vol. III. p. 242.

[‡] Mean of 17 crania without regard to sex =81·1.

Measurements of the Palate.

	G_1		G_2		$100G_2/G_1$	
	♂	♀	♂	♀	♂	♀
Congo	50·3	50·2	38·9	35·9	76·8	70·8
Gaboon, 1864... ...	52·1	50·1	39·5	37·4	76·1	74·8
Northern Negroes ...	53·1	—	39·7	—	74·8	—
English (Moorfields)...	53·0	50·0	39·7	37·0	74·9	74·1
Bavarian	44·3?	42·2?	33·2?	32·1?	74·4?	76·0?
Würtemberger ...	51·7	50·2	38·1	37·2	74·2	74·6
French	50·2	—	37·2	—	74·1	—
Zulu	52·0	—	37·9	—	73·1	—
Theban Mummies ...	56·2	52·1	40·8	38·7	72·6	74·3
Aino	53·0	51·4	38·2	37·4	72·1	72·8
Naqada	55·8	53·9	40·3	38·9	71·9	72·3
Kaffir	52·1	—	37·2	—	71·6	—
English (Whitechapel)	53·2	49·1	36·8	35·2	69·2	71·2
Angoni	51·9	—	35·8	—	69·1	—

Some explanation is requisite as to the manner in which this table has been reached. G_1 is defined in the Frankfurt Concordat as the palate length measured from point of the *spina nasalis posterior* to the inner wall of the alveolar rim between the mesial incisors. The inconvenience of this definition is obvious, the alveolar rim is often worn away between the mesial incisors and the spine is often broken off so that the measurement cannot be made. In the series of the above table attempts were made to measure G_1 as thus defined in the Naqada, Congo and Gaboon 1864 series*. The measurements themselves suggest that it was actually used in determining the Würtemberger, Aino, Zulu, Kaffir and Angoni series. Macdonell in his two English series, which were much worn, measured his G_1 from an imaginary surface tangential to the inner alveolar surfaces of the mesial incisors, as suggested by E. Schmidt (see *Biometrika*, Vol. I. p. 429). This for the Naqada crania made a difference of 4·9 and 4·0 mm. for ♂ and ♀ respectively. As the best approximation available these have been added to Macdonell's values of G_1 for the two English series. With a view of using crania with broken spines, Schmidt suggests that the palate shall be measured from the base of the spine. This appears to have been done in the French and Northern Negro series. The average length of the spine in the European male (English) is 3·6 and in the Negro male (Gaboon, 1864) 3·7 mm.; these values have therefore been added respectively to the French and Northern negro series. In the Theban mummies as already indicated (*Biometrika*, Vol. I. p. 430) both terminations of the palate length have been modified in Schmidt's sense and accordingly (*loc. cit.*) 9·9 mm. has to be added to G_1 in the male and 8·0 mm. in the female. Finally the Bavarian measurements seem to have been made *sui generis* and G_2 is clearly

---

* The Gaboon 1880 series only provided seven male and nine female crania for the palate index and has been disregarded.

not our measurement. As it cannot be corrected, it is idle to correct G_1; and by a lucky chance the index fits well for the male with what we might anticipate.

The table is most hypothetical and its chief service must be to emphasise the need of precise standardisation of the palate measurements. As it stands, it may suffice to show that possibly in palate measurements also the Congo-Gaboon negro is differentiated from the Kaffir-Zulu group. The general want of systematic differentiation between negro and European for palate measurements may either prove that the palate is not a markedly racial character, or indicate that our hypothetical corrections have screened the true values.

Orbits. The orbit is another character which is distinctly of racial importance, but which it is very hard to discuss. There is not the least doubt that in facial appreciation orbital form is a most important factor, and the orbital index ought to be most suggestive in its bearing on racial differences. Unfortunately one component of it, the breadth of the orbit, is almost as vague as the length of the palate. This point has been discussed by both Fawcett and Macdonell*, and a few additional remarks may be made here. It is probable that most writers have taken the dacryon as the start on the nasal side of the greatest breadth. There are great advantages in simplicity of measurement if this be done, and possibly it will be best to standardise the measurement in this manner, but when it is done it must be admitted at once that we are only getting an artificial orbital index. The physiognomical effect of the orbit is far better expressed by the original words of the Frankfurt Concordat—the greatest breadth from side to side of orbit wherever found, measuring from margin to margin—, only as there happens to be no margin on the nasal side, we are thrust back on the alternative of either giving up the definition, or defining what is meant by the 'margin' on the nasal side. Those who give up the margin use the dacryon or, occasionally, a purely arbitrary point leading to endless personal equation in the measurement, and to perfectly idle results, as they can be compared with no other series of measurements. If we retain the idea of margin we must put in an imaginary margin on the nasal side, and measure the greatest diameter from this. Two possibilities occurred to the biometric workers:

(i) To produce with a pencil the margin from the last sensible points of the upper and lower orbital rims as a curve on the nasal wall, starting without breach of continuity with the curvatures indicated by the upper and lower orbital rims, so that these rims meet again without breach of continuity on the nasal wall. The method is not difficult in practice and does not admit of as much personal freedom as the description may suggest to the reader. It was adopted by C. D. Fawcett in her measurement of the Naqada crania. I will term it the curvature method.

(ii) To join with a pencil line the last sensible points on the upper and lower orbital rims, the line to be the curve of shortest length on the nasal wall between

---

* *Biometrika*, Vol. I. pp. 430, 431 and Vol. III. p. 201.

these two points, i.e. the geodesic. This 'false' margin gives usually abrupt changes of curvature at the terminals of the upper and lower orbital rims. I will term this the 'geodesic' method.

In either (i) or (ii),—which endeavour to maintain the Frankfurt Concordat idea of greatest breadth—the greatest measurement was to be taken, wherever found, an end being on the imaginary margin thus defined. While Fawcett used (i), Macdonell tells us he used (ii)*. He declares that in practice it is identical with following the curvature of the lower orbital rim to its furthest inward point,—but this is limited again by the statement that he does not cross the naso-frontal suture. This really appears to be not (ii) but a third definition, namely to produce the lower orbital rim from its last sensible point *without change of curvature* up to the naso-frontal suture and to measure from some point on this production. I admit that in many cases there is little change of curvature at the terminal of the lower orbital rim when the geodesic is drawn as described in (ii), but I do not think Macdonell's method is absolutely the same as Fawcett's geodesic method especially for male orbits, and thus we have new difficulties in forming racial comparisons.

In Fawcett's paper the differences obtained in the orbital breadth by using (i) and (ii) for one series of crania are given as 1·9 mm. for males and 1·6 mm. for females. What difference, however, is there between the curvature method and the lacrymal point method? The extraordinary uniformity of the breadth measured by the lacrymal point method in prehistoric Egyptians, Theban mummies and modern Copts (see *Biometrika*, Vol. I. pp. 426, 427) shows us that we may safely take for Egyptians, prehistoric to modern, the difference between the curvature method and the lacrymal point method as 4·8 mm. for males and 4·3 mm. for females. Hence to reduce Fawcett and Macdonell's measurements to a lacrymal point measurement we must roughly subtract 4·8 mm. and 4·3 mm. from Fawcett's values and 2·9 mm. and 2·7 mm. from Macdonell's values; the orbital index will then have to be taken as a ratio of means. I believe that this is the best that can be done; I should have preferred to try and get a true orbital index as defined in the Frankfurt Concordat, but there are too many series to modify and too little is known of how they have really been measured.

Dr Benington followed Macdonell in his orbital measurements and accordingly his maximum breadths O_1 have to be reduced by 2·9 mm. and 2·7 mm. respectively for male and female. I have reduced his averages by these and recalculated the orbital indices from the modified averages.

Before we look at the table thus modified we may compare the English (Macdonell) and Negro (Benington) results which were presumably measured by practically the same methods:

---

* *Biometrika*, Vol. III. p. 201.

	O_1		O_2		$100\ O_1/O_2$	
	♂	♀	♂	♀	♂	♀
English (Moorfields)...	42·0	40·9	32·8	32·7	77·9	80·0
English (Whitechapel)	43·0	41·1	33·4	33·7	77·8	82·1
Congo	40·8	40·9	34·6	33·5	84·9	81·9
Gaboon, 1864... ...	41·5	39·6	35·0	33·7	84·6	85·2
Gaboon, 1880... ...	42·1	40·1	35·2	34·1	83·7	85·2

These results suggest that the Congo-Gaboon type of negro is certainly not broader in the orbit than the European English, but the orbit, especially in the male, appears to be sensibly higher and thus the general appearance of the orbit will be rounder. I think this confirms the impression which arises from photographs of these negroes. Possibly the greater development of the superciliary ridges in the English male will account for some of the difference, which is more marked in the males than in the females.

In the following table the best has been done to correct divergent measurements. The one measurement which is fairly reliable is the height of the orbit and it is extraordinary how little is the racial variation. In the female the racial differences do not cover more than about 2 mm. and in the male about 3 mm. Even the breadth of orbit has really a small range, about 3 mm. for either sex, if we can trust our corrected values. An appreciation of this limited variability will show how essential it is to restrict all personal equation within a fraction of a millimetre, and how hopeless it is to compare measurements which are not exactly standardised.

Measurements on the Orbit. (Corrected.)

	O_1		O_2		$100\ O_1/O_2$	
	♂	♀	♂	♀	♂	♀
Congo	37·9	38·2	34·6	33·5	91·3	87·7
Gaboon, 1864 ...	38·6	36·9	35·0	33·7	90·9	91·3
Gaboon, 1880 ...	39·2	37·4	35·2	34·1	89·8	91·9
Angoni	38·0	—	33·3	—	88·1	—
Northern Negro ...	39·6	—	34·2	—	87·2	—
French	38·2	—	33·3	—	87·3	—
Theban Mummies ...	38·1	37·0	33·0	32·3	86·5	89·8
Würtemberger ...	39·8	38·0	34·1	32·2	85·9	85·1
Zulu	38·7	—	33·2	—	85·8	—
Kaffir	39·8	—	34·0	—	85·5	—
Copts	38·2	36·9	32·5	32·5	85·2	86·7
Aino	40·9	39·8	34·9	33·9	85·3	85·2
Naqada	38·1	37·2	32·3	32·0	84·8	86·0
Bavarian	39·9	38·3	33·7	33·2	84·5	86·6
English (Moorfields)...	39·1	38·2	32·8	32·7	83·9	85·6
English (Whitechapel)	40·1	38·4	33·4	33·7	83·3	87·9

The feeling that remains on my mind after study of this list is the sad pity of it,—it is so futile!—one feels quite certain that the French and the Würtemberger are displaced, and that a very little standardisation of the breadth measurement would have provided us with as instructive a table as those for the acroplatic and the gnathic indices. The table represents the measurement of hundreds of orbits and hours and hours of tedious work, and the result for want of careful standardisation is almost worthless. No one is likely to believe that the French and a considerable section of the German population have as round an orbit as Zulu and Kaffir. The table, were it above suspicion, would probably show us the Congo-Gaboon group at one end, the European races at the other, the Kaffir-Zulu group following the Congo-Gaboon negroes and separated from the Europeans by the Egyptian and primitive races. But as it stands, it tells us nothing more than our previous table—that of our few standardised measurements—namely, that the Congo-Gaboon group and the English are at opposite ends of the orbital scale. The table provides one great and important lesson: the urgent need for effective standardisation in craniometric work. If craniologists, instead of trying to do something a little bit different from their colleagues, would only strive to do exactly the same thing, they would double the speed at which their science would develop. At present all we can say is that all work hitherto done on palate and orbit is practically worthless for interracial comparisons!

Lastly, we turn to the nose where we have the height, breadth and nasal index for a number of races and the mesodacryal chord and arc, as well as the mesodacryal indices for a few races.

Nasal Characters. (Table A.)

	Nasal Height		Nasal Breadth		Nasal Index	
	♂	♀	♂	♀	♂	♀
Zulus	47·2	—	27·3	—	58·1	—
Angonis	48·5	—	28·0	—	58·0	—
Kaffirs	48·6	—	28·0	—	56·3	—
Gaboon, 1880	47·9	45·3	27·1	25·7	56·5	56·9
Northern Negroes	49·5	—	27·3	—	55·4	—
Gaboon, 1864	48·2	45·1	26·6	24·7	55·3	54·9
Congo	47·2	45·0	26·0	25·0	55·2	55·8
Naqadas	48·9	46·7	25·1	24·3	51·1	52·3
Aino	50·5	47·4	25·6	24·7	50·7	50·5
Theban Mummies	50·6	48·4	25·3	24·2	50·1	50·2
Copts	50·7	45·3	24·9	24·6	49·4	54·6
Long Barrow	49·4	47·0	24·1	22·8	49·0	49·1
Bavarian	50·9	48·2	24·8	23·7	48·7	49·2
English (Moorfields)	50·4	48·0	24·0	23·4	47·6	48·7
English (Whitechapel)	51·2	48·7	24·3	23·2	47·6	47·8
Würtemberger	51·5	45·7	23·9	23·0	46·6	50·3
French	51·2	—	23·1	—	45·3	—
Scottish	52·4	48·7	23·2	23·2	44·5	44·1

Nasal Characters. (Table B.)

	Mesodacryal Chord		Mesodacryal Arc		Mesodacryal Index α		Mesodacryal Index β		Simotic Index	
	♂	♀	♂	♀	♂	♀	♀	♂	♀	♂
Gaboon, 1880	25·5	22·7	—	—	—	—	—	—	—	—
Congo	24·4	23·5	32·9	23·2	39·9	41·5	34·5	36·8	25·6	25·2
Gaboon, 1864	24·5	23·1	33·5	32·3	45·4	46·7	42·7	44·8	30·8	28·7
Zulus	24·3	—	—	—	—	—	—	—	—	—
Angoni	24·3	—	—	—	—	—	—	—	—	—
Kaffirs	24·1	—	—	—	—	—	—	—	—	—
Egyptian (Series E) ...	21·5	20·4	34·5	32·4	57·0	55·9	61·3	59·3	44·8	38·6
English (Whitechapel)	22·6	20·9	35·1	31·9	53·9	52·2	56·2	53·4	50·7	47·4

Taking in the first place Table A of nasal characters, we see that the high nasal index of the negro is due to two sources. It occurs chiefly because the breadth of the nose is markedly greater in Negroes than in Europeans, and partly because Europeans have a somewhat longer nose. The Congo-Gaboon group lies closer in breadth to the primitive and prehistoric group, than do the Zulu-Kaffirs. Here again it would seem that the Congo-Gaboon group is the link between the Southern (and Northern ?) negroes and the Egyptian type, just as on the other hand the Egyptians link up the prehistoric Naqadas with the Europeans. The marked gap, however, between the Congo-Gaboon and Egyptian series indicates that we are on different branches of the developmental tree.

The Congo-Gaboon crania are, however, nearer the Egyptian than the "full-blooded" negro crania are, while modern Europeans are more or less continuously linked up through the Nile valley and the Congo-Gaboon group with Kaffir and Zulu. Thus the nasal index appears to emphasise the same form of relationship as the gnathic and the acroplatic indices have already indicated.

Table B is unfortunately very defective. We have only just realised the importance of measurements on the nasal bridge. Unfortunately Dr Benington only measured the mesodacryal arc on the male Congo series, and we have reason to believe that the females were not measured by exactly the same convention. Accordingly Miss K. M. Ryley kindly undertook not only to re-measure the Congo, but to measure also the Gaboon 1864 series, and, for purposes of comparison, fifty crania of each sex of the Egyptian (E) series and the English (Whitechapel) series.

In dealing with the nose the straightforward characters to measure are chords and arcs, e.g. the mesodacryal chord and mesodacryal arc. We have already seen that a dacryal index can be formed = 100 (arc − chord)/chord, but this does not give a readily appreciable idea of the shape of the bridge of the nose. If we want to realise this a better measure is the ratio of the subtense to the chord. We might proceed to find this exactly as we have dealt with the occipital index from chord and arc of opisthion to lambda. But the subtense in that case is small relative to the chord and an approximate formula suffices. This is not true in the case of the nose, although more nearly true in the case of the Negro than in that of the European nose. For this reason a circular arc is by no means a good approximation to the nasal bridge. In fact in certain noses the subtense is more than half the chord and accordingly to use a circular arc supposes more than a semicircle! After an examination of moulds of the bridge of the nose, it seemed to me that the bridge of the nose could be fairly closely represented in section by the common catenary with axis in the median plane. We might thus pass from arc and chord to subtense and chord by a simple table without much trouble and with sufficient accuracy for practical purposes.

The equation to the common catenary is

$$y = \tfrac{1}{2}c\,(e^{+\frac{x}{c}} + e^{-\frac{x}{c}}),$$

c being the parameter.

Therefore $$R_1 = \text{arc/chord} = \tfrac{1}{2}\,(e^{+\frac{x}{c}} - e^{-\frac{x}{c}})\Big/\frac{x}{c}$$
$$= \sinh u/u,$$

if u be used to denote x/c.

$$R_2 = \text{subtense/chord} = (y - c)/2x$$
$$= R_1 \times \tfrac{1}{2}\tanh\tfrac{1}{2}u.$$

Hence if we use tables of the hyperbolic sine and cosine we can determine corresponding values of R_1 and R_2. A table of corresponding values is given on pp. 338—9*. $100\,(R_1 - 1)$ is the Dacryal Index β, and $100\ R_2$ the Dacryal Index α, and the table is entered by these indices α and β. In other words we have a means of passing from the measured arc and chord to a *theoretical* subtense. There is little doubt that the ratio of this theoretical subtense to the actual chord gives a fair approximation to the real shape of the bridge of the nose.

It will be seen at once that our mesodacryal indices α and β include more than the nasal bones, for the nasal processes of the superior maxillary bones also contribute. But from the standpoint of the physiognomy of the living it is more than doubtful whether it is the form of the nasal bones only which is concerned in our classification of the nasal bridge—a feature which most certainly plays a large part in racial appreciation. In order to judge fully of the differentiation in this character between Negro and European races we have also measured the nasal bones alone, and obtained the index introduced by Mérejkowsky, and which I venture to term the *Simotic Index*†. In this case the shortest horizontal chord

---

* Computed by Miss Julia Bell, M.A., from Gudermann's, Glaisher's and Newman's tables of the hyperbolic functions.

† C. de Mérejkowsky: "Sur un nouveau caractère anthropologique," *Bulletins de la Société d'Anthropologie*, T. v. Troisième série, Paris, 1882, pp. 293—304. Mérejkowsky calls this index "indice de la racine du nez."

of the nose (n) is measured from nasal suture to nasal suture, and also the minimum subtense (p), i.e. the line from this chord in the median plane perpendicular to the nasal bridge. The ratio $100p/n$ is the simotic index and measures the snubbiness or flatness of the nasal bones at the bridge. A special instrument for taking the measurements has been devised by Mérejkowsky himself and a simometer of this type was kindly lent to us by Professor Thane. The instrument does not strike one as mechanically efficient. The distances to be measured are extremely small, one reading in the differential measurement of the subtense being often a fraction of a millimetre, and the bearings for such fine measurements were quite inadequate. These we improved, but the addition of fine micrometer screws and an absolute not a differential determination of the subtense would be requisite for really accurate work. The results are given for what they are worth, and they may be useful in comparing the value of various rhinal indices.

The table on p. 318 brings together all the measurements made by Miss Ryley* and reduced by Miss Bell.

Now some very noteworthy results flow from this table. It will be evident that the Mesodacryal Index α has remarkably little variation within the race compared with either the Mesodacryal Index β or the Simotic Index. It therefore has some advantages in use over the two latter. It measures, however, a quite different character to the Simotic Index and we see at once that the racial order is not the same for simotic and mesodacryal measurements. That is to say an order based purely on the nasal bones may not be the same as one based on the entire bridge of the nose.

Judging first by the simotic index we have the following results:

(*a*) The Negro races have flatter nasal bones than Egyptian or European.

(*b*) There is a marked sexual difference, the women being in every case more platyrhinal than the men, the difference however being less marked in the negroid than European and Egyptian races.

(*c*) From the standpoint of variability the Negroes are far more variable than Egyptian or European.

From this view of the matter we are again confronted by the probability that it is the European who has been more stringently selected from a primitive stock than the Negro, and that in the nasal bones the female has retained rather more of the primitive character—nasal flatness—than the male.

---

* All the measurements on which the constants of this Table are based for the four races were taken by Miss Ryley. This was done because (i) the measurements are extremely delicate and it is desirable that they should be made by one person for all races investigated in order to avoid errors due to personal equation and (ii) because we feel convinced from our re-measurements that Dr Benington's mesodacryal arc, the "Tape Dacry" of his Tables I and II, has not been measured in the same manner for the Congo male and female crania—an interval of several months elapsed between the two sets of measurements made by Dr Benington and Miss Thomson respectively—and that the "Tape Dacry" column of Table II for the female Congo skulls must be cancelled.

Table of Biometric Constants of Nasal Characters showing their Mean Values and Variability in Four Races.

Character	Congo		Gaboon 1864		Egyptian (Series E)		English (Whitechapel)	
	♂	♀	♂	♀	♂	♀	♂	♀
Mesodacryal Chord: Mean ...	24·43± ·26	23·52± ·26	24·50± ·23	23·09± ·30	21·48± ·19	20·36± ·16	22·57± ·19	20·89± ·22
,, ,, S. D. ...	2·67± ·18	2·09± ·19	2·38± ·16	2·87± ·21	2·01± ·14	1·69± ·11	1·97± ·13	2·29± ·15
,, ,, C. of V....	10·92± ·75	8·89± ·79	9·70± ·66	12·43± ·92	9·35± ·69	8·30± ·61	8·71± ·64	10·96± ·82
Mesodacryal Arc: Mean ...	32·85± ·39	32·16± ·40	33·49± ·28	32·29± ·41	34·52± ·36	32·36± ·24	35·13± ·32	31·90± ·32
,, ,, S. D. ...	4·08± ·28	3·21± ·28	2·91± ·20	3·78± ·29	3·77± ·25	2·53± ·17	3·34± ·23	3·31± ·22
,, ,, C. of V. ...	12·43± ·86	10·00± ·89	8·68± ·59	11·72± ·91	10·91± ·81	7·81± ·57	9·50± ·70	10·38± ·77
Mesodacryal Index *a*: Mean ...	39·85± ·61	41·48± ·71	45·41± ·58	46·71± ·82	57·00± ·89	55·93± ·56	53·89± ·79	52·15± ·78
,, ,, S. D. ...	6·32± ·43	5·71± ·51	6·04± ·41	7·49± ·58	9·31± ·63	5·91± ·40	8·23± ·56	8·16± ·55
,, ,, C. of V.	15·85±1·11	13·75±1·24	13·30± ·91	16·03±1·27	16·34±1·13	10·56± ·72	15·28±1·05	15·65±1·08
Mesodacryal Index *β*: Mean ...	34·54± ·87	36·84±1·03	42·70± ·87	44·78±1·27	61·26±1·49	59·29± ·92	56·15±1·27	53·35±1·24
,, ,, S. D. ...	9·03± ·62	8·24± ·73	9·11± ·61	11·58± ·90	15·58±1·05	9·60± ·65	13·29± ·90	13·03± ·88
,, ,, C. of V.	26·13±1·90	22·37±2·08	21·34±1·50	25·87±2·13	25·43±1·82	16·20±1·11	23·67±1·64	24·42±1·70
Simotic Chord: Mean... ...	9·44± ·21	9·46± ·33	9·16± ·26	9·55± ·28	10·58± ·18	10·76± ·19	9·52± ·17	8·98± ·18
,, ,, S. D. ...	2·17± ·15	2·52± ·23	2·78± ·19	2·49± ·20	1·88± ·13	1·99± ·13	1·78± ·12	1·93± ·13
,, ,, C. of V. ...	23·05±1·67	26·64±2·61	30·31±2·22	26·08±2·18	17·77±1·24	18·51±1·29	18·72±1·31	21·53±1·52
Simotic Subtense: Mean ...	2·38± ·08	2·31± ·11	2·87± 11	2·68± ·10	4·67± ·10	4·11± ·09	4·74± ·11	4·14± ·07
,, ,, S. D. ...	·86± ·06	·86± ·08	1·14± ·08	·90± ·07	1·07± ·07	·90± ·06	1·13± ·08	·77± ·05
,, ,, C. of V. ...	36·03±2·78	37·38±3·88	39·73±3·11	33·59±2·92	22·83±1·62	21·94±1·55	23·79±1·69	18·54±1·29
Simotic Index: Mean ...	25·55± ·78	25·19±1·25	30·81± ·89	28·73± ·96	44·77± ·84	38·59± ·73	50·67±1·20	47·36± ·91
,, ,, S. D.	8·04± ·55	9·66± ·89	9·24± ·63	8·70± ·68	8·75± ·59	7·68± ·52	12·52± ·85	9·50± ·64
,, ,, C. of V. ...	31·46±2·37	38·34±4 00	30·00±2·22	30·27±2·58	19·55±1·37	19·91±1·40	24·72±1·77	20·06±1·41

If we turn to the factors on which the simotic index is based, the simotic chord and subtense, we note:

(*d*) That both are far more variable in the Negro than in the European or Egyptian; but

(*e*) That the simotic chord does not differ very widely in English and Negro, both being somewhat less than the Egyptian.

(*f*) That it is the subtense that is widely different in the Negro, its value being a little more than half that of European or Egyptian, while its relative variation is 35—40 °/₀ compared with their 18—24 °/₀—i.e. not quite double. Roughly therefore we may say that the nasal span is the same for all these races and the selection has been that of size of nasal bones covering the span from one nasal process of the superior maxilla to the other. There are indeed among the negro skulls several in which the simotic index could not be measured, because there were practically no nasal bones to measure, e.g. Gaboon 1864, Nos. 10, 11, and 81, and Congo 70 (cf. Remarks on folding sheets of measurements). The significance of this is considerable when we consider the nasal bones of the higher apes*.

Turning to the mesodacryal characters we see:

(*a*) That the mesodacryal chord is larger in the Negroes than in Egyptian or English series, i.e. from the physiognomic standpoint the orbits are farther apart. On the other hand for the mesodacryal arc, while the English and Egyptian males have a somewhat larger value than the Negroes, there is no sensible difference in the females. Thus from the standpoint of the nasal bridge as a whole, it is the interorbital distance, the mesodacryal chord, rather than the arc which is the essentially differentiating feature†. Roughly therefore what the European has gained in nasal bones, he must have lost in superior maxillary processes.

(*b*) The two mesodacryal indices amply suffice to differentiate markedly the Negro from the European or Egyptian crania, but the relatively small difference between European and Egyptian, unlike that of the simotic index, places the English nearer to the Negro than the Egyptian. This point is of considerable interest, because a large negroid admixture has been frequently asserted, without much basis than that of general impression, to exist in these and other series of Egyptian crania.

We see, however, that in both mesodacryal indices, in mesodacryal chord and in simotic chord the English crania stand nearer to the Negro than the Egyptian

---

* An investigation of the mesodacryal and simotic characters of these apes is now in progress. A female gorilla skull in the Laboratory possesses an almost similar absence of nasal bones at the bridge.

† A familiar illustration would be the suspension of the same lengths of clothes line between two sets of posts at different distances apart, the droop corresponding to the mesodacryal subtense would be greater for the nearer pair of posts.

does; and this inverted order has been observed in some other cranial features*. It is of course far from the rule generally, but it has some bearing on the question of whether the Egyptians really did mix freely with the adjacent negro populations. An examination of the three nasal indices given in this paper will show how the Egyptians and English lie far closer together than Negroes and Egyptians, and this although we are dealing only with the least negroid of the negro races!

(*c*) Another noteworthy feature is that while the simotic index is greater for the males than the females in all races, in the two negro races both mesodacryal indices are greater for the females; while for the English and Egyptians the males have higher indices. In other words the sexual order is *changed*. Taking the bridge of the nose from dacryon to dacryon the negro male is flatter than the female, but if this holds for primitive human types in general, then the male in the advanced races has been more intensely selected than the female, with the result that she is now slightly more flatnosed.

To sum up it appears to me:

(i) That both mesodacryal and simotic indices are markedly differential racial characters.

(ii) That the mesodacryal index α, besides being physically more comprehensible than the mesodacryal index β, has a less percentage range of variation within the individual race. It is easily found by aid of the table now issued.

(iii) That the fuller study of the bridge of the nose promises results of much value for their bearing on selective evolution.

(iv) That notwithstanding the mechanical defects of Mérejkowsky's instrument definite results of much interest can be obtained from it, and it would be quite easy to devise a better instrument†.

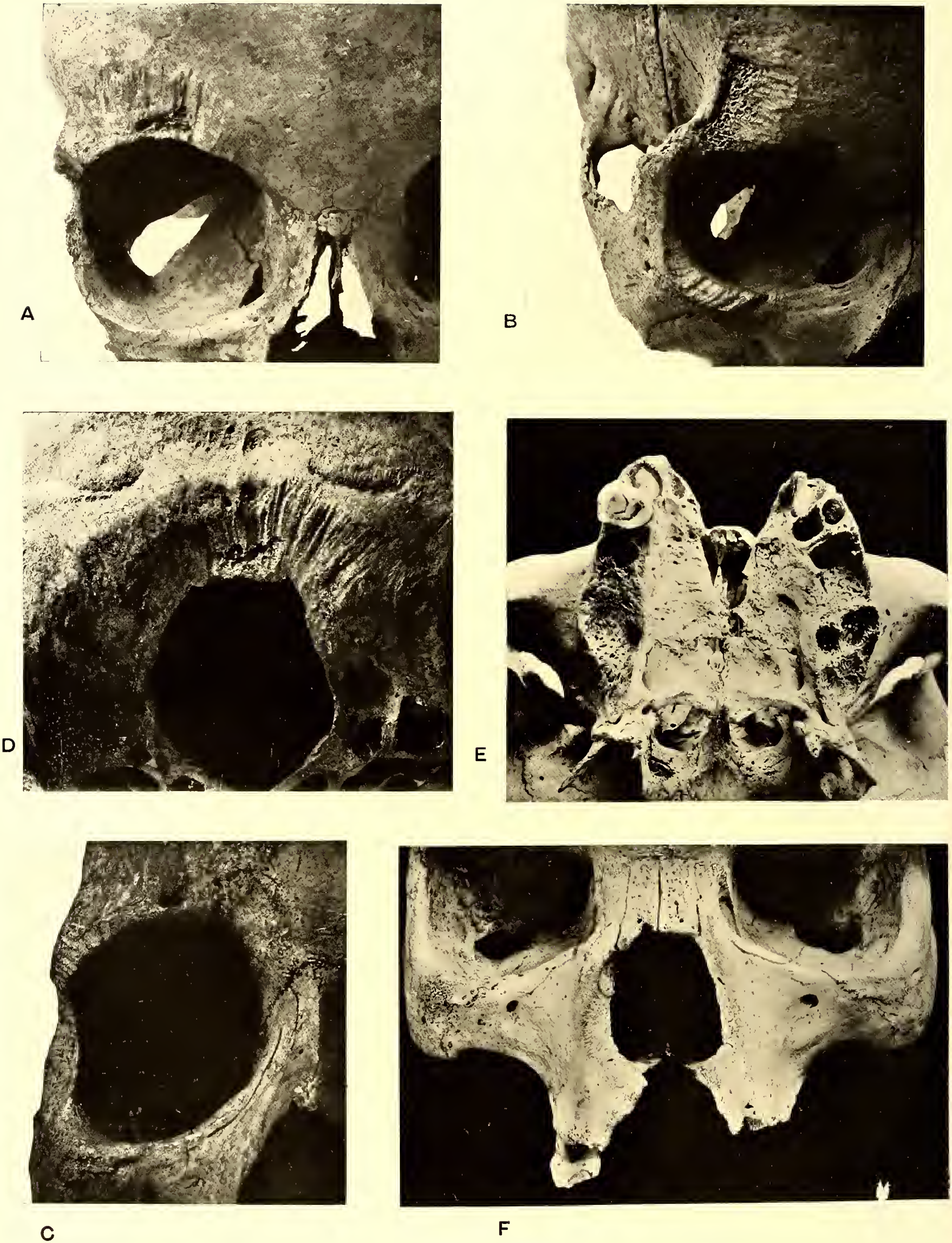
(v) That Mérejkowsky has undoubtedly led the way to a series of measurements which if carried out on a long enough series of good sized homogeneous groups would be extremely valuable for racial evolution.

With the nose I conclude our investigation of the differences between the crania of negroid and other races. The reader may question why other absolute lengths or indices have not been considered. The answer to this question is that in a number of cases they have been investigated but they led to no marked differences. In the bulk of these cases I do not believe that the absence of

---

* Cf. *Biometrika*, Vol. VIII. p. 136.

† Taking a rough model of the bridge of the nose in plasticene and constructing from it by aid of a small pair of dividers and a piece of paper the triangle whose base is the lacrymal diameter and sides the distances from the ends of this base to the nearest point on the median plane of the bridge a very good approach to the mesodacryal index α in the living subject can be made. Without any plasticene model at all the reconstruction of the triangle by three measurements on the skull gives quite closely both simotic and mesodacryal indices.



Fernand Vaz Crania.

A, B and C, filed or gnawed orbits. D, filed or gnawed foramen magnum. E and F, Cranium 82, 1864 Series. Cleft of palate and complete absence of praemaxilla.

differentiation is wholly real, indeed I think it may often be traced under the chaos produced by personal equation in measurement. A not uninteresting example of this is the Zygomatic Upper Face Index, or the ratio of the distance of the nasion from the alveolar point to the maximum zygomatic breadth ($100\ G'H/J$)—this measurement has been much approved by some craniologists. But its range is very small throughout the races of man, and personal equation both in the matter of the alveolar point, and in placing the callipers on the zygomatic arches brings chaos into the racial table. When the racial range is small even moderate personal equation produces serious effects. The following table will illustrate the point:

Zygomatic Upper Face Index, $100\ G'H/J$ (*found from means of absolute values*).

	♂	♀		♂	♀
Congo	50·1	48·5	Northern Negroes ...	52·5	—
Gaboon, 1880... ...	50·8	51·9	English, Moorfields ...	52·8	52·6
Aino	50·8	50·5	Sweden, Stone Age...	53·0	—
Gaboon, 1864... ...	51·5	52·1	Theban Mummies ...	53·6	54·4
Kaffirs...	51·5	—	Würtemberger ...	53·6	52·9
Reihengräber... ...	52·0	—	Naqada	53·8	56·3
Zulus	52·2	—	English, Whitechapel	53·8	54·8
Long Barrow ...	52·2	—	Scottish	54·4	55·1
French	52·3	—	Copts	54·6	52·6
Bavarian	52·4	52·9			

The difference between the two English series alone would suffice to carry the North African Negroes to a place fourth or fifth on the list right among their racial fellows. The fact that, only roughly, the female order is the same as the male shows that nothing is final in the list, but generally the appearance of the Negroes at one end of the scale and the Europeans at the other, with the prehistoric races towards the centre, indicates that with longer series and more careful standardisation the zygomatic upper face index might be of value. At present the racial range, under five points for males, is too small to indicate profitable results in the face of small series and large personal equation in measurement.

(5) *On the Variability of the Negro Skull.* It is of interest to consider whether there is less variation in the Negro than in the European skull. The first of the accompanying tables gives the coefficients of variation for the absolute measurements of the Congo and Gaboon 1864 series compared with Naqada, Aino, Theban mummy, English, Bavarian and other European data. The following table gives the standard deviations of the indices. When one examines the gaps in these tables one notices the vast amount of laborious arithmetic still waiting to be done on actually measured series! Such work is too often shirked by the craniologist, but yet it is absolutely needful if we are to determine the relative variability of the various races of man. Without a knowledge of such variability how can we determine the range within which selection can act or has acted? Taking first the table of coefficients of variation for the absolute measurements, we

have only introduced the French* and Scottish data to show what differences may be supposed to arise when the material is very heterogeneous. A careful study of this table shows (i) that the two Negro series are at least as variable as, perhaps on the whole rather more variable, than the prehistoric Naqada, (ii) compared with the English, it is difficult to say which series is decisively more variable. In regard to the second table of the standard deviations of indices, the conclusion must, I think, be that there is no single index in the Negro series of which we can say the variation is markedly less than in either European or primitive races. There is, however, one character in which the Negro differs markedly from the European, the occipital index; in this he appears to be at least three times as variable. No great stress can at present be laid on the point, because the occipital index has at present been ascertained for far too few series, but undoubtedly it is *suggestive*. The indication given would be that the European crania are a stringent selection with regard to occipital development, and that the negro stand closer to more variable and less developed primitive stock. As another marked negroid character we may examine the nasal index. There appears again to be a greater variability in the negroid group; it is, perhaps, less marked, but it is quite significant. Again suggestion, not proven hypothesis, arises if we look at the components of the nasal index, NH and NB of the table of absolute length variations. We see that the nasal height of the Negro is sensibly more variable than that of the English. There is no very great difference in the variability of nasal breadth. The suggestion is therefore indicated that the nasal differentiation of European and Negro is due to a selection of nasal height in the former, and not to a selection of nasal breadth in the latter. Thus, as in the case of the occipital index, the indications are towards the theory that the European rather than the Negro has been more stringently selected from an original stock†. Such indications, however, cannot reach the plane of probable hypotheses until far larger series of Negro crania have been dealt with and much fuller biometric reductions have been made on European and primitive race crania. At present all we can say is that there is no evidence whatever for the Negro having small variation, or for his being a stringently selected offshoot of a more widely varying primitive type. It is the European who appears to differ from him by the action of more stringent selection.

(6) *Special Crania and Anatomical Peculiarities.* As usual a careful examination was made for anatomical anomalies, and all such are recorded in the "Remarks" column of the folding plates of measurements. It is possible that externally visible anomalies do not occur so frequently in the Negro as in European races; the reason being that deformed infants have in many negro tribes been, until recently, promptly destroyed. However, a collection of photographs

---

* These are the Paris Catacomb crania and not the French crania of the previous tables, which were those of French prisoners of war dying in Germany.

† Pigmentation investigations—both as to hair, skin and eye-pigment—seem to support the suggestion that the white races could have arisen from a xanthous variant of a dark race, and there is no good evidence at present for any really dark skinned variation having arisen in a white race.

Variability of Absolute Lengths. Coefficients of Variation.

	Congo		Gaboon, 1864		Naqada		Theban Mummies		Aino		English, Whitechapel		English, Moorfields		Bavarian		French*		Scottish †	
Organ ...	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
Capacity	9·4	8·9	7·8	10·3	7·7	6·9	8·1	8·2	6·9	6·8	8·3	8·7	9·0	8·3	7·7	8·2	7·4	7·1	—	—
F ...	3·7	3·5	3·1	3·6	3·3	3·2	—	—	—	—	3·3	3·5	3·2	3·4	—	—	—	—	—	—
L ...	3·8	3·5	3·1	3·5	3·2	3·1	—	—	3·2	3·1	3·3	3·5	3·0	3·3	3·4	3·6	3·8	3·7	4·0	4·0
B ...	3·6	4·6	2·5	3·6	3·3	3·5	—	—	2·8	2·7	3·8	3·5	3·7	3·8	3·9	3·4	4·2	3·7	4·1	3·7
B′ ...	4·0	4·0	3·3	4·3	5·3	4·5	—	—	—	—	4·3	4·6	4·2	4·2	—	—	—	—	4·6	4·0
H ...	3·1	3·6	3·6	4·5	4·0	3·7	—	—	—	—	4·2	4·0	5·0	3·8	—	—	—	—	—	—
OH ...	3·6	4·0	—	—	3·9	3·5	—	—	3·7	3·2	3·7	4·1	4·1	4·0	4·5	3·9	—	—	—	—
LB ...	4·7	3·7	3·8	4·2	4·9	4·7	—	—	—	—	4·1	4·1	4·6	4·5	—	—	—	—	—	—
Q ...	3·3	3·5	2·9	3·6	3·3	2·7	2·7	2·8	—	—	3·7	4·0	4·1	3·8	—	—	—	—	—	—
S ...	3·0	3·3	3·4	3·9	3·2	3·5	2·7	2·9	—	—	3·6	3·9	3·2	3·7	—	—	—	—	—	—
S_3 ...	5·8	7·1	9·0	9·6	—	—	—	—	—	—	—	—	6·1	6·4	—	—	—	—	—	—
U ...	3·4	2·8	2·3	2·9	2·5	2·3	—	—	—	—	2·9	2·9	2·7	3·3	2·9	3·1	—	—	—	—
G′H ...	6·5	8·2	9·0	7·8	6·1	6·9	—	—	—	—	5·5	7·1	6·0	5·7	—	—	—	—	—	—
GB ...	5·9	4·7	4·5	6·7	5·2	4·8	—	—	—	—	5·6	5·4	4·7	5·8	—	—	—	—	—	—
J ...	5·5	3·5	3·8	4·9	4·2	4·8	—	—	—	—	4·3	4·1	3·6	3·5	—	—	—	—	—	—
NH ...	6·0	6·4	6·2	6·8	6·1	6·8	—	—	—	—	5·1	5·6	5·2	5·7	—	—	—	—	6·1	5·9
NB ...	7·9	7·0	8·3	8·0	7·9	7·3	—	—	—	—	8·9	7·1	7·9	8·2	—	—	—	—	7·6	8·4
O_1 ...	3·8	3·5	3·5	4·3	5·0	5·3	—	—	—	—	4·4	3·8	3·5	4·1	—	—	—	—	4·8	4·3
O_2 ...	6·0	5·4	6·0	5·5	7·2	6·7	—	—	—	—	6·1	4·4	6·5	6·7	—	—	—	—	7·6	7·7
G_1 ...	8·9	5·8	6·8	8·0	6·5	7·4	—	—	—	—	5·7	6·5	8·8	6·2	6·4	6·8	—	—	—	—
G_2 ...	10·2	6·8	7·4	8·9	9·3	8·6	—	—	—	—	7·8	7·7	5·9	7·6	—	—	—	—	—	—
GL ...	5·7	4·0	4·6	5·0	4·8	5·1	—	—	—	—	4·7	5·0	4·3	6·1	—	—	—	—	—	—
W_1 ...	—	—	4·8	3·9	7·5	6·8	—	—	—	—	5·6		—	—	—	—	—	—	—	—
W_2 ...	—	—	6·6	7·9	7·6	7·2	—	—	—	—	8·7		—	—	—	—	—	—	—	—
h ...	—	—	12·6	12·4	9·9	8·5	—	—	—	—	11·7		—	—	—	—	—	—	—	—
f ...	—	—	5·2	5·4	5·7	5·3	—	—	—	—	6·2		—	—	—	—	—	—	—	—

* Crania from the Paris Catacombs, and covering many centuries; very heterogeneous.

† Sir William Turner's series; all periods and all districts combined and therefore very variable.

Variability of Indices (Standard Deviations).

	Congo		Gaboon, 1864		Naqada		Egyptian		Aino		English, Whitechapel		English, Moorfields		Bavarian		French *		Scottish *		Northern Negroes	
Organ ...	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
$100\,B/L$ ...	2·6	2·5	2·8	3·4	2·8	3·1	3·4‡	3·4‡	2·4	2·5	3·3	3·0	3·0	2·4	3·5	3·0	4·4	4·2	—	—	2·8	3·5
$100\,H/L$ ...	2·9	2·5	2·4	3·1	2·7	3·0	—	—	—	—	3·2	2·8	3·5	2·8	—	—	3·5	3·7	—	—	—	—
$100\,B/H$ ...	4·6	4·2	4·3	5·3	4·5	4·7	—	—	—	—	4·6	3·8	4·7	4·0	—	—	4·7	4·3	—	—	—	—
$100\,G'H/GB$	4·8	5·1	5·6	5·0	4·5	4·2	—	—	—	—	5·4	6·3	4·8	4·0	3·3	3·3	—	—	—	—	—	—
$100\,NB/NH$	4·9	4·8	4·9	4·2	4·2	4·9	—	—	—	—	4·6	3·9	4·1	4·0	4·4	4·6	—	—	3·8	4·6	—	—
$100\,O_2/O_1$ ...	4·9	3·7	5·4	4·7	5·0	4·7	—	—	—	—	4·2	4·3	4·2	5·7	6·7	5·2	—	—	5·5	6·8	—	—
$100\,G_2/G_1$ ...	6·3	4·9	6·5	6·1	7·4	5·9	—	—	—	—	6·4	6·6	5·6	4·6	—	—	—	—	—	—	—	—
$Oc.\ I$ ...	7·9	4·6	6·8	8·5	—	—	—	—	—	—	—	—	2·2	2·9	—	—	—	—	—	—	—	—
$100\,Fmb/Fml$	6·0	5·8	8·2	7·0	—	—	—	—	—	—	—	—	6·6	6·1	—	—	—	—	—	—	—	—
Mesodacryal Index a	6·3	5·7	6·0	7·5	—	—	9·3†	5·9†	—	—	8·2	8·2	—	—	—	—	—	—	—	—	—	—
Profile Angle	3°·7	3°·4	—	—	2°·9	3°·7	—	—	—	—	3°·9	2°·8	4°·0	2°·3	2°·8	3°·6	—	—	—	—	—	—

* Both these series are very heterogeneous.

† Egyptian, Series E. ‡ Thebans.

of Nigerian natives recently shown me by Dr Hugh Stannus indicates that most well-known European anatomical deformities occur, if possibly with lessened frequency, even in the adult Negro. A striking example of this occurs in No. 82 of the Gaboon 1864 series reproduced as E and F on our Plate. It is an interesting case of cleft palate with a complete absence of praemaxilla*. But such anatomical defects externally obvious during life, appear to occur rarely in collections of negro crania.

A certain post-mortem injury to these negro crania is referred to by Professor Keith in his note on the Congo series: "Some of the crania show marks of having been cut or notched in the supra-orbital region": see our p. 294. This notching was noticed originally by Dr Benington in the two Gaboon series, and he was surprised to find it again in the Congo skulls at the Royal College. It is not, however, confined to the supra-orbital, or indeed to the orbital regions. Fig. A (Plate XXVI) shows it on the supra-orbital region, Fig. B on the supra- and sub-orbital regions, and Figs. A and C along the external margin. Again Fig. D shows it on the occipital edge of the foramen magnum. It is not easy to understand how the notching came about. Sometimes the markings have almost the appearance of the teeth-gnawings of a rodent; at other times they seem to be due to a rough knife, which might possibly have been used to clear away persistent fragments of flesh. It is noteworthy that they are common to both the Congo and the Fernand Vaz series, and they are not common to the large series of prehistoric and historic Egyptian crania in the Biometric Laboratory. In the latter series a few such markings have been found almost invariably about the foramen magnum, and suggest rough handling by the native finder, who, being paid for the skull, wished to separate it rapidly by aid of his knife from the trunk. In one such case at least it was obvious that the separation of a fused atlas had been the cause of the notching. Whether due to man or to rodent, no doubt some explanation will be forthcoming now that attention is drawn to it and inquiries can be made on the spot.

General Peculiarities. In the Congo series many of the crania show inferior frontal protuberance (i.e. bulging of the temporal area of the frontal); for example No. 78. This condition is associated with a marked Sylvian depression, which lies between the inferior frontal protuberance and the temporal protuberance of the squamous.

There is often a median eminence in the metopic area of the frontal more or less marked and lying between the regions of the frontal eminence. Frontal processes of the squamous: these are varied giving fronto-squamosal sutures of different lengths, e.g. No. 73, where this suture measures 6 mm. on *R*, while on the *L* a slender frontal process of the squamous meets a similar process of the frontal giving the "pterion in *K*."—The *K* may be vertical as in No. 73, or

---

* Compare Plate G, Fig. E, and Plate I, Figs. 10 and 13 of the *Treasury of Human Inheritance*, Vol. I. pp. 121—133.

horizontal as in No. 83 on the *R.* Epipterics may be associated with the frontal processes of the squamous as in No. 66 on the *L.*

The form of nose varies largely. In many cases, e.g. No. 73, the upper parts of the nasal processes of the superior maxillary bones and the nasal bones themselves are of uniform width. In others the nasal bones are very much reduced and the nasal processes may even meet above them, e.g. No. 70. The form of nose in No. 83 is peculiar. Here the nasal processes of the superior maxillary bones nearly meet, and only a small pointed process of the frontal intervenes. This just meets the small triangular nasal bones (13 mm. long and 10 mm. wide at edges) which are nearly horizontal.

Ossicles and wormians are often of considerable interest; thus in Fig. (i), p. 327, a tracing is provided of a posterior sagittal ossicle occurring in No. 82 and in Fig. (ii), p. 328, we have the large paired ossicles of the lambda (pre-interparietals) occurring in No. 88. Both these crania are female.

Turning to the Gaboon 1864 series, we note several instances of the "metopic crest" or median frontal crest of Schwalbe. This is associated with bregmatic eminence in certain cases, and then the crania have a peculiar formation resembling in transverse section that of the Eskimo* (e.g. No. 86, Table III. B). A good many instances of local metopic eminence also occur (e.g. No. 13, Table III. A).

Fig. (iii) is a tracing of a marked irregularity in the posterior portion of the sagittal suture which seems worthy of graphic reproduction.

A marked feature of both the Congo and Gaboon series is the drawing or filing of the incisors.

Classifying our peculiarities as in previous papers we have the following results:

(i) *Peculiarities of Form.*

Post-coronal depression occurs in: Gaboon 1864 series in three females (10, 54, 74) and in ten males (13, 14, 25, 28, 32, 62, 65, 67, 40 A. and a skull without number marked as (?)); Gaboon, 1880 in one male (3) and one female (37). In the Congo series it occurs in five males (28, 34, 50, 51, 57), seven females (65, 76, 77, 82, 84, 85, 90) and one non-adult (12). It thus does not appear in the Negro as in the European† to be a markedly female character; it occurs in about 12 °/ₒ of female and 14 °/ₒ of male negro crania. In the case of the English crania (Whitechapel) post-coronal depression or constriction occurred in 20 °/ₒ of female and 11 °/ₒ of male crania.

Flattening of the Obelion seems very frequent in the negro skull and passes into depression. In Gaboon 1864 it occurred in 13 males (11, 27, 28, 30, 33, 45, 47, 50, 57, 61, 65, 67, 79) and 14 females (4, 10, 12, 17, 21, 22, 24, 38, 39, 46, 48,

---

* See *Biometrika*, Vol. VIII. p. 181.

† Macdonell, *Biometrika*, Vol. III. p. 218.

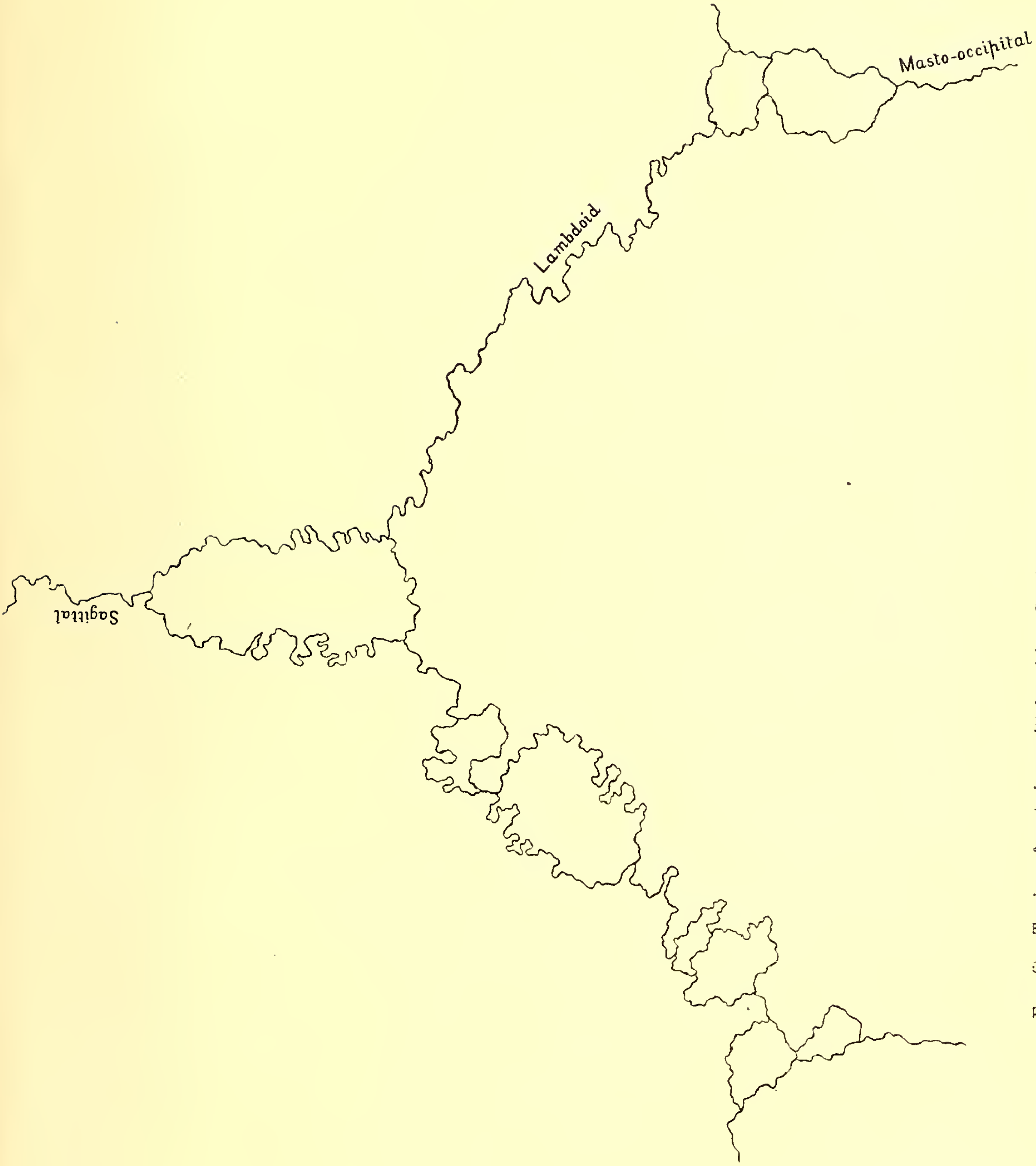


Fig. (i). Tracing of posterior sagittal ossicle. Cranium 82, R. C. of S. Series. Table II, Females.

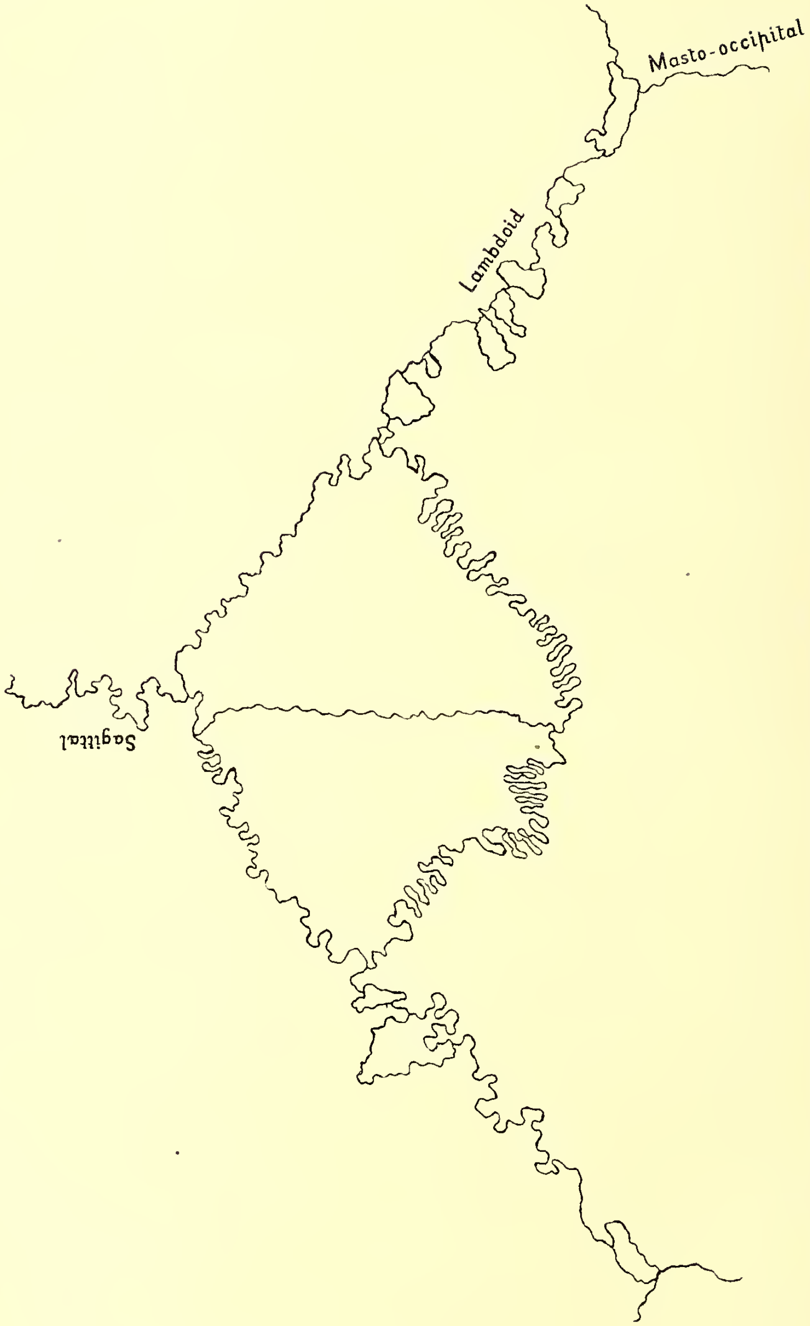


Fig. (ii). Tracing of large paired ossicles of lambda (pre-interparietals). Cranium 88, R. C. of S. Series. Table II, Females.

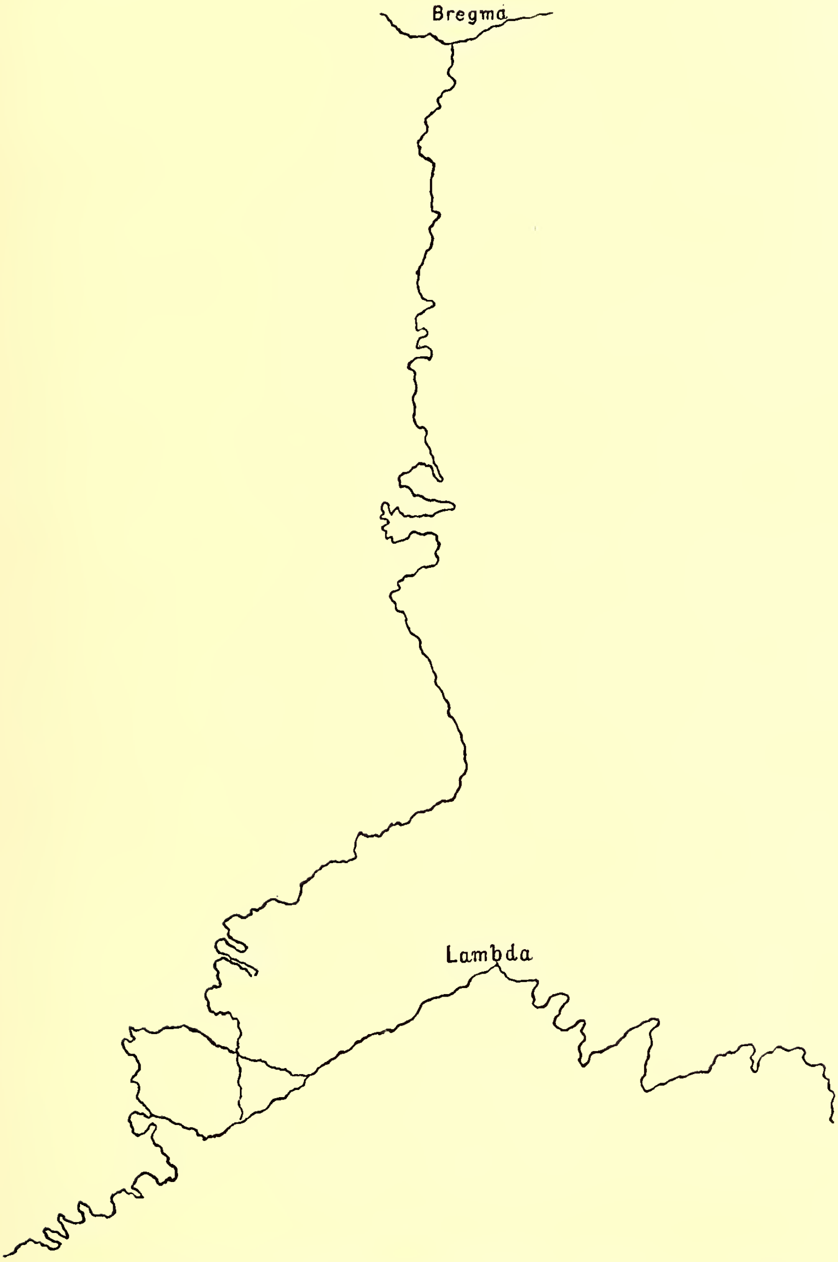


FIG. (iii). Tracing of sagittal and lambdoid sutures. Cranium 75, Table III B.

54, 78, 86), one or two having the obelion depressed (e.g. 38). In Gaboon 1880, the flattening occurred in six males (1, 14, 26, 30, 12^a, 20) and in six females (9, 10, 17, 36, 38, 13). In the Congo series it occurs in 23 males (20, 23, 24, 26, 27, 29, 30, 34, 36, 37, 39, 40, 41, 43, 46, 47, 52, 53 (depression), 54, 56, 57, 58, 59), seven females (66, 68, 74, 84, 86, 87, 91) and two non-adults (11, 12). It has thus been noted in 36 °/₀ of males and 30 °/₀ of females. The Whitechapel English crania showed depression of the obelion in 6 °/₀ of male and 8 °/₀ of female crania, only one case was recorded of flattening. Thus unless the standard was extremely different, flattening of the obelion appears almost a racial character in the Congo-Gaboon group. While bathrocephaly was relatively frequent in the English crania and of a marked type, slight *bathrocephaly* was noticed in only one male skull (59) of the Gaboon 1864 series and in two male skulls (20, 24) of the Gaboon 1880 series and in none of the Congo series.

Temporal Ridges. Pronounced or marked ridges have been held to be a negro feature. In Gaboon 1864 it appears in two females (5, 39) and ten males (8, 27, 36, 55, 61, 65, 71, 76, 77 and (?)). In Gaboon 1880 it occurs in two males (5, 26). In the Congo series in ten males (26, 28, 35, 43, 44, 47, 49, 52, 55, 57), in two females (66, 91) and two non-adults (10 and 13).

Marked *frontal eminences* occurred in the Gaboon 1864 series in five females (22, 38, 46, 54, 64) and five males (14, 18, 61, 65, 85); in the Gaboon 1880 series in one female (15) and in the Congo in six males (23, 29, 30, 41, 46, 48).

Inferior frontal protuberances, often accompanied by Sylvian depressions, occur in Gaboon 1864 series in six females (2, 4, 26, 42, 52, 84) and in six males (13, 34, 71, 80, 85, 89). In Gaboon 1880 series no case is noted, but in the Congo series such inferior frontal protuberances occur in 5 females (65, 72, 78, 80, 86), in 11 males (26, 28, 31, 35, 37, 40, 42, 46, 54, 56, 60) and in 4 non-adults (10, 12, 13, 14).

Sylvian Depression occurs in 7 females (2, 4, 26, 42, 52, 74, 84) and in 7 males (13, 34, 71, 76, 77, 80, 85) of the Gaboon 1864 series. It occurs in two females (10, 39) and three males (20, 23, 24) of the Gaboon 1880 series; while in the Congo series it was noted in two females (78, 88), seven males (17, 23, 26, 35, 56, 60, 62) and two non-adults (12, 13). Thus the Sylvian depression has been noted in 12 °/₀ of females and 14 °/₀ of males.

Marked *metopic eminences* were met with in four females (22, 38, 64, 86) and five males (13, 43, 55, 61, 85) of the Gaboon 1864 series; while they were found in two females (9, 21) of the 1880 Gaboon crania. The Congo series provide the same eminences in four males (15, 28, 29, 43) and four females (12 (young), 66, 68, 87). A 'metopic crest' occurred in a female (5) of the Gaboon 1864 series, and in six males of the same series (27, 44, 63, 65, 77, 79). Such 'crests' were not recorded as occurring in the Congo series.

Marked *bregmatic eminences* were noted in two females (70, 86) of the earlier Gaboon series and a post-bregmatic eminence in the male (32) of the Congo series.

Marked *parietal eminences* occurred in two males (19, 46) of the Congo series. Parietal depression in one male (14) of the 1864 Gaboon series.

Superior occipital bulging was found in one female (10) of the Gaboon 1864 series and in seven males (7, 8, 18, 40 A, 50, 55, 69). It was not recorded as occurring in the Gaboon 1880 series, but in the Congo we find it in one non-adult (12) and in four males (20, 33, 39, 51).

A markedly *retreating forehead* occurs in one male (55) of the 1864 series; in one female (80) and three males (26, 40, 44) of the Congo series. A *prominent glabella* was only recorded in one case a male (55) of the Congo series. A markedly 'infantile' forehead was noted in one female (35) of the 1864 Gaboon series.

Supra-orbital ridges were recorded in two males (45, 59) of the Gaboon 1864 series. *Supra-mastoid ridges* in two males (45, 53) of the same series, and in thirteen males of the Congo series (24, 26, 27, 29, 33, 34, 36, 37, 40, 41, 43, 48, 61).

Subnasal prognathism of a marked character was found in five females (4, 5, 9, 23, 78) and eight males (1, 32, 33, 37, 44, 45, 83, 89) of the Gaboon 1864 series; in two females (12^b, 21) and three males (4, 28, 35) of the 1880 series. It occurred in one female (14) and fourteen males (17, 19, 21, 22, 23, 41, 45, 46, 47, 50, 51, 57, 63, 64) of the Congo series. This negro characteristic therefore is to be found in a marked condition in 21 °/ₒ of males but only 9 °/ₒ of females. Markedly flat and broad noses were especially noticed in one female (87) and twelve males (26, 28, 29, 33, 37, 38, 39, 42, 43, 48, 49, 59) of the Congo series. The base of nose was depressed in two males (31, 36) of the Gaboon 1864 and in two males (28, 31) of the Gaboon 1880 series, or in 1 °/ₒ of females and 14 °/ₒ of males we find flattening or depression chiefly of the bridge*. Special peculiarities in the arrangement of the nasal bones (see p. 319) were noted in four females of the 1864 series (21, 29, 39, 81) and in five males (3, 11, 25, 59, 61). In the Congo series nasal peculiarities occur in four females (65, 67, 70, 83) and five males (29, 37, 42, 49, 59). Rounding of the margin of the pyriform aperture, a peculiar negro character, does not occur in a marked manner in many of the Congo-Gaboon series. It was recorded especially in one male (43) of the Gaboon 1864 series and in two females (79, 80) of the Congo series. Many of the peculiarities in the negro nose thus recorded may appear slight, but the general impression formed is that the Negro nose is subject to more variations than the European.

(ii) *Anomalies of the Sutures.* Only a few crania are recorded as *metopic*, e.g. one male (34) and one non-adult (14) of the Congo series and one female (74) of the Gaboon 1864 series. The spheno-occipital synchondrosis was not obliterated in three young females (2, 20, 70) of the 1864 series and a remarkable lateral deviation of the posterior part of the sagittal suture in a female (75) of the same

---

* This is an individual anatomical confirmation of the difference indicated on p. 318 above, that the negro females have a higher mesodacryal index β than the males.

series (see Fig. (iii), p. 329). Cases of the parieto-squamosal suture being straight and almost horizontal were noted in three Congo males (19, 21, 39).

Frontal processes of the squamous, generally bilateral, were very frequent. They occurred in nine females (10, 12, 60, 70, 75, 78, 82, 86, 88) and seven males (18, 40, 53, 71, 77, 89, (?)) of the Gaboon 1864 series, in two males (5, 32) and two females (19, 37) of the 1880 series; and in two females (66, 73) and three males (32, 50, 57) of the Congo series. Temporal processes of the squamous occurred in one Congo male (90). The pterion* was in '*K*' in three males (8, 31, 73) of the Gaboon 1864 series, in two males (3, 31) and one female (33) of the Gaboon 1880 series; and in one female (83) and one male (45) of the Congo series. There was a marked *sutura notha* in one male (79) of the Gaboon 1864 series, and a *fronto-squamosal suture* occurred in two males (69, 89) of the same series.

(iii) *Interparietals.* A tripartite interparietal, similar to that figured *Biometrika*, Vol. III. Plate XXXVII, occurred in a male (58) of the Gaboon 1864 series. A large simple interparietal occurred also in a female (8) of the Gaboon 1880 series, and another simple interparietal in a male (28) of the Congo series.

Epiterics occur in two females (54, 91) and three males (7, 36, 50) of the Gaboon 1864 and in one male (14) and one female (10) of the 1880 series. In the Congo series they are recorded in seven males (16, 34, 38, 40, 43, 47, 52), four females (66, 69, 70, 74) and one non-adult (14).

(iv) *Ossicles.* Ossicles of the lambda occur in two females (10, 70) and five males (7, 25, 30, 61, (?)) of the Gaboon 1864 series; and in two females (70, 88 (paired), see Fig. (ii), p. 328) and three males (27, 41, 48) of the Congo series. Ossicles of the lambdoid suture are recorded in ten males (3, 8, 14, 15, 25, 34, 47, 50, 57, 61) and seven females (16, 20, 22, 56, 60, 70, 74) of the Gaboon 1864 series; in six males (4, 5, 14, 22, 23, 27) and two females (10, 39) of the Gaboon 1880 series; and in two females (83, 91), thirteen males (16, 17, 20, 21, 22, 23, 25, 27, 28, 34, 38, 47, 56) and one non-adult (14) of the Congo series. Thus ossicles of the lambdoid suture are recorded in 25 °/₀ of male and 12 °/₀ of female crania. Macdonell found no such sexual difference in his English series, recording them for about 9 °/₀ in female and 11 °/₀ in male crania.

Posterior sagittal ossicles were recorded in one female (16) of the Gaboon 1864 series, in one female (4) of the Gaboon 1880 series, and in one male (34) and one female (82, see Fig. (i), p. 327) of the Congo series.

Ossicles in the masto-occipital suture occurred in four females (10, 29, 56, 82) and in one male (53) of the Gaboon 1864 series; they occurred in one female (83), two males (22, 23) and one non-adult (14) of the Congo series.

Ossicles in the masto-parietal suture occurred in two females (70, 82) of the Gaboon 1864 series. *Ossicles in the parieto-squamosal suture* in one female (10)

---

* Flattening of the pterion was only recorded in two crania (30, 31) of the Gaboon 1880 series, both males.

of the 1864 series and one male (7) of the same series. *Ossicles of the spheno-malar suture* in one female (79) and two males (20, 34) of the Congo series, and in one non-adult (14) of the same series ossicles in the squamous suture.

Ossicles at the asterion occur in four females (16, 20, 60, 91) and five males (1, 14, 49, 53, 61) of the 1864 Gaboon crania; in one male (23) and one female (39) of the 1880 Gaboon crania, and in one female (89), two males (34, 38) and one non-adult (14) of the Congo series.

Lastly we may note that a deep *canine fossa* occurred in two males (27, 40) of the Congo series.

There can be no question that every record of anatomical peculiarities varies with the individual observer, who is likely to be more interested in one character than another; nay, the same observer will have his attention concentrated by the occurrence of a very marked anomaly on its appearance in less marked forms in examining further skulls; or again he will seek for certain forms of anomaly because he has read that they are racial features of the group he is studying. There is at present unfortunately a complete want of standardisation in the matter of recording anatomical variations, and no illustrative classification for the guidance of the craniologist. Still admitting that selection plays a large part in all such records, and that the percentages of anomalies of any kind must be very rough, there does seem sufficient justification in the record we have given to show that in the matter of these anomalies and peculiarities, which admit less of the application of metrical methods, there is nothing to indicate that the negro skull of the Congo-Gaboon group is in any way less variable than the skull of any European people.

(7) *Conclusions.* Such conclusions as we draw must be by way of suggestion; one might hope that they may act as incitement to further investigation, above all as some inducement towards the procuring of really ample material and its adequate measurement and reduction. No final conclusions can be drawn until at least 100 crania of each sex for each local race have been obtained and measured by the same individual or by standardised methods. We have endeavoured to indicate the need of this by demonstrating how little profit can be obtained from existing measurements of such important characters as the orbit and palate.

We may summarise the indications of the present paper as follows:

(i) We do not feel justified in differentiating the Congo from the Gaboon crania on the basis of the present material. In some respects the Gaboon 1880 series gives for the males a type which approaches slightly closer to the Angoni and so to the Zulu-Kaffir group than to the Gaboon 1864 series, which is at all points close to the Congo series. This difference is not, however, usually confirmed by examination of the females, and we believe is largely due to the smallness of the 1880 series, which contains some rather large male crania. We may safely look upon the Gaboon and Congo as forming a fairly representative group*.

(ii) This group differs sensibly from the Kaffir-Zulu group of Shrubsall, but negro crania from the north coast (our Northern negroes) seem to approach these southern negroes more closely than they do the Congo-Gaboon group.

(iii) If we wish to determine characteristic negro features, but few *absolute* measurements are of any service. Thus the usual tape arcs, vertical, sagittal and horizontal, are of small differential value. There are, however, some exceptions to this rule.

(*a*) The Negro skull is higher than the European, when we take the basio-bregmatic height. This excess, however, is not due to the development of the upper part of the cranium, there is sensible equality of the auricular heights. The increased height lies between the auricular passages and the basion. It is in fact due to the more massive character of the base of the negro skull. A good measure of this result is found in the index $100\, OH/H$, which is about 85 for Negroes and 87—88 for Europeans.

(*b*) Notwithstanding that little of an orderly evolutionary scale can be made out from absolute measurements—diameters or arcs—the available data show that both for the Gaboon-Congo and the Northern Negro groups there is a great deficiency of cranial capacity—a difference of 100 to 150 cm.^{3}—between Negro and modern European races.

(*c*) The nature of this difference in cranial capacity appears to be principally occipital in origin. There is some difference in the average of tape measurements from nasion to bregma between English and Negro†, but the chief difference lies in the section from lambda to opisthion. If we measure the occipital prominence by the occipital index, we find the English cranium stands at 58 to 59 and the Negro at 68 to 69, the flatter the cranium from opisthion to lambda the higher is this index. There is, I think, no doubt that relatively to the Negro the occipital portion of the skull in the European is greatly developed‡.

(iv) Certain indices—as distinguished from absolute measurements—seem to be of special value in differentiating European and Negro crania.

(*a*) In the first place the ordinary cephalic indices ($100\, B/L$ and $100\, H/L$) are small service, but a new index $100\, (B-H)/L$, which I have called the acroplatic index, is not only positive and fairly high for Europeans but is insensible or actually negative for Negroes. In this they agree with the primitive and

---

* At the same time the mesodacryal and simotic indices show that the two series are by no means wholly alike.

† To some extent indicated in the relative frequency of "markedly retreating foreheads" (see p. 331 above) notwithstanding equality of auricular height.

‡ This has been already indicated in the previous paper on cranial contours (*Biometrika*, Vol. VIII. pp. 135, 136) and may be illustrated graphically by superposing Fig. XV in tissue on Fig. XXVII.

prehistoric peoples; the development of the highly civilised European peoples has been in the direction of increased breadth at the expense of cranial height.

(*b*) The gnathic index gives almost as uniform and suggestive results as the acroplatic index. The Europeans stand at one end of the list, then come the prehistoric, historic and modern Egyptians, then primitive races like the Long Barrow folk and the Aino, then the Negroes, although the full-blooded Negroes are in this respect not clearly differentiated from the Congo-Gaboon group.

(*c*) The Upper Face Index is less clear in its indications, the Kaffir-Zulu group being to some extent displaced. As in the case of the Zygomatic Upper Face Index, this may possibly be due to personal equation. But still the general order—Europeans, Egyptians, Primitive Races, Negroes—appears though with slight confusion.

(*d*) Nasal characters, as we might anticipate, provide a perfectly good scheme of classification. We find full-blooded Negroes, Gaboon-Congo group, primitive and prehistoric peoples and then the European races in excellent sequence. Measurements on the mesodacryal arc and chord are at present too few to be of real service, but they suggest that these characters and the mesodacryal index may ultimately be of much service for differentiation. The simotic index of Mérejkowsky fully justifies its introduction. It places the English at one end of the scale, the Congo at the other with Egyptians and Gaboon crania as intermediate links. The flatness of the Negro nose is seen to be due to defect in simotic subtense, the chord being almost the same as in European crania.

(*e*) We have already referred to the importance of the occipital index, and the failure of orbital and palate indices—in the present absence of standardisation—to be of real value.

(v) The Negro mandible is in all respects broader, where comparison is possible, than that of the primitive races or the European. Relatively to the primitive races and the European, however, it appears (the evidence is still slender) to lack height, or chin development.

(vi) The profile angle measured by the Ranke goniometer and craniophor—too often neglected by craniologists—justifies its existence even on the small data available. We find the European races head the list, then come the Egyptians, followed by primitive races, Congo Negroes and North Africans. Unfortunately there are no data for the Kaffir-Zulu group, but other characters would suggest that this group will not be found far from our North African Negroes. It is much to be desired that the use of the profile angle should be extended.

(vii) No measures of the variation of the Kaffir-Zulu group have yet been taken, but the variability of the Congo-Gaboon group, whether it be judged by biometric measures or by the appreciation of anatomical peculiarities, appears in no wise less than that of European or of primitive races. Indeed we may even suggest without danger that it is greater. This appears to be markedly the case in the occipital index and sensibly the case in the nasal index, and in nasal height. Even in 'chin height' the variability of the Gaboon series in either sex exceeds the variability of the English, rendered as it is heterogeneous and therefore exaggeratedly variable by grouping both sexes together. It seems to me therefore that in the characteristic negro features, lack of occipital development, lack of chin height and low nasal index, the Negro is markedly more variable than the European. Accordingly the suggestion can be made that the European and not the Negro has been developed by a more stringent selection in these respects from an original prototype.

(viii) It has been sometimes suggested that the Negro is incapable of development, for he is less variable than the European. This, as far as physical cranial characters are concerned, is not the fact—he is at least as variable as the European, if not more so. The statement must not be interpreted as meaning more than that the Congo-Gaboon group presents as wide a range for the working of natural selection as the European group; it does not signify that a few years of European education or civilised environment could convert the Negro cranium physically or develop it mentally into an instrument equivalent to the European cranium. Physically the European selection has been the work of long centuries, and the thoughtful anthropologist will hardly doubt that mental characters take as long to develop as the physical. But the point is an important one—the Congo-Gaboon group is not one of a stringently selected nature; it appears to have something of the wide variability of a non-differentiated race. It is the European races which suggest stringent selection with regard to these peculiarly negroid characters.

C. D. Fawcett in comparing modern North African Negroes with the primitive Egyptian Naqadas found that in many respects the Naqadas were nearer to European races than to this negro group, which the present investigation shows to be closer to the Kaffir-Zulu than to the Congo-Gaboon group. But now that the latter group has been investigated we find that it stands between the Egyptian group and the full-blooded Negroes. Roughly the links are:

Europeans Kaffir-Zulus
| |
Egyptians——Congo-Gaboon Group

Further we see that the Egyptians and the Congo-Gaboon Negroes are more closely allied than Europeans as a whole or Kaffir-Zulus as a whole to the prehistoric and primitive races—even to such widely separated primitive races as the Long Barrow folk and the Aino. It is not suggested that Europeans have developed under selection from a race of the Egyptian type or Kaffirs and Zulus from a race of the Congo-Gaboon type; but the suggestion is made that the Egyptian and Congo-Gaboon types are possibly lower branches of the respective stems which link European and Negro together through some trunk type in itself closely related to our prehistoric and primitive races. Roughly the scheme suggested is of the kind:

TABLE I.

Remarks
s. Central incisors drawn. Metopic eminence
Large epipterics 36 mm. long on both sides, and another 12 mm. long inserted behind
ne on R. Small ossicles in lambdoid suture [marked
rs. Sylvian depression. Small ossicle in lambdoid suture. Subnasal prognathism
s ? Central incisors drawn
mm. long on R. Straight and almost horizontal parieto-squamosal suture on L.
asal prognathism. Parietal eminences marked [suture
rs. Flattening of obelion. Superior occipital bulging. Small ossicles in lambdoid
nathism. Ossicles in lambdoid suture and a small one in L. spheno-malar suture.
mosal suture straight and almost horizontal on L.
nathism. Ossicles in R. and L. masto-occipital and a small one in lambdoid suture
n. Subnasal prognathism. Frontal eminences. Flattening of obelion. Sylvian de-
Ossicles in lambdoid and masto-occipital sutures
n. Flattening of obelion. Supra-mastoid ridge
d. Small ossicles in lambdoid suture
rs drawn. Low retreating forehead. Marked temporal ridges. Sylvian depression and
ntal protuberance. Supra-mastoid ridge. Flattening of obelion. Flat nose
rs drawn. Supra-mastoid ridge. Deep canine fossa. Depression of obelion. Small
mbda and others in lambdoid suture
ence. Marked temporal ridges and temporo-malar angle. Inferior frontal protube-
ple interparietal 85 mm. broad and 43 mm. high with small ossicle on L. in lambdoid
st-coronal depression. Broad nose (formed as in No. 29)
rs drawn. Broad nose largely composed of nasal processes of Sup. Max. Supra-
ge. Flattening of obelion. Frontal eminences united in median line (i.e. continuous
nces very marked. Flattening of obelion [with metopic eminence)
rmed as in No. 29). Marked temporo-malar angle and inferior frontal protuberance
rs drawn. Nasal bones wider at lower margin than at root. Eminence 20 mm.
bregma. Slender frontal process of squamous on L.
n. Superior occipital bulging. Supra-mastoid ridge. Flat nose. L. orbit sliced
t-coronal depression and very flat vertex. Flattening of obelion. Two small posterior
cles and others in lambdoid and R. and L. spheno-malar sutures and at asterion.
eric on R. Supra-mastoid ridge. L. orbit sliced
es marked. Sylvian depression between inferior frontal protuberance and slight
otuberance [of obelion. Supra-mastoid ridge
. Top of nasal bones and nasal processes of Sup. Max. uniform width. Flattening
almost entirely formed of nasal processes of Sup. Max. Flattening of obelion. Supra-
ge. Inferior frontal protuberance [and L. asterion
nasal bones wide at root. Small epipteric on L. Ossicles in lambdoid suture and R.
n. Flat nose. Nasal bones uniform. Flattening of obelion. Superior occipital
rieto-squamous sutures straight and almost horizontal
rs drawn. Deep canine fossa. Epipterics 28 mm. long on R., 25 mm. on L. Low
rehead. Inferior frontal protuberance. Flattening of obelion. Supra-mastoid ridges.
sliced
rs drawn. Frontal eminences. Flattening of obelion. Supra-mastoid ridge. Lower
onal suture obliterated. Nose more prominent than usual. Subnasal prognathism.
mbda 45 × 20 mm.
s drawn. Broad flat nose (formed as in No. 29), L. pterion in K. Inferior frontal
e. All sutures open
(35 mm. long) on L. and almost obliterated on R. Marked temporal ridges. Metopic
Flat nose. Nasal bones uniform width. Supra-mastoid ridges. Well marked inion.
f obelion [of coronal suture obliterated
head. Marked temporal ridges. Heavier mastoid processes than usual. Lower parts
. L. pterion in K. Subnasal prognathism
s drawn. Broad low forehead with marked development of frontal eminences. Inferior
berance. Parietal eminences marked. Flattening of obelion. Subnasal prognathism.
of coronal suture obliterated
es marked. Flattening of obelion. Ossicles in lambdoid suture. Epipteric 15 mm.
Sutures open. Subnasal prognathism [ossicle of lambda
s drawn. Frontal eminences. Broad flat nose. Slight supra-mastoid ridge. Small
s very marked. Flat nose (formed as in No. 29)
n. Subnasal prognathism. Post-coronal depression. Coronal and sagittal sutures
ated. Frontal processes of squamous on R. and L. Nasal bones uniform width. R.
ess longer than L. and re-curved [depression. Subnasal prognathism
rs drawn. Lateral incisors filed. Superior occipital bulging. Lateral post-coronal
es marked. Flattening of obelion. Epipteric 20 mm. long on R., defective on L.
s drawn. Depression of obelion. Obliteration of coronal suture
coronal suture. Flattening of obelion. Slight inferior frontal protuberance
s drawn. Prominent glabella and marked temporal ridges. Mastoid processes and
ramen magnum and both orbits and temporal regions appear to have been gnawed or
ged with some sharp instrument
Obliteration of sutures at pterion. Sylvian depression and inferior frontal protube-
tening of obelion. Small ossicles in lambdoid suture
drawn. Post-coronal depression. Flattening of obelion. Frontal processes of squamous
L. Temporal ridges marked. Subnasal prognathism. Obliteration of coronal and
drawn. Nasal bones uniform width. Flattening of obelion [sagittal sutures
Broad nose with nasal bones and nasal processes of Sup. Max. uniform width at root.
obelion. Obliteration of coronal, sagittal and lambdoid sutures
s drawn. Nasal bones prominent. Slight Sylvian depression and inferior frontal
nines drawn. Supra-mastoid ridges [protuberance
athism. Nasal bones uniform width. Slight Sylvian depression
drawn. Nasal bones uniform width. Subnasal prognathism
drawn. L. mastoid process immature. Orbits sliced or gnawed. Subnasal prognathism

MALES.

No.	C	F	L'	L	B	B'	H	OH	LB	P	Q	Q_1	S	S_1	S_2	S_3	S_3'	U	G'H	GB	J	NH	NB	O_1 R.	O_1 L.	O_2 R.	O_2 L.	G_1'	G_1	G_2	GL	fml	fmb	Dacry	Tape Dacry	B_2	PT	St	B/L	H/L	B/H	G'H/GB	NB/NH	O_2/O_1 L.	O_2/O_1 R.	G_2/G_1	Remarks
15	1208	170	171	172 N.	124	95	133	111	93	88°	295	295	354	127	120	107	96	479	51	90	113	39	24	40	39	33	33	39	41	36	88	36	28	24	30	24	104	112	72·1	77·3	93·2	56·7	61·5	84·6	82·5	87·8	About 18 years. Central incisors drawn. Metopic eminence
16	1430	175	175	175	139	98	128	113	92	81°	312	312	360	120	123	117	101	502	61	84	113	44	24	41	40	35	35	45	48	34	93	37	30	22	30	22	112	118	79·4	73·1	108·6	72·6	54·5	87·5	85·4	70·8	18—20 years. Large epipterics 36 mm. long on both sides, and another 12 mm. long inserted behind the larger one on R. Small ossicles in lambdoid suture
17	1195	168	169	168	132	93	135	112	90	81	297	296	350	120	120	110	98	475	67	87	117	47	22	39	39	33·5	34	41	43	35	89	32	29	19	30	22	97	116	78·6	80·4	97·8	77·0	46·8	87·2	85·9	81·4	About 20 years. Sylvian depression. Small ossicle in lambdoid suture. Subnasal prognathism marked
18	1175	173	174	174	135	99	126	107	93	89°	296	296	347	122	117	108	91	488	60	85	120	47	24	43	42	31	31	41	43	33	90	34	34	22	30	21	110	114	77·6	72·4	107·1	70·6	51·1	73·8	72·1	76·7	About 20 years? Central incisors drawn
19	1388	178	d	180	142	89	131	d	95	d	d	d	358	123	120	115	100	500	69	d	d	48	26	d	d	d	d	d	d	38	92	39	31	25	31	22	111	119	78·9	72·8	105·4		54·2	—	—	—	Epipteric 27 mm. long on R. Straight and almost horizontal parieto-squamosal suture on L. Slight subnasal prognathism. Parietal eminences marked
20	1415?	170	172	172	139	94	131	113	91	86°	311	311	355	123	127	105	92	489	55	85	116	43	24	41	40	33	32	39	42	32	87	36	27	24	30	21	109	115	80·8	76·2	100·1	64·7	55·8	80·0	80·5	76·2	About 20 years. Flattening of obelion. Superior occipital bulging. Small ossicles in lambdoid suture
21	1235	167	167	169 N.	134	96	134	116	93	81°	312	312	350	125	125	100	89	484	63	85	115	43	25	39	38	33	32	44	47	31	95	35	31	25	30	22	114	122	79·3	79·3	100·0	74·1	58·1	84·2	84·6	66·0	Subnasal prognathism. Ossicles in lambdoid suture and a small one in L. spheno-malar suture. Parieto-squamosal suture straight and almost horizontal on L.
22	1180	167	167	168 N.	135	97	130	112	90	82°	305	302	355	130	127	95	87	480	59	86	121	44	23	38	38	32·5	31	45	47	36	91	33	28	24	32	25	104	115	80·4	77·4	103·8	68·6	52·3	81·6	85·5	76·6	Subnasal prognathism. Ossicles in R. and L. masto-occipital and a small one in lambdoid suture
23	1438	173	174·5	176	148	95	132	113	98	78°	318	315	355	133	117	105	93	512	61	91	121	44	24	41	40	34	35	48	53	40	102	35	27	22	32	28	115	122	84·1	75·0	112·1	67·1	54·5	87·5	82·9	75·5	Incisors drawn. Subnasal prognathism. Frontal eminences. Flattening of obelion. Sylvian depression. Ossicles in lambdoid and masto-occipital sutures
24	1615	190	193	193	147	100	135	116	104	82	326	326	386	135	135	110	99	537	66	104	d	52	28	45	44	35	35	d	d	43	102	35	27	26	40	32	112	124	76·2	69·9	108·9	63·5	53·5	79·6	77·8	—	Incisors drawn. Flattening of obelion. Supra-mastoid ridge
25	d	180	d	180	139	98	d	d	d	d	d	d	362	130	119	d	100	508	61	96	d	50	30	40	40	39	38	d	d	32	d	d	d	26	35	25	105	110	77·2	—	—	63·5	60·0	95·0	97·5	—	Much damaged. Small ossicles in lambdoid suture
26	1400	179	181	180	144	95	132	115	104	78°	320	319	357	125	122	110	99	515	66	104	136	50	26	43	41	36	35	53	60	42	107	36	30	25	40	31	106	111	80·0	73·3	109·1	63·5	52·0	85·4	83·7	70·0	Central incisors drawn. Low retreating forehead. Marked temporal ridges. Sylvian depression and inferior frontal protuberance. Supra-mastoid ridge. Flattening of obelion. Flat nose
27	1315	183	186	185	141	102	130	110	104	88°	305	300	364	117	141	106	94	518	62	107	d	48	28	46	45	35	36	47	51	43	100	36	31	26	39	28	108	116	76·2	70·3	108·5	58·0	58·3	80·0	76·1	84·3	Central incisors drawn. Supra-mastoid ridge. Deep canine fossa. Depression of obelion. Small ossicle of lambda and others in lambdoid suture
28	1458	183	183	185	138	101	135	113	102	81	310	310	370	125	130	115	101	515	72	93	134	51	24	42	41	36	36	48	52	38	105	39	35	25	35	25	117	119	74·6	73·0	102·2	77·4	47·1	87·8	85·7	73·1	Metopic eminence. Marked temporal ridges and temporo-malar angle. Inferior frontal protuberance. Simple interparietal 85 mm. broad and 43 mm. high with small ossicle on L. in lambdoid suture. Post-coronal depression. Broad nose (formed as in No. 29)
29	1320	178	178	179	142	105	127	116	100	85°	322	320	363	117	141	105	96	510	71	100	134	50	28	42	41	34	35	47	50	42	104	33	27	32	50	23	115	124	79·3	70·9	111·8	71·0	56·0	85·4	81·0	84·0	Central incisors drawn. Broad nose largely composed of nasal processes of Sup. Max. Supra-mastoid ridge. Flattening of obelion. Frontal eminences united in median line (i.e. continuous with metopic eminence)
30	1088	163	165	165	131	90	132	109	91	82	300	300	355	125	127	103	90	471	66	90	d	48	27	38·5	38	32	33	d	d	40	90	33	30	23	35	23	110	114	79·4	80·0	99·3	73·3	56·3	86·9	83·1	—	Frontal eminences very marked. Flattening of obelion
31	1355	175	d	176	135	95	136	d	97	d	d	d	366	126	131	109	94	498	63	92	d	47	28	40	40	d	33	47	d	35	100	37	29	28	38	25	104	108	76·7	77·3	99·3	68·5	59·6	82·5	—		Broad nose (formed as in No. 29). Marked temporo-malar angle and inferior frontal protuberance
32	1190	171	174	173	134	90	132	129	100	79	294	292	350	125	125	100	86	485	67	97	127	50	20	42	42	33	33	51	55	38	101	37	30	25	34	29	104	106	77·5	76·3	101·5	69·1	52·0	78·6	78·6	69·1	Central incisors drawn. Nasal bones wider at lower margin than at root. Eminence 20 mm. behind the bregma. Slender frontal process of squamous on L.
33	1415	181	183	184	143	97	135	117	97	79°	324	322	360	125	135	100	88	515	65	99	153	48	28	43	41	37	36	47	d	42	89	41?	28	24	30	34	114	117	77·7	73·4	105·9	63·6	58·3	87·8	86·1	—	Incisors drawn. Superior occipital bulging. Supra-mastoid ridge. Flat nose. L. orbit sliced
34	1410	177	177	178	140	98	133	110	103	86	306	306	363	120	143	100	87	510	66	93	124	49	26	39	39	35	34	46	50	36	95	37	30	27	35	25	114	120	78·7	74·7	105·3	71·0	53·1	87·2	89·8	72·0	Metopic. Post-coronal depression and very flat vertex. Flattening of obelion. Two small posterior sagittal ossicles and others in lambdoid and R. and L. spheno-malar sutures and at asterion. Small epipteric on R. Supra-mastoid ridge. L. orbit sliced
35	1232	168	169	170	139	98	133	113	100	81	315	310	350	118	122	110	96	485	63	100	130	49	26	41	39	32	32	50	53	40	97	33	29	24	30	31	110	116	81·8	78·2	104·5	63·0	53·1	82·1	78·1	75·5	Temporal ridges marked. Sylvian depression between inferior frontal protuberance and slight temporal protuberance
36	1438	183	183	185	141	98	137	118	101	84	320	319	370	132	128	110	98	520	67	98	d	51	28	40	41	35	35	d	d	40	98	37	30	22	30	24	109	120	76·2	74·1	102·9	68·4	54·9	85·4	87·5	—	Incisors drawn. Top of nasal bones and nasal processes of Sup. Max. uniform width. Flattening of obelion. Supra-mastoid ridge
37	1305	173	174	174	137	97	135	114	94	85	311	309	368	132	126	110	97	495	62	96	123	47	30	41	40	34·5	35	d	d	d	91	33	26	26	30	d	109	118	78·7	77·6	101·5	64·6	63·8	87·5	84·2	—	Very flat nose almost entirely formed of nasal processes of Sup. Max. Flattening of obelion. Supra-mastoid ridge. Inferior frontal protuberance
38	d	176	177	177	134	96	141	119	100	87°	318	315	370	128	137	105	95	492	d	d	130	d	26	42	39	34	33	d	d	d	d	34	31	30	45	d	100	116	75·7	79·7	95·1	—	—	84·6	81·0	—	Flat nose with nasal bones wide at root. Small epipteric on L. Ossicles in lambdoid suture and R. and L. asterion
39	1340	180	181·5	182	142	96	131	113	96	80°	309	309	368	127	121	120	100	510	66	96	123	45	27	41	40	36	36	48	50	36	100	36	30	28	38	27	100	113	78·0	72·0	108·4	68·8	57·8	90·0	87·8	72·0	Incisors drawn. Flat nose. Nasal bones uniform. Flattening of obelion. Superior occipital bulging. Parieto-squamous sutures straight and almost horizontal
40	1295	180	180·5	181	141	98	130	109	102	77°	303	299	364	115	133	116	100	512	63	100	134	48	28	45	43	39	40	51	54	49	102	36	31	26	38	34	116	116	77·9	71·8	108·5	63·0	58·3	93·0	86·7	90·7	Central incisors drawn. Deep canine fossa. Epipterics 28 mm. long on R., 25 mm. on L. Low retreating forehead. Inferior frontal protuberance. Flattening of obelion. Supra-mastoid ridges. Both orbits sliced
41	1200	170	171	171	132	94	134	111	100	80	299	299	348	115	133	100	90	485	64	89	125	47	24	41	40	33	33	53	53	40	99	35	30	19	30	27	105	111	77·2	78·4	98·5	71·9	51·1	82·5	80·5	75·5	Central incisors drawn. Frontal eminences. Flattening of obelion. Supra-mastoid ridge. Lower parts of coronal suture obliterated. Nose more prominent than usual. Subnasal prognathism. Ossicle of lambda 45 × 20 mm.
42	1600	186	189	188	149	100	135	118	101	87	327	327	380	130	140	110	97	537	60	99	d	49	24	41	40	35	35	49	51	37	96	40	35	30	39	27	123	132	79·3	71·8	110·4	60·6	49·0	87·5	85·4	72·3	Central incisors drawn. Broad flat nose (formed as in No. 29), L. pterion in K. Inferior frontal protuberance. All sutures open
43	1390	185	185	185	130	101	136	114	103	80	305	302	374	128	132	114	92	520	68	99	135	48	28	42·5	41	36	37	d	d	39	103	40	31	29	39	33	105	113	70·3	73·5	95·6	68·7	58·3	90·3	84·7		Large epipteric (35 mm. long) on L. and almost obliterated on R. Marked temporal ridges. Metopic eminence. Flat nose. Nasal bones uniform width. Supra-mastoid ridges. Well marked inion. Flattening of obelion
44	1585	183	186	187	143	100	145	126	104	84°	334	334	370	130	130	110	101	523	70	98	136	53	25	42	42	37	37	51	54	40	105	39	32	22	38	31	113	121	76·5	77·5	98·6	71·4	47·2	88·1	87·7	74·1	Retreating forehead. Marked temporal ridges. Heavier mastoid processes than usual. Lower parts of coronal suture obliterated
45	1300	177	179	179	137	97	135	114	103	77°	312	310	365	128	128	99	96	508	63	97	d	46	27	42	42	34	35	55	60	42	107	37	31	23	30	34	114	116	76·5	75·5	101·5	65·0	58·7	83·3	81·0	70·0	Incisors drawn. L. pterion in K. Subnasal prognathism
46	1445	178	181	180	140	97	138	114	100	76·5	317	312	370	125	127	118	102	520	66	95	125	50	29	42	40	37	38	d	d	39	102	36	30	23	31	27	108	114	81·1	76·7	105·8	69·5	58·0	95·0	88·1	—	Central incisors drawn. Broad low forehead with marked development of frontal eminences. Inferior frontal protuberance. Parietal eminences marked. Flattening of obelion. Subnasal prognathism. Lower parts of coronal suture obliterated
47	1438	178	180·5	179	140	97	141	123	97	82°	325	325	372	135	137	100	93	510	63	d	d	45	25	d	d	d	d	d	d	32	96	34	30	26	40	26	112	115	78·2	78·8	99·3	—	55·5	—	—	—	Temporal ridges marked. Flattening of obelion. Ossicles in lambdoid suture. Epipteric 15 mm. long on L. Sutures open. Subnasal prognathism
48	1255?	173	175	174	135	101	130	110	98	88°	301	297	346	123	133	110	82	494	56	100	d	47	27	39·5	40	34	34	d	d	d	91	36	32	28	40	d	104	111	77·6	74·7	103·8	56·0	57·4	85·0	86·1	—	Central incisors drawn. Frontal eminences. Broad flat nose. Slight supra-mastoid ridge. Small ossicle of lambda
49	1345	174	175	174	140	107	131	111	96	82°	307	306	358	122	126	100	95	510	65	102	130	45	24	41	40	33	33	45	48	40	95	36	28	24	30	30	114	122	80·5	75·3	106·9	63·7	53·3	82·5	80·4	83·3	Temporal ridges very marked. Flat nose (formed as in No. 29)
50	1335	183	185	184	136	95	137	111	105	89	304	304	371	125	135	111	99	509	61	93	129	47	26	40	40	30	30	49	52	34	96	33	26	25	35	25	99	115	73·9	74·5	99·3	65·6	55·3	75·0	75·0	65·4	Incisors drawn. Subnasal prognathism. Post-coronal depression. Coronal and sagittal sutures much obliterated. Frontal processes of squamous on R. and L. Nasal bones uniform width. R. mastoid process longer than L. and re-curved
51	1505	182	186·5	184	144	98	136	115	100	84	314	314	373	125	139	109	98	522	70	96	d	51	23	d	42	37	39	46	51	39	94	34	33	24	35	30	108	111	78·3	73·9	100·0	73·0	45·1	92·9	—	76·5	Central incisors drawn. Lateral incisors filed. Superior occipital bulging. Lateral post-coronal depression. Subnasal prognathism
52	1308	170	175·5	174	139	97	134	119	97	84	314	311	362	123	134	105	97	495	65	96	133	40	25	42	42	36	35	40	50	40	93	37	30	25	35	28	106	116	79·9	77·0	103·7	67·7	54·3	83·3	85·7	80·0	Temporal ridges marked. Flattening of obelion. Epipteric 20 mm. long on R., defective on L.
53	1215	164	167	166	135	94	134	112	95	80°	303	300	336	120	120	96	89	483	63	99	129	41	26	40	40	35	35	d	d	41	95	37	32	24	35	30	108	112	81·3	80·7	100·7	63·6	63·4	87·5	87·5	—	Central incisors drawn. Depression of obelion. Obliteration of coronal suture
54	d	170	178	178	140	96	d	115	d	87°	317	314	375	133	125	117	102	512	57	88	124	45	26	40·5	39	33	32	d	d	39	d	d	d	25	35	30	110	115	78·7	—	—	64·8	57·8	82·1	81·5	—	Obliteration of coronal suture. Flattening of obelion. Slight inferior frontal protuberance
55	1430	189	194	194	145	105	144	110	111	83	317	317	375	132	133	110	91	535	70	—	—	53	26	—	—	—		51	54	43	105	41	30	26	40	30	112	120	74·7	74·2	100·7	—	49·1	—	—	79·6	Central incisors drawn. Prominent glabella and marked temporal ridges. Mastoid processes and margin of foramen magnum and both orbits and temporal regions appear to have been gnawed or filed or damaged with some sharp instrument
56	1595	187	187·5	187	147	103	140	123	100	90°	334	333	385	133	138	114	100	541	60	92	—	45	28	43	42	36·5	36	—	—	—	92	37	30	26	35	—	110	130	78·6	74·9	105·0	65·2	62·2	85·7	84·9	—	Incisors drawn. Obliteration of sutures at pterion. Sylvian depression and inferior frontal protuberance. Flattening of obelion. Small ossicles in lambdoid suture
57	1150	169	169	171	136	97	132	113	96	79	310	310	353	120	125	108	96	487	61	98	131	47	28	41	39	34	35	52	46	40	98	31	26	24	30	32	107	116	79·5	77·2	103·0	62·3	59·6	89·7	82·9	87·0	Central incisors drawn. Post-coronal depression. Flattening of obelion. Frontal processes of squamous on R. and L. Temporal ridges marked. Subnasal prognathism. Obliteration of coronal and sagittal sutures
58	1335	182	182	182	137	97	125	109	93	80°	293	293	363	123	125	105	99	513	60	100	—	47	23	40	40	36	36	—	—	42	93	39	34	28	40	29	106	108	75·3	68·7	109·6	60·0	48·9	90·0	90·0	—	Central incisors drawn. Nasal bones uniform width. Flattening of obelion
59	1443	182	182	182	142	100	134	120	98	87°	323	320	376	138	131	107	95	522	61	100	—	45	31	41	40	35	34	49	52	41	98	37	31	27	35	29	101	107	78·0	73·6	100·0	61·0	68·9	85·0	85·4	78·9	Incisors drawn. Broad nose with nasal bones and nasal processes of Sup. Max. uniform width at root. Flattening of obelion. Obliteration of coronal, sagittal and lambdoid sutures
60	1300	177	180	178	134	98	138	117	94	86	310	310	360	125	120	115	97	498	64	93	133	48	23	40	41	37	39		—	48	86	37	28	24	35	41	110	117	75·3	77·5	97·1	68·8	47·9	95·1	92·5	—	Central incisors drawn. Nasal bones prominent. Slight Sylvian depression and inferior frontal protuberance
61	1315	182	183·5	182	134	102	134	113	104	87°	306	305	362	130	130	102	92	510	60	91	—	47	24	41	39	32	33	45	48	38	96	38	34	24	35	—	105	112	73·6	73·6	100·0	66·0	51·1	84·6	78·0	79·2	Incisors and canines drawn. Supra-mastoid ridges
62	1125	167	168	167	138	96	128	110	94	78°	308	305	345	125	117	103	90	485	62	93	125	47	26	40	40	32	32	47	52	42	97	31	26	25	32	28	105	115	82·6	76·6	107·8	66·7	55·3	80·0	80·0	80·8	Subnasal prognathism. Nasal bones uniform width. Slight Sylvian depression
63	1300	172	174	172	137	93	136	110	99	82°	303	300	351	115	135	101	92	492	63	88	121	47	28	40	40	34	34	44	—	36	92	35	32	20	30	27	105	115	79·7	79·1	100·7	71·6	59·6	85·0	85·0	—	Central incisors drawn. Nasal bones uniform width. Subnasal prognathism
64	1260	170	172	170	134	92	132	109	95	77	293	293	350	125	110	115	97	486	65	95	128	46	27	40·5	40	36	37		—	45	97	35	29	18	25	27	108	102	78·8	77·6	101·5	68·4	58·7	92·5	98·9	—	Central incisors drawn. L. mastoid process immature. Orbits sliced or gnawed. Subnasal prognathism

TABLE II.

Q_1	O_2/O_1 L.	O_2/O_1 R.	G_2/G_1	REMARKS
312	78·6	78·6	—	Central incisors drawn. Lateral post-coronal depression. Inferior frontal protuberance (see Text). Nasal processes of sup. max. form almost entire nose
297	81·0	75·0	71·2	Central incisors drawn. Pronounced temporal ridges. Flattening of obelion. Ossification of post. 3rd of sagittal. Frontal processes of squamous. (Frontal-squamosal suture 15 mm. R., 18 mm. L. with epipteric.) Flat nose equally nasal and maxillary. Slight metopic eminence (see Text)
316	82·9	81·0	—	Central incisors drawn. Parieto-squamous suture straight and almost horizontal. Small nasal bones, arch almost entirely maxillary
299	86·7	86·7	80·4	Metopic eminence. Flattening of obelion. Obliteration of upper parts of coronal and sagittal sutures. Broad nose chiefly maxillary
320	81·0	78·0	67·3	Incisors not drawn. Single epipteric 24 mm. long R. Two epipterics 24 mm. and 15 mm. long L.
287	84·6	80·0	—	Incisors drawn. Epipteric bones R. and L. Ossicle of lambda 27 × 27. Nasal processes of sup. max. meet above and exclude very small nasal [bones from frontal
295	82·5	81·7	74·5	Incisors drawn
287	89·7	—	—	Marked temporo-malar angle and inferior frontal protuberance. Nasal bones uniform width
304	82·1	81·3	70·8	Incisors drawn. Frontal processes of squamous R. and L. (see Text). Top of nasal bones and nasal processes of sup. max. uniform width
294	82·1	83·3	65·3	Incisors not drawn. Broad flat nose with nasal and maxillary portions as above. Small epipteric on L. Flattening of obelion
284	—	83·3	—	Incisors not drawn. Upper part of nose mostly maxillary
309	83·1	84·6	66·1	Incisors drawn. Nasal bones uniform width. Post-coronal depression
285	87·2	80·0	69·8	Incisors drawn. Nasal bones uniform width. Post-coronal depression
281	87·8	81·0	72·9	Central incisors drawn. Sylvian depression. Inferior frontal and temporal protuberances (see Text). Obliteration of sagittal suture. Small nasal bones
300	78·1	82·5	—	Incisors drawn. Nasal bones large. Lower margin of pyriform aperture rounded. Ossicles in R. spheno-malar suture
305	74·4	72·5	74·0	Incisors drawn. Retreating forehead. Inferior frontal protuberance. Sagittal suture obliterated. Lower margin of pyriform aperture rounded
275	79·5	78·0	73·9	Incisors not drawn. Small nasal bones
292	79·1	84·7	—	Incisors drawn. Post-coronal depression. Posterior sagittal ossicle 40 × 15 (see Figure in text)
301	86·3	87·7	66·0	Incisors drawn. R. pterion in K (horizontal). R. orbit sliced. Ossicles in lambdoid and masto-occipital sutures. Small triangular nasal bones nearly horizontal. Large nasal processes of sup. max. (see Text)
292	82·9	82·9	74·0	Orbits scratched. Post-coronal depression and flattening of obelion. Incisors drawn. Nose equally nasal and maxillary
—	86·0	84·1	—	Incisors drawn. Nose as above. Post-coronal depression
297	78·6	78·6	71·8	Incisors drawn. Nose as above. Inferior frontal protuberance. Flattening of obelion
305	81·4	83·3	78·7	Left central incisor filed, other absent (not drawn). Epipteric on L. Metopic eminence. Flattening of obelion. Very flat nose, principally nasal bones which are of uniform width
296	76·5	75·6	61·6	Large paired ossicles of lambda 97 × 47. Sylvian depression. Incisors drawn. Nasal bones small
303	86·8	85·9	65·3	Inferior frontal protuberance. Ossicles at asterion. Nasal bones uniform width
300	85·7	81·4	—	Coronal suture obliterated at pterion. Temporal processes of squamous on both sides (probably pterion in K). Some post-coronal depression. Maxillary portion of nose large. Left orbit filed
294	80·5	78·6	—	Marked temporal ridges. Nasal bones uniform width. Flattening of obelion. Ossicles in lambdoid suture

—	—	—	—	8 to 10 years. Very broken and fragile. Marked temporal ridges and inferior frontal protuberance
296	85·7	88·9	73·2	10 years? Flattening of obelion. Teeth or saw marks above R. orbit. Nasal bones prominent
303	94·6	94·6	—	♂, 12—13 years. Post-coronal depression. Superior occipital bulging. Metopic eminence. Flattening of obelion. Sylvian depression. Inferior frontal protuberance
312	91·7	91·9	74·0	♂. About 12 years. Sylvian depression. R. orbit sliced. Temporal ridges marked. Inferior frontal protuberance
303	91·7	91·7	69·4	? ♂, 14—15 years. Metopic. On R. two epipteric bones, and three ossicles in hind part of squamous; one, now wanting, in masto-occipital suture; four ossicles in lambdoid suture, one (at L. asterion) also wanting. R. inferior frontal protuberance. Subnasal prognathism

ROYAL COLLEGE OF SURGEONS. CONGO SKULLS.

TABLE II.

FEMALES.

No.	C	F	L'	L	B	B'	H	OH	LB	P	Q	Q_1	S	S_1	S_2	S_3	S_3'	U	G'H	GR	J	NH	NB	O_1 R.	O_1 L.	O_2 R.	O_2 L.	G_1'	G_1	G_2	GL	fml	fmb	Dacry	Tape Dacry	B_4	PT	ST	B/L	H/L	B/H	G'H/GR	NB/NH	O_2/O_1 L.	O_2/O_1 R.	G_2/G_1	Remarks
65	1327	170	172	172	135	92	136·5	116	97·5	84°	313	312	348	132	118·5	98·5	89	485	61	91	120	45	24·5	42	42	33	33	50·5	52·5	—	96	37	26	23	37	31	109	116	78·5	79·4	98·9	67·0	54·4	78·6	78·6	—	Central incisors drawn. Lateral post-coronal depression. Inferior frontal protuberance (see Text). Nasal processes of sup. max. form almost entire nose
66	1266	174	175	175	134	93	128	110	94	77°	299	297	356	119	127	110	96	501	54·5	100	128	41·5	27	44	42	33	34	49	52	37	97	38	29	27	43	32	107	113·5	76·6	73·1	104·7	54·5	65·1	81·0	75·0	71·2	Central incisors drawn. Pronounced temporal ridges. Flattening of obelion. Ossification of post. 3rd of sagittal. Frontal processes of squamous. (Frontal-squamosal suture 15 mm. R., 18 mm. L. with epipteric.) Flat nose equally nasal and maxillary. Slight metopic eminence (see Text)
67	1445	182	182·5	182·5	140	88	134	115	95·5	87	317	316	375	118	132	125	108	512	60	98·5	122	45	27·5	42	41	34	34	48	51		91·5	40	32	26	43	33·5	108·5	112	76·7	73·4	104·5	60·9	61·1	82·9	81·0	–	Central incisors drawn. Parieto-squamous suture straight and almost horizontal. Small nasal bones, arch almost entirely maxillary
68	1285·5	172	172·5	172·5	130	96	130	110	98	87	302	299	354	127	116	111	98	495	61	92	129	49	24	41·5	41·5	36	36	43	46	37	90	34	28	28	50	31	109·5	117	75·4	75·4	100·0	66·3	49·0	86·7	86·7	80·4	Metopic eminence. Flattening of obelion. Obliteration of upper parts of coronal and sagittal sutures. Broad nose chiefly maxillary
69	—	173·5	176	175	136	92	131	120	91	78	320	320	370	123	139	109	98	495	56	86	119·5	39·5	22	41	39·5	32	32	51	52	35	99	36	30	25	39	30	110	113	77·7	74·9	103·8	65·1	55·7	81·0	78·0	67·3	Incisors not drawn. Single epipteric 24 mm. long R. Two epipterics 24 mm. and 15 mm. long L.
70	1123	164	165	163	122	92	121	104·5	88	84	287	287	341	123	106	112	96	472	49	91	—	42	24	40	39	32	33	—	—	40	87	32	27	25	40	35	111	116	74·8	74·2	100·8	53·8	57·1	84·6	80·0	—	Incisors drawn. Epipteric bones R. and L. Ossicle of lambda 27 × 27. Nasal processes of sup. max. meet above and exclude very small nasal bones from frontal
71	1182	167	167·5	167	137	93	126·5	105	92	80	295	295	337	122	117	98	87	486	53·5	92	—	45	26	41	40	33·5	33	45	47	35	85	37·5	30	23	44	32	107	113	82·0	75·7	108·3	58·2	57·8	82·5	81·7	74·5	Incisors drawn
72	—	164	165·5	165	130	87	123	102	90	—	288	287	331	107	114	111	95	481	—	83	—	44	23		39	37	35	–				39	30	24	40	—	96	105	78·8	74·5	105·7	—	52·3	89·7		—	Marked temporo-malar angle and inferior frontal protuberance. Nasal bones uniform width
73	1150	166	167	166	127	92	128	111	91	84	303	304	344	119·5	110	114·5	98	478	55	87	114	44	24	40	39	32·5	32	47	48	34	90	32	27	24	38	29	106	113·5	76·5	77·1	99·2	63·2	54·5	82·1	81·3	70·8	Incisors drawn. Frontal processes of squamous R. and L. (see Text). Top of nasal bones and nasal processes of sup. max. uniform width
74	1068	164	165	165	128	88	116	106	89	80°	292	294	332	115	111	107	91	475	63	87	117	46	25	39	39	32·5	32	47·5	49	32	97	33	25	25	40	28	105	106	77·6	70·3	110·3	72·4	54·3	82·1	83·3	65·3	Incisors not drawn. Broad flat nose with nasal and maxillary portions as above. Small epipteric on L. Flattening of obelion
75	—	171·5	171·5	172	124	90	129	106	97	79	287	284	346	130	105	111	96	481	65	96	—	46	28	42	—	35	33	–		38	98	34	29	27	40	31	102	108	72·1	75·0	96·2	67·7	60·9	—	83·3	—	Incisors not drawn. Upper part of nose mostly maxillary
76	1393	177	177·5	178	137	91	129	113	94	81°	301	309	362	132	122	108	97	499	67	89	122·5	47	25	39	38·5	33	32	53·5	56	37	98	38	31	23	38	29	103	118	77·0	72·5	106·2	75·3	53·2	83·1	84·6	66·1	Incisors drawn. Nasal bones uniform width. Post-coronal depression
77	1130	167	167·5	167	122	90	120·5	104	94	75°	285	285	343	121	112	111	97	475	57	93	120	45·5	28	40	39	32	34	57	53	37	98	30	28	26·5	41	31	101·5	105	73·1	72·2	101·2	61·3	61·5	87·2	80·0	69·8	Incisors drawn. Nasal bones uniform width. Post-coronal depression
78	1205	174	175	175	127	90	130	105	98	82°	281	281	343	118	122	103	92	496	59	91	127	46	25	42	41	34	36	46·5	48	35	93	38	32·5	25	41	31	103	107	72·6	74·3	97·7	64·8	54·4	87·8	81·0	72·9	Central incisors drawn. Sylvian depression. Inferior frontal and temporal protuberances (see Text). Obliteration of sagittal suture. Small nasal bones
79	—	175	173	174	129	92	129	110	96	80	303	300	357	116	126	115	97	491	52·5	90	—	41	23	40	41	33	32	-	—	—	95	34	27	22	36	—	106	111	74·1	74·1	100·0	58·3	56·1	78·1	82·5	—	Incisors drawn. Nasal bones large. Lower margin of pyriform aperture rounded. Ossicles in R. spheno-malar suture
80	1135	170	171·5	170	129	86	132	114	92	82	308	305	354	119	125	111	101	486	56·5	91	115	42	25	40	39	29	29	45·5	50	37	91	32	27	22	33	30	101·5	109	75·9	77·6	97·7	62·1	59·5	74·4	72·5	74·0	Incisors drawn. Retreating forehead. Inferior frontal protuberance. Sagittal suture obliterated. Lower margin of pyriform aperture rounded
81	948	158	159	158	117	86	122	105	86	79	270	275	327	102	117	110	97·5	448	58·5	86	—	42	24	41	39	32	31	45	46	34	94	30	28	22	37	29	97	105	74·1	77·2	95·9	68·0	57·1	79·5	78·0	73·9	Incisors not drawn. Small nasal bones
82	—	173·5	176	175·5	137·5	93·5	123	105	96	83	292	292	352	117	138	98	89	505	55	93		46	27	42·5	43	36	34	—	--	41	92	38	30	24	41	—	109	114	78·3	70·1	111·8	59·1	58·7	79·1	84·7		Incisors drawn. Post-coronal depression. Posterior sagittal ossicle 40 × 15 (see Figure in text)
83	1267	169	171	170	134	94·5	129	110	90	80°	302	301	348	117	115	115·5	98·5	494	52·5	88	–	40	26	40·5	40	35·5	34·5	46	47	31	91	38	32	26	43	27	108	114	78·8	75·9	103·9	59·7	65·0	86·3	87·7	66·0	Incisors drawn. R. pterion in K (horizontal). R. orbit sliced. Ossicles in lambdoid and masto-occipital sutures. Small triangular nasal bones nearly horizontal. Large nasal processes of sup. max. (see Text)
84	1242	179·5	180	179	130	95	126	105	92	82	292	292	361	124	122	115	96	502	65	99	123·5	48	28	41	41	34	34	49	50	37	93	36·5	27	25	40	33	104	110·5	72·6	70·4	103·2	65·7	58·3	82·9	82·9	74·0	Orbits scratched. Post-coronal depression and flattening of obelion. Incisors drawn. Nose equally nasal and maxillary
85	—	185	—	185·5	135	100	—	—	101	—	--	—	358	129	109	120	100	515	66	95		51	21	44	43	37	37	—	—	37	97	38	28	25·5	43	30	116	115	72·8	—	—	69·5	47·5	86·0	84·1	—	Incisors drawn. Nose as above. Post-coronal depression
86	1152	162·5	163	162	130	98	127	109	94	74	298	297	332	119	123	90	82	477	63	95	121	49	23	42	42	33	33	50	51·5	37	101	39	30	23·5	39	32	109·5	116	80·2	78·4	102·4	66·3	46·9	78·6	78·6	71·8	Incisors drawn. Nose as above. Inferior frontal protuberance. Flattening of obelion
87	1165	171·5	172·5	172	135	98	133	113	93	83°	306	305	357	127	126	104	92	491	65	97·5	123·5	49	26	42	43	35	35	47	47	37	92	35	26	26	40	32	115	118	78·5	77·3	101·5	66·7	53·7	81·4	83·3	78·7	Left central incisor filed, other absent (not drawn). Epipteric on L. Metopic eminence. Flattening of obelion. Very flat nose, principally nasal bones which are of uniform width
88	1230	172·5	173·5	173	135	99	125	106	91	77	299	296	360	122	117	121	108·5	495	58	97	124·5	44	25	41	40·5	31	31	52	56	34·5	96	32	30	25	41	30·5	108	115	78·0	72·3	108·0	59·8	56·3	76·5	75·6	61·6	Large paired ossicles of lambda 97 × 47. Sylvian depression. Incisors drawn. Nasal bones small
89	1228	170	170	170	128	95	132	113	90·5	80	303	303	356	125	113	118	102	483	60	90·5	122	44	26	39	38	33·5	33	47	49	32	92	34	28	24	34	28	98	112	75·3	77·6	96·9	66·3	59·1	86·8	85·9	65·3	Inferior frontal protuberance. Ossicles at asterion. Nasal bones uniform width
90	1257	173	174	174	137	95·5	129	107	96	84°	301	300	352	122	122	108	95·5	505	66	94	129	48	24	43	42	35	36	49	50	—	92	38	33	26	40	35	111	115	78·7	74·1	106·2	70·2	50·6	85·7	81·4	—	Coronal suture obliterated at pterion. Temporal processes of squamous on both sides (probably pterion in K). Some post-coronal depression. Maxillary portion of nose large. Left orbit filed
91	1125	167	169	169	129	92·5	126	109·5	93	79	295	294	349	119	130	100	90	488	62	97	121	46	22	42	41	33	33	51	53	—	96	31	27	25	45	32	107	113	76·3	74·6	102·4	64·0	48·2	80·5	78·6		Marked temporal ridges. Nasal bones uniform width. Flattening of obelion. Ossicles in lambdoid suture

Not Adult.

No.	C	F	L'	L	B	B'	H	OH	LB	P	Q	Q_1	S	S_1	S_2	S_3	S_3'	U	G'H	GR	J	NH	NB	O_1 R.	O_1 L.	O_2 R.	O_2 L.	G_1'	G_1	G_2	GL	fml	fmb	Dacry	Tape Dacry	B_4	PT	ST	B/L	H/L	B/H	G'H/GR	NB/NH	O_2/O_1 L.	O_2/O_1 R.	G_2/G_1	Remarks
10	1150	169		168	124	90	133	—		—		–	350	115	135	100	86	470	—		—	—			—	—	—	—	—	—	—	35	25	—	—	—	97	106	73·8	79·2	93·2	—	—	—	—	—	8 to 10 years. Very broken and fragile. Marked temporal ridges and inferior frontal protuberance
11	—	165	162	165	133	88	122	105	90	90°	297	296	328	110	113	105	90	472	54	73	103	42	20	36	35	32	30	39	41	30	82	38	34	16	25	25	97	118	80·6	73·9	109·0	74·0	47·6	85·7	88·9	73·2	10 years? Flattening of obelion. Teeth or saw marks above R. orbit. Nasal bones prominent
12	1290	165	166	165	133	89	122	107	85	88	304	303	335	118	117	100	90	479	58	86	109	42	24	37	37	35	35	40	-	37	82	38	28	18	22	25	102	114	80·6	73·9	109·0	67·5	57·1	94·6	94·6		♂, 12—13 years. Post-coronal depression. Superior occipital bulging. Metopic eminence. Flattening of obelion. Sylvian depression. Inferior frontal protuberance
13	1390	174	173	174	136	94	130	114·5	95	87	313	312	350	123	125	102	90	492	60	83	112	43	24	37	36	34	33	45	50	37	93	41	30	23	30	30	100	112	78·2	74·7	104·6	72·3	55·8	91·7	91·9	74·0	♂. About 12 years. Sylvian depression. R. orbit sliced. Temporal ridges marked. Inferior frontal protuberance
14	1375	172	173	173	132	95	132	115	93	82°·5	305	303	353	120	130	103	91	482	62	86	—	47	21	36	36	33	33	41	49	34	93	39	30	25	32	20	99	111	76·3	76·3	100·0	72·1	44·7	91·7	91·7	69·4	?♂, 14—15 years. Metopic. On R. two epipteric bones, and three ossicles in hind part of squamous; one, now wanting, in masto-occipital suture; four ossicles in lambdoid suture, one (at L. asterion) also wanting. R. inferior frontal protuberance. Subnasal prognathism

TABLE III *A*.

Remarks
s. Large superciliary ridges. Marked mastoid processes and inion. Nasal bones promi-
bnasal prognathism. Ossicles at R. asterion
s. Nasal bones flat. Ossicles in lambdoid suture (now wanting at R. and L. asterion).
aping and fragile
s. Superior occipital bulging. Epipteric 41 mm. long on L., and another ossicle 32 mm.
d this, in parieto-squamosal suture. Ossicle of lambda 22 × 10 mm. Small mastoids and
orepart of central incisors sliced off. Dentition good
s. Well-marked temporal ridges. Pterion in K on R. Fronto-squamosal suture 15 mm.
. Ossicles in lambdoid suture. Small inion. Some superior occipital bulging
teration of coronal and sagittal sutures. Flattening of obelion. Nose peculiarly formed.
es absent save for small median rudiment, and their place taken by broad nasal processes
ax. between which frontal sends down a pointed process 7 mm. long
of sagittal suture. R. lateral incisor filed. Post-coronal depression. Marked metopic
Sylvian depression and inferior frontal protuberance. Frontal processes of squamous
quamosal suture 9 mm. long on R. and 10 mm. on L.)
etal depression on both sides. Ossicles in lambdoid suture and at asterion. Frontal
s well marked. Coronal suture obliterated. Post-coronal depression
s. Small ossicles in lambdoid suture. R. lateral incisor filed, other incisors wanting.
caries
s. Marked frontal eminences. Frontal process of squamous and epipteric (somewhat
d) R. and L. Superior occipital bulging [lambdoid suture
l bones much reduced. Post-coronal depression. Small ossicle of lambda and others in
ttal and coronal sutures entirely obliterated. Flattening of obelion. Marked temporal
orehead rises to a marked eminence similar to the condition in No. 5 (Female,
s. Lateral post-coronal depression. Flattening of obelion [Table III *B*.)
ression of obelion and median sagittal area. Ossicle of lambda 55 mm. wide and 35 mm.
ewhat obliterated)
s. Small recurved mastoids. Root of nose depressed. Pterion in K on L.
s. Post-coronal depression. Obliteration of posterior half of sagittal suture. Subnasal
sm [ture sliced
s. Flattening of obelion. Subnasal prognathism. R. orbit and margin of piriform aper-
s. R. orbit sliced. Sylvian depression and inferior frontal protuberance. Ossicle in
suture
t of nose depressed. Marked fronto-malar angle and temporal ridges. L. orbit sliced.
20 mm. long on L.
s. Fine large skull said to have been "prepared" by Owen. Thick cranial walls.
on of coronal and sagittal sutures. Frontal processes of squamous on R. and L.
Metopic eminence. Very prognathous. Lower margin of piriform aperture rounded
s. Subnasal prognathism. Form of frontal as in No. 5
L. Flattening of obelion. Subnasal prognathism. Supra-orbital and supra-mastoid ridges
s. Flattening of obelion. Sutures open. Small symmetrically placed ossicles in lambdoid
R. and L.
. incisor filed—others wanting. Ossicles at R. and L. asterion
7 mm. long on R. Symmetrically placed ossicles in lambdoid suture on R. and L.
of obelion. Superior occipital bulging
s. Obliteration of sagittal and lambdoid sutures. Ossicles in R. masto-occipital suture
terion. Frontal process of squamous on R. Supra-mastoid ridges
s. Retreating forehead. Temporal ridges marked. Metopic eminence. Slight superior
bulging
s. Flattening of obelion. Obliteration of lower ends of coronal and parts of sagittal and
sutures and small ossicles in the last [cephaly. Marked inion
s. Superciliary and supra-orbital ridges marked. Nasal bones prominent. Slight bathro-
rs. Flattening of obelion. Flat nose. Frontal and metopic eminences. Ossicle of
Others in lambdoid suture, and R. and L. asterion
bdoid suture entirely and coronal and sagittal largely obliterated. Post-coronal depression
ars. Teeth good. Third molars uncut. Temporal ridges marked. Form of frontal as in
s. Some obliteration of coronal and sagittal sutures. Flattening of obelion. Post-coronal
. Temporal ridges marked. Frontal eminences and metopic crest [obelion
rn. Obliteration of coronal and sagittal sutures. Post-coronal depression. Flattening of
ars. Slight superior occipital bulging. Pterion defective on L. Fronto-squamosal suture
g on R.
s. Sagittal suture obliterated and lower ends of coronal. Small frontal processes of squamous
L. Sylvian depression between inferior frontal protuberance and temporal protuberance
s. R. squamous broken. L. pterion in K [(most marked on R.)
dges marked. Sylvian depression
idges marked. Sylvian depression. Obliteration of coronal suture. Frontal process of
on L. Form of frontal as in No. 5
onal, sagittal and lambdoid sutures obliterated. Teeth much worn and alveolar rim
Sylvian depression and inferior frontal protuberance
l lambdoid sutures much obliterated and antero-inferior angle of parietal synostosed with
and frontal
rognathism. Sutures open. Frontal processes of squamous (Fronto-squamosal suture
ng on L., 12 mm. on R.). Inferior frontal protuberance
-coronal depression. Temporal ridges marked. Frontal process of squamous on L.
f lambda 44 × 63 mm.
ing. Teeth absent and alveolar margin much absorbed
l depression. Slight superior occipital bulging
ognathism
onal and lambdoid sutures obliterated. Flattening of obelion. Marked "Sutura Notha."
rehead rising to an eminence as in No. 5
sal prognathism. Mental prominence strong [and frontal eminences
ure much obliterated. Sylvian depression and inferior frontal protuberance. Metopic

1864 SERIES. CRANIA FROM FERNAND VAZ, GABOON.

TABLE III *A*.

MALES.

No.	C	F	L	B	B'	H	LB	Q	S	S_1	S_2	S_3	S_3'	U	$G'H$	GB	J	NH	NB	O_1	O_2	G_1'	G_1	G_2	GL	fml	fmb	Dacry	B_2	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1	G_2/G_1	W_1	W_2	h_1	f	Remarks
1	1233	173	173	133	96	134	98	310	358	128	130	100	87	490	76	97	125	50	26	39	37	53	56	38	104	36	29	25	30	123	76·9	77·5	99·3	78·4	52·0	94·9	67·9	—	—	—	—	40—50 years. Large superciliary ridges. Marked mastoid processes and inion. Nasal bones prominent. Subnasal prognathism. Ossicles at R. asterion
3	—	175	176	145	96	131	97	325	355	130	115	110	97	508	70	96	127	50	21	41	37	50	53	38	100	40	30	21	28	123	82·4	74·4	110·7	72·9	42·0	90·2	71·7	—	—	—	—	40—50 years. Nasal bones flat. Ossicles in lambdoid suture (now wanting at R. and L. asterion). Sutures gaping and fragile
7	1515	181	182	138	97	136	101	312	370	122	138	110	95	505	67	97	131	46	26	41	32	—	—	40	105	35	30	24	29	112	75·8	74·7	101·5	69·1	56·5	78·1	—	—	—	—	—	30—40 years. Superior occipital bulging. Epipteric 41 mm. long on L., and another ossicle 32 mm. long behind this, in parieto-squamosal suture. Ossicle of lambda 22 × 10 mm. Small mastoids and inion. Forepart of central incisors sliced off. Dentition good
8	1400	178	178	137	97	134	94	310	365	120	140	105	95	505	69	91	125	47	23	41	37	53	56	36	100	33	20	25	29	122	77·0	75·3	102·2	75·8	48·9	90·2	64·3	—	—	—	—	30—40 years. Well-marked temporal ridges. Pterion in K on R. Fronto-squamosal suture 15 mm. long on L. Ossicles in lambdoid suture. Small inion. Some superior occipital bulging
11	1468	186	188	131	98	139	101	302	383	138	134	111	99	505	66	96	134	49	33	43	33	51	55	44	100	38	33	22	33	103	69·7	73·9	94·2	68·8	67·4	76·7	80·0	—	—	—	—	Aged. Obliteration of coronal and sagittal sutures. Flattening of obelion. Nose peculiarly formed. Nasal bones absent save for small median rudiment, and their place taken by broad nasal processes of Sup. Max. between which frontal sends down a pointed process 7 mm. long
13	1380	177	178	135	92	133	98	295	364	130	130	104	95	498	61	98	128	43	25	43	35	50	54	40	98	36	29	23	30	112	75·8	74·7	101·5	62·2	58·1	81·4	74·1	113	98	30	46	Obliteration of sagittal suture. R. lateral incisor filed. Post-coronal depression. Marked metopic eminence. Sylvian depression and inferior frontal protuberance. Frontal processes of squamous (Fronto-squamosal suture 9 mm. long on R. and 10 mm. on L.)
14	1352	175	177	131	93	139	97	302	370	130	135	105	95	495	66	92	124	45	25	41	34	47	—	35	92	35	29	21	25	113	74·0	78·5	94·2	71·7	55·6	82·9	—	110	90	31	42	Aged. Parietal depression on both sides. Ossicles in lambdoid suture and at asterion. Frontal eminences well marked. Coronal suture obliterated. Post-coronal depression
15	1342	174	175	131	99	133	104	295	350	125	125	100	86	492	65	92	129	49	27	42	34	50	51	42	106	33	28	21	28	113	74·9	76·0	98·5	70·7	55·1	81·0	82·4	117	96	36	47	30—40 years. Small ossicles in lambdoid suture. R. lateral incisor filed, other incisors wanting. Extensive caries
18	1430	176	178	136	95	142	100	312	367	135	130	102	92	504	68	93	118	51	25	41	35	48	51	43	102	39	28	26	30	116	76·4	79·8	95·8	73·1	49·0	85·4	84·3	110	102	37	47	30—40 years. Marked frontal eminences. Frontal process of squamous and epipteric (somewhat obliterated) R. and L. Superior occipital bulging
25	1395	173	173	134	100	131	98	312	357	125	122	110	98	500	68	96	130	55	28	41	39	48	51	38	99	37	30	28	28	121	77·5	75·7	102·3	70·8	50·9	95·1	74·5	109	83	26	44	Aged. Nasal bones much reduced. Post-coronal depression. Small ossicle of lambda and others in lambdoid suture
27	1415	185	186	138	96	142	101	315	388	135	137	116	102	515	67	100	134	46	27	44	33	47	50	36	101	32	27	25	32	111	74·2	76·3	97·2	67·0	58·7	75·0	72·0	113	87	—	46	Aged. Sagittal and coronal sutures entirely obliterated. Flattening of obelion. Marked temporal ridges. Forehead rises to a marked eminence similar to the condition in No. 5 (Female, Table III *B.*)
28	1333	178	179	133	93	136	97	310	366	130	128	108	95	496	64	91	123	43	24	41	33	41	46	—	—	38	29	22	—	120	74·3	76·0	97·8	70·3	55·8	80·5	—	—	—	—	—	40—50 years. Lateral post-coronal depression. Flattening of obelion
30	1412	170	172	135	96	136	99	308	348	130	122	96	88	488	51	100	140	46	26	43	38	44	46	—	94	42	31	23	—	111	78·5	79·1	99·3	51·0	56·5	88·4	—	—	—	—	—	Aged. Depression of obelion and median sagittal area. Ossicle of lambda 55 mm. wide and 35 mm. high (somewhat obliterated)
31	1268	166	172	137	94	132	100	308	348	120	120	99	90	485	60	86	132	46	29	39	31	53	56	37	101	35	30	25	29	120	79·7	76·7	103·8	69·8	63·0	79·5	66·1	110	99	29	45	30—40 years. Small recurved mastoids. Root of nose depressed. Pterion in K on L.
32	1435	179	182	138	98	139	106	315	360	122	130	102	95	508	71	97	—	47	22	40	34	52	55	41	110	38	29	21	33	115	75·8	76·4	99·3	73·2	46·8	85·0	74·6	—	—	—	—	30—40 years. Post-coronal depression. Obliteration of posterior half of sagittal suture. Subnasal prognathism
33	1445	185	186	140	98	140	—	323	385	126	136	123	101	520	68	—	—	48	27	43	36	—	—	41	100	34	30	23	32	116	75·3	75·3	100·0	—	56·3	83·7	—	—	—	—	—	30—40 years. Flattening of obelion. Subnasal prognathism. R. orbit and margin of piriform aperture sliced
34	1420	173	177	137	96	138	100	311	358	125	125	108	95	495	67	100	130	47	30	40	33	48	53	40	103	37	32	24	27	117	77·4	78·0	99·3	67·0	63·8	82·5	75·5	—	—	—	—	30—40 years. R. orbit sliced. Sylvian depression and inferior frontal protuberance. Ossicle in lambdoid suture
36	1300	173	173	136	98	133	90	310	357	127	115	115	92	495	59	90	125	44	24	42	31	48	51	37	90	38	31	25	32	115	78·6	76·9	102·3	65·6	54·6	73·8	72·6	—	—	—	—	Aged. Root of nose depressed. Marked fronto-malar angle and temporal ridges. L. orbit sliced. Epipteric 20 mm. long on L.
40	1450	180	187	142	99	140	94	314	392	128	138	126	110	520	68	94	124	46	26	42	34	48	50	39	98	34	29	23	36	116	75·9	74·9	101·4	72·3	56·5	81·0	78·0	114	92	32	45	40—50 years. Fine large skull said to have been "prepared" by Owen. Thick cranial walls. Obliteration of coronal and sagittal sutures. Frontal processes of squamous on R. and L.
43	1500	182	183	145	94	135	105	325	372	139	111	122	105	525	70	105	—	50	27	40	36	53	56	40	103	32	31	24	34	110	79·2	73·8	107·4	66·7	54·0	90·0	80·0	103	101	29	51	40 years. Metopic eminence. Very prognathous. Lower margin of piriform aperture rounded
44	1324	170	171	133	91	135	96	303	345	120	125	100	91	484	67	90	124	50	29	40	37	52	54	37	101	39	30	24	30	105	77·8	79·0	98·5	74·4	58·0	92·5	68·5	105	85	35	45	30—40 years. Subnasal prognathism. Form of frontal as in No. 5
45	1465	180	183	133	97	132	100	312	370	132	138	100	89	508	68	96	127	47	25	41	33	46	48	39	100	35	30	24	36	113	72·7	72·1	100·8	70·8	53·2	80·5	81·3	113	97	31	43	Epipteric on L. Flattening of obelion. Subnasal prognathism. Supra-orbital and supra-mastoid ridges
47	1375	181	182	132	98	138	105	317	370	125	145	100	92	502	64	97	130	50	25	40	35	48	53	41	105	36	28	20	30	114	72·5	75·8	95·7	66·0	50·0	87·5	77·4	113	97	41	48	30—40 years. Flattening of obelion. Sutures open. Small symmetrically placed ossicles in lambdoid suture on R. and L.
49	1279	170	175	138	99	131	93	308	350	113	129	108	96	490	72	91	130	50	27	41	36	46	50	40	92	34	31	29	29	116	79·8	75·7	105·3	79·1	54·0	87·8	80·0	117	90	32	44	30 years. R. incisor filed—others wanting. Ossicles at R. and L. asterion
50	1275	175	176	131	93	132	95	297	362	123	110	120	101	485	65	93	123	49	26	40	34	44	—	33	95	37	27	23	25	106	74·4	75·0	99·2	69·9	53·1	85·0	—	104	88	28	44	Epipteric 17 mm. long on R. Symmetrically placed ossicles in lambdoid suture on R. and L. Flattening of obelion. Superior occipital bulging
53	1478	183	184	130	93	140	102	315	377	130	135	112	98	506	53	95	132	43	24	41	32	45	47	39	96	37	30	24	29	111	70·7	76·1	92·9	55·8	55·8	78·1	83·0	—	—	—	—	40—50 years. Obliteration of sagittal and lambdoid sutures. Ossicles in R. masto-occipital suture and L. asterion. Frontal process of squamous on R. Supra-mastoid ridges
55	1375	176	184	135	100	137	104	310	360	115	133	112	101	492	72	102	138	51	28	42	33	52	56	45	106	38	35	27	30	110	73·4	74·5	98·5	70·6	54·9	78·6	80·4	109	98	41	52	30—40 years. Retreating forehead. Temporal ridges marked. Metopic eminence. Slight superior occipital bulging
57	1205	172	173	135	101	131	100	308	359	120	128	105	94	493	64	97	126	45	27	40	34	47	—	37	101	30	26	27	28	118	78·0	75·7	103·1	66·0	60·0	85·0	—	111	89	33	47	30—40 years. Flattening of obelion. Obliteration of lower ends of coronal and parts of sagittal and lambdoid sutures and small ossicles in the last
59	1434	183	186	134	93	138	105	305	370	118	141	111	98	507	67	94	133	49	23	44	36	52	58	40	105	40	31	22	33	113	72·0	74·2	97·1	71·3	46·9	81·8	69·0	—	—	—	—	40—50 years. Superciliary and supra-orbital ridges marked. Nasal bones prominent. Slight bathrocephaly. Marked inion
61	1517	182	184	140	97	145	102	327	380	140	125	115	101	512	76	96	128	52	28	42	34	43	—	43	102	36	26	28	30	111	76·1	78·8	96·6	79·2	53·9	81·0	—	—	—	—	—	40—50 years. Flattening of obelion. Flat nose. Frontal and metopic eminences. Ossicle of lambda. Others in lambdoid suture, and R. and L. asterion
62	1277	169	170	135	98	125	98	303	340	117	95	128	103	486	64	102	128	47	28	41	34	49	51	45	95	39	32	25	30	116	79·4	73·5	108·0	62·8	59·6	82·9	88·2	—	87	26	44	Aged. Lambdoid suture entirely and coronal and sagittal largely obliterated. Post-coronal depression
63	1730	191	192	141	103	140	104	345	396	135	139	122	106	531	72	97	127	50	29	41	37	50	—	42	109	37	32	27	30	124	73·4	76·0	96·6	74·2	58·0	90·2	—	123	100	36	41	About 20 years. Teeth good. Third molars uncut. Temporal ridges marked. Form of frontal as in No. 5
65	1318	172	173	134	94	138	102	303	353	119	123	111	95	494	68	95	128	47	25	41	35	48	—	38	98	35	30	21	32	107	77·5	79·8	97·1	71·6	53·2	85·4	—	—	—	—	—	40—50 years. Some obliteration of coronal and sagittal sutures. Flattening of obelion. Post-coronal depression. Temporal ridges marked. Frontal eminences and metopic crest
67	1126	168	171	133	90	126	96	295	340	115	105	120	102	475	53	90	120	40	27	40	34	—	—	—	93	30	30	22	—	108	77·8	73·7	105·6	58·9	67·5	85·0	—	—	—	—	—	Weather worn. Obliteration of coronal and sagittal sutures. Post-coronal depression. Flattening of obelion
69	1230	181	182	135	96	132	101	310	360	130	116	114	97	495	63	90	—	47	27	40	33	47	—	39	106	30	28	25	32	103	73·1	72·5	100·8	70·0	57·5	82·5	—	105	83	35	40	About 40 years. Slight superior occipital bulging. Pterion defective on L. Fronto-squamosal suture 9 mm. long on R.
71	1400	181	184	135	99	137	106	315	362	125	130	107	94	505	74	97	134	52	27	45	36	49	—	37	100	38	29	23	26	120	73·4	74·5	98·5	76·3	51·9	80·0	—	114	100	36	45	30—40 years. Sagittal suture obliterated and lower ends of coronal. Small frontal processes of squamous on R. and L. Sylvian depression between inferior frontal protuberance and temporal protuberance (most marked on R.)
73	1515	182	183	138	99	134	100	315	372	120	140	112	99	511	70	102	139	48	29	41	34	51	55	42	101	39	29	29	36	117	75·4	73·2	103·0	68·6	60·4	82·9	76·4	—	—	—	—	40—50 years. R. squamous broken. L. pterion in K
76	1520	187	187	137	103	135	100	320	371	135	125	111	95	523	70	99	140	50	25	45	36	48	50	41	—	39	31	27	32	122	73·3	72·2	101·5	70·7	50·0	80·0	82·0	121	93	30	46	Temporal ridges marked. Sylvian depression
77	1230	176	176	136	96	131	99	310	362	115	128	119	105	500	70	99	129	47	29	42	37	50	—	—	98	34	26	31	—	108	77·3	74·4	103·8	70·7	61·7	88·1	—	—	100	41	46	Temporal ridges marked. Sylvian depression. Obliteration of coronal suture. Frontal process of squamous on L. Form of frontal as in No. 5
80	1210	170	170	135	91	127	99	295	340	125	115	100	87	486	66	97	133	49	25	41	34	42	49	46	104	36	37	25	31	113	79·4	74·7	106·3	68·0	51·0	82·9	93·9	—	—	—	—	Aged. Coronal, sagittal and lambdoid sutures obliterated. Teeth much worn and alveolar rim defective. Sylvian depression and inferior frontal protuberance
87	1505	178	179	133	98	148	104	310	370	125	135	110	99	497	66	92	131	51	27	41	35	47	49	39	94	35	36	27	29	112	74·3	82·7	89·9	71·7	52·9	85·4	79·6	—	100	30	48	Sagittal and lambdoid sutures much obliterated and antero-inferior angle of parietal synostosed with sphenoid and frontal
89	1460	176	179	136	94	136	107	310	360	120	130	110	97	500	70	96	134	51	27	44	36	46	53	35	108	35	31	26	31	109	76·0	76·0	100·0	72·9	52·9	81·8	66·0	121	96	31	45	Subnasal prognathism. Sutures open. Frontal processes of squamous (Fronto-squamosal suture 20 mm. long on L., 12 mm. on R.). Inferior frontal protuberance
?	1452	177	180	135	97	135	100	300	364	122	175	67	65	500	68	96	127	47	25	41	33	44	49	35	100	40	34	24	26	109	75·0	75·0	100·0	70·8	53·2	80·5	71·4	117	99	30	44	Aged. Post-coronal depression. Temporal ridges marked. Frontal process of squamous on L. Ossicle of lambda 44 × 63 mm.
11A	1322	179	180	134	102	130	—	315	364	120	127	117	98	503	53	91	128	45	25	43	35		—	—	96	34	27	24	—	117	74·4	72·2	103·1	58·2	55·6	81·4	—	111	95	28	44	Sutures gaping. Teeth absent and alveolar margin much absorbed
40A	1412	182	183	134	94	135	103	305	372	125	133	114	100	512	57	—	—	51	27	43	40	47	51	37	98	37	34	26	33	117	72·4	73·0	99·3	—	52·9	93·0	72·6	121	97	30	41	Post-coronal depression. Slight superior occipital bulging
69A	1230	174	175	130	93	130	103	310	360	130	116	114	97	495	67	96	129	48	27	41	38	47	—	39	99	30	28	23	32	103	77·7	77·7	100·0	69·8	56·3	92·7	—	105	83	35	46	
37	1345	179	181	140	93	126	99	307	357	125	120	112	98	508	77	99	127	54	31	42	40	56	62	44	107	34	28	23	31	112	77·4	69·6	107·9	77·8	57·4	95·2	71·0	—	—	—	—	Subnasal prognathism
79	1305	184	188	132	100	137	100	312	379	130	132	117	102	510	75	105	133	51	28	43	38	45	48	38	99	36	33	25	31	108	70·2	72·9	96·4	71·4	54·9	88·4	79·2	116	97	33	45	Sagittal, coronal and lambdoid sutures obliterated. Flattening of obelion. Marked "Sutura Notha." Sloping forehead rising to an eminence as in No. 5
83	1410	178	188	131	100	145	106	315	360	130	130	100	95	498	73	99	125	53	28	42	34	52	57	42	108	38	33	28	32	116	69·7	77·1	90·3	73·7	52·8	81·0	73·7	116	106	36	44	40? Subnasal prognathism. Mental prominence strong
85	1368	180	180	133	94	130	99	310	364	137	130	97	89	505	57	85	125	46	29	38	37	—	—	—	102	33	31	27	—	116	73·9	72·2	102·3	67·1	63·0	97·4	—	—	97	—	37	Coronal suture much obliterated. Sylvian depression and inferior frontal protuberance. Metopic and frontal eminences

TABLE III*B*.

S.	REMARKS
10	Basilar synchondrosis. Permanent incisors deep in alveoli. Slight Sylvian depression r frontal protuberance
11	nal and lambdoid sutures much obliterated. Teeth somewhat worn. Sylvian depression r frontal protuberance. Subnasal prognathism. Flattening of obelion
13	d. Sagittal, coronal and lambdoid sutures much obliterated. Subnasal prognathism. poral ridges. "Front bombé." Metopic crest rising to a marked eminence somewhat the form in the Esquimaux
12	Subnasal prognathism. R. orbit sliced
11	Teeth absent and alveolar margin decayed. Small mastoid processes. Flattening of ateral post-coronal depression. Frontal processes of squamous R. and L. Two small ambda and others in R. and L. parieto-squamosal and masto-occipital sutures. Superior lging. Great width of base
13	. Obliteration of sagittal suture. Flattening of obelion. Left lateral incisor filed. cesses of squamous (Fronto-squamosal suture 12 mm. long on L. and 7 mm. on R.)
13	Upper wisdom teeth in sockets; erupted in mandible. Small posterior sagittal ossicle ssicles in lambdoid suture and asterion
11	stoid processes somewhat larger than is usual in this series, and nasal bones more prominent ttening of obelion
13	th orbits sliced, also margin of foramen magnum and mastoid processes cut and zygomatic oved as if by some sharp instrument
12	metopic eminences. Wisdom teeth not erupted. Basilar synchrondrosis. Small lambdoid suture and asterion
12	ning of obelion. Nasal bones wider at lower margin than at root
11	ight metopic and more marked frontal eminences. Flattening of obelion. Ossicle in uture
13	rs. Subnasal prognathism
13	Some obliteration of sagittal and lambdoid sutures and flattening of obelion
11	absent and alveolar margin absorbed. Sagittal suture obliterated. Sylvian depression r frontal protuberance
12	iced. Ossicles in R. and L. masto-occipital sutures. Small nasal bones
13	Infantile forehead. Coronal suture obliterated at lower ends
11	al suture obliterated. Obelion depressed. Teeth absent and alveolar margin absorbed. mastoid processes. Metopic and frontal eminences and a slight metopic crest rising to eminence (resembling No. 5)
11	absorption of alveolar margin. Sutures open. Flattening of obelion. Temporal ridges rominent nasal bones
11	al suture completely obliterated, and coronal at pterion. Broad flat nose
10	lvian depression between temporal protuberance and slight inferior frontal protuberance
11	Some obliteration of coronal suture. Molars well ground. Flattening of obelion. nasal bones
12	e much obliterated. Flattening of obelion. Very small mastoid processes
12	
13	ssion and inferior frontal protuberance. Some obliteration of coronal, sagittal and utures. Flat nose
13	ipteric on L. Post-coronal depression. Flattening of obelion. Frontal eminences
12	s. Ossicles in lambdoid and masto-occipital sutures
16	absent and alveolar margin much absorbed. Sutures distinct. Interparietal tripartite. ale and R. os triangulare separate (see Plate XXXVII, *Biometrika*, Vol. III. Part II. special 19)
13	sses of squamous (Fronto-squamosal sutures 12 mm. long on R. and 7 mm. on L.). les in lambdoid suture and R. and L. asterion
12	s. Frontal and metopic eminences. Mastoid processes larger than usual in this series toid deeply grooved
11	
11	absent and alveolar rim much absorbed. Obliteration of coronal suture
12	bda 28 mm. wide and 16 mm. high. Others in lambdoid and R. and L. masto-parietal rontal processes of squamous. (Fronto-squamosal sutures 18 mm. long R. and L.) gmatic eminence
11	lar synchondrosis. Second molar just appearing
12	s. Metopic. Slight post-coronal depression. Sylvian depression. Ossicles in lambdoid
13	absent. Alveolar margin absorbed. Lower ends of coronal suture obliterated. Frontal squamous R. and L. Posterior portion of sagittal suture irregular and joins lambdoid point 40 mm. to L. of median line. (See figure in text.)
12	Male? Flattening of obelion. Mandible relatively prominent. Subnasal prognathism. cess of squamous on L.
12	e entirely and sagittal largely obliterated. Formation of nose as in No. 11 a male of Antero-inferior angle of parietal synostosed with sphenoid and frontal
11	l processes of squamous R. and L. Small ossicles in masto-parietal and masto-occipital left of palate with complete absence of praemaxilla, but anterior to the socket of the R. e socket of a rudimentary tooth—not seen on L. The posterior portions of the palate the median line. (See Plate accompanying this paper.)
12	sion and inferior frontal protuberance
11	l teeth, well ground. Front incisors slightly filed. Flattening of obelion. Flat nose. d mastoid processes. Frontal processes of squamous R. and L. Slight metopic crest ery marked bregmatic eminence (as in No. 38)
12	ses of squamous (Fronto-squamosal suture 22 mm. long on R., 17 mm. long on L.)
12	Third molar just appearing. Small epipteric on R. and ossicles at R. and L. asterion
11	years. Sutures open. Very prognathous

1864 SERIES. CRANIA FROM FERNAND VAZ, GABOON.

TABLE III *B*.

FEMALES.

No.	*C*	*F*	*L*	*B*	*B'*	*H*	*LB*	*Q*	*S*	S_1	S_2	S_3	S_3'	*U*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1	O_2	G_1'	G_1	G_2	*GL*	*fml*	*fmb*	Dacry	B_2	*St*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH*	O_2/O_1	G_2/G_1	W_1	W_2	h_1	*f*	Remarks
2	—	163	163	123	85	—	—	288	342	120	108	114	96	452	50	71	95	35	19	36	31	32	35	28	-	—	25	19	23	102	75·5	—	—	70·4	54·3	86·1	80·0	—	—	—	—	Very young. Basilar synchondrosis. Permanent incisors deep in alveoli. Slight Sylvian depression and inferior frontal protuberance
4	1039	163	165	125	90	115	93	277	325	105	117	103	92	456	61	89	118	41	25	40	35	44	48	35	95	33	26	21	28	103	75·8	69·7	108·7	68·5	61·0	87·5	72·9	—	—	—	—	Sagittal, coronal and lambdoid sutures much obliterated. Teeth somewhat worn. Sylvian depression and inferior frontal protuberance. Subnasal prognathism. Flattening of obelion
5	1277	175	175	130	92	130	98	288	365	125	131	109	99	489	70	102	124	49	24	39	34	50	52	40	104	32	30	22	33	108	74·3	74·3	100·0	68·6	49·0	87·2	76·9	—	—	—		Dentition good. Sagittal, coronal and lambdoid sutures much obliterated. Subnasal prognathism. Marked temporal ridges. "Front bombé." Metopic crest rising to a marked eminence somewhat resembling the form in the Esquimaux
9	1198	166	168	126	94	130	94	288	350	120	122	108	95	471	71	90	120	50	24	39	37	48	52	38	97	33	26	21	28	101	75·0	77·4	96·9	78·9	48·0	94·9	73·1	–	—	—	–	30—40 years. Subnasal prognathism. R. orbit sliced
10	1165	165	166	131	92	128	90	295	347	120	116	111	99	475		98	125	47	29	40	39	—	—	—	—	30	30	25	—	109	78·9	77·1	102·3	—	61·7	97·5	—		—	—	—	40—50 years. Teeth absent and alveolar margin decayed. Small mastoid processes. Flattening of obelion. Lateral post-coronal depression. Frontal processes of squamous R. and L. Two small ossicles of lambda and others in R. and L. parieto-squamosal and masto-occipital sutures. Superior occipital bulging. Great width of base
12	1292	163	163	143	97	133	93	285	357	125	130	97	84	488	61	84	127	42	23	40	33	46	50	41	95	30	26	22	26	121	87·7	81·6	107·5	72·6	54·8	82·5	82·0		—	—	—	30—40 years. Obliteration of sagittal suture. Flattening of obelion. Left lateral incisor filed. Frontal processes of squamous (Fronto-squamosal suture 12 mm. long on L. and 7 mm. on R.)
16	1553	182	183	141	95	125	90	322	378	135	135	108	99	508	61	93	118	45	22	38	32	48	50	37	95	33	27	21	33	119	77·1	68·3	112·8	65·6	48·9	84·2	74·0	–	—	—	—	Young adult. Upper wisdom teeth in sockets; erupted in mandible. Small posterior sagittal ossicle and small ossicles in lambdoid suture and asterion
17	1134	163	166	129	92	130	98	297	342	125	116	101	92	464	63	87	116	47	25	40	33	43	—	38	96	34	29	22	31	109	77·7	78·3	99·2	72·4	53·2	82·5	—	105	97	30	45	30 years. Mastoid processes somewhat larger than is usual in this series, and nasal bones more prominent. Flattening of obelion
19	1142	166	167	131	91	124	94	300	335	110	135	90	81	472	—	—	—	43	25	38	35	-	—	—	—	—	—	23	—	109	78·4	74·3	105·7	-	58·1	92·1		-	—	—	—	40 years. Both orbits sliced, also margin of foramen magnum and mastoid processes cut and zygomatic arches removed as if by some sharp instrument
20	1260	172	172	126	95	135	98	305	350	120	125	105	92	475	67	87	112	46	21	40	37	45	47	34	95	37	29	21	30	111	73·3	78·5	93·3	77·0	45·7	92·5	72·3		—	—	—	Frontal and metopic eminences. Wisdom teeth not erupted. Basilar synchondrosis. Small ossicles in lambdoid suture and asterion
21	1335	178	179	135	96	136	100	310	363	125	120	118	101	503	68	99	123	48	26	43	36	46	47	36	98	39	30	24	27	—	75·4	76·0	99·3	68·7	54·2	83·7	76·6			--	—	Aged. Flattening of obelion. Nasal bones wider at lower margin than at root
22	1295	171	173	122	96	137	99	302	350	127	118	105	90	480	67	95	122	50	27	42	36	49	52	37	100	34	28	24	31	106	70·5	79·2	89·1	70·5	54·0	85·7	71·2		—	—	—	30 years. Slight metopic and more marked frontal eminences. Flattening of obelion. Ossicle in lambdoid suture
23	1300	174	175	132	86	127	94	295	300	115	130	115	95	488	64	90	115	46	26	39	34	48	51	40	101	33	27	25	29	107	75·4	72·6	103·9	71·1	56·6	87·2	78·4	105	86	—	42	About 30 years. Subnasal prognathism
24	1270	161	165	135	93	138	98	308	348	118	132	98	92	478	61	81	120	47	24	39	33	45	48	30	93	32	29	22	25	105	81·8	83·6	97·8	75·3	51·1	84·6	62·5	105	90	34	46	40—50 years. Some obliteration of sagittal and lambdoid sutures and flattening of obelion
26	1276	170	172	133	—	131	89	290	345	117	118	110	97	488	—	—	—	43	25	—	—	—		—	—	—	28	—	—	113	17·3	76·2	101·5	—	58·1	—	-	–	—	—	—	Aged. Teeth absent and alveolar margin absorbed. Sagittal suture obliterated. Sylvian depression and inferior frontal protuberance
29	1220	173	175	130	88	130	93	295	363	125	125	113	104	485	66	93	118	46	23	40	32	52	55	37	104	31	27	23	28	105	74·3	74·3	100·0	71·0	50·0	80·0	67·3		—		—	Both orbits sliced. Ossicles in R. and L. masto-occipital sutures. Small nasal bones
35	1283	178	178	131	91	132	102	304	355	130	135	90	85	492	66	87	120	47	25	36	31	47	50	36	101	36	28	23	28	113	73·6	74·2	99·2	75·9	53·2	86·1	72·0			–	—	40—50 years. Infantile forehead. Coronal suture obliterated at lower ends
38	1137	172	174	123	91	133	96	292	350	122	116	118	105	470	—	86	122	44	24	41	36	—	—	—	-	30	28	22	—	104	70·7	70·4	92·5	—	54·6	87·8	—	—	82	31	45	Aged. Coronal suture obliterated. Obelion depressed. Teeth absent and alveolar margin absorbed. Very small mastoid processes. Metopic and frontal eminences and a slight metopic crest rising to a bregmatic eminence (resembling No. 5)
39	1060	156	157	129	84	126	92	288	330	110	113	107	86	452	64	89	122	44	27	38	33	51	54	—	99	31	24	20	29	99	82·2	80·3	102·4	71·9	61·4	86·8	—	105	81	28	43	Aged? Some absorption of alveolar margin. Sutures open. Flattening of obelion. Temporal ridges marked. Prominent nasal bones
41	1093	169	169	131	96	123	93	283	340	120	115	105	91	479	53	94	124	43	22	40	35	47	—	37	91	34	30	28	28	110	77·5	72·8	106·5	56·4	51·2	87·5	—	113	91	32	47	Aged. Sagittal suture completely obliterated, and coronal at pterion. Broad flat nose
42	1025	162	162	125	90	124	95	272	322	114	101	107	90	460	57	80	122	45	27	37	31	42	46	38	97	38	26	24	29	98	77·2	76·5	100·8	66·3	60·0	83·8	82·6	—	--	—	—	40 years. Sylvian depression between temporal protuberance and slight inferior frontal protuberance
46	1192	171	172	135	98	126	96	308	336	122	114	100	91	486	62	92	121	45	26	39	33	—	—	36	97	34	28	25	25	117	78·5	73·3	107·1	67·4	57·8	84·6	—	–	97	36	47	40—50 years. Some obliteration of coronal suture. Molars well ground. Flattening of obelion. Broad flat nasal bones
48	1266	179	180	125	93	133	101	292	367	130	120	117	99	491	62	86	113	46	26	40	35	44	49	38	96	32	27	18	28	105	69·4	73·9	94·0	72·1	56·5	87·5	77·6	107	89	25	40	Coronal suture much obliterated. Flattening of obelion. Very small mastoid processes
51	1220	167	167	132	97	131	95	294	343	120	125	98	89	475	64	93	127	44	27	42	34	49	52	33	94	33	22	26	27	113	79·0	78·4	100·8	68·8	61·4	81·0	63·5	113	91	31	47	Aged
52	1587	181	181	137	94	141	100	310	376	137	130	109	99	513	68	96	128	48	25	38	33	50	54	37	98	36	32	25	28	118	75·7	77·9	97·2	70·8	52·1	86·8	68·5	—	—	—	—	Sylvian depression and inferior frontal protuberance. Some obliteration of coronal, sagittal and lambdoid sutures. Flat nose
54	1205	173	173	128	99	133	102	296	355	122	136	97	87	485	68	93	124	47	27	41	35	45	52	39	103	34	28	29	33	105	74·0	76·9	96·2	73·1	57·5	85·4	75·0	115	99	30	45	Metopic. Epipteric on L. Post-coronal depression. Flattening of obelion. Frontal eminences marked
56	1290	170	172	128	93	136	106	292	343	125	121	97	92	475	67	96	126	49	23	41	34	54	57	42	109	35	32	24	30	96	74·4	79·1	94·1	69·8	46·9	82·9	73·7	113	96	30	45	About 30 years. Ossicles in lambdoid and masto-occipital sutures
58	1215	177	177	127	93	125	99	285	342	112	168	68	63	490	60	94	125	48	26	42	35	-	—	—	97	38	32	26		104	71·8	70·6	101·6	63·8	54·2	83·3	—	110	89	32	43	Aged. Teeth absent and alveolar margin much absorbed. Sutures distinct. Interparietal tripartite. Os pentagonale and R. os triangulare separate (see Plate XXXVII, *Biometrika*, Vol. III. Part II. special skull W. 219)
60	1262	170	173	130	86	132	94	292	362	122	130	110	97	478	65	96	121	44	27	41	31	—	—	38	101	32	31	18	31	108	75·1	76·3	98·5	67·7	61·4	75·6	—	—	—		—	Frontal processes of squamous (Fronto-squamosal sutures 12 mm. long on R. and 7 mm. on L.). Small ossicles in lambdoid suture and R. and L. asterion
64	1290	177	177	127	97	133	98	305	353	127	125	101	88	485	64	97	120	46	25	42	34	46	47	42	100	38	26	25	31	114	71·8	75·1	95·5	66·0	54·4	81·0	89·4	105	101	35	48	About 20 years. Frontal and metopic eminences. Mastoid processes larger than usual in this series and R. mastoid deeply grooved
66	1236	173	173	129	91	132	99	297	345	120	118	107	95	480	67	92	—	44	24	40	34	51	54	39	105	33	27	22	30	111	74·6	76·3	97·7	72·8	54·6	85·0	72·2	110	100	29	47	
68	1175	168	168	136	86	123	92	303	315	122	118	105	95	486	61	87	121	46	26	37	35	46	50	—	95	35	29	22	—	106	81·0	73·2	110·6	70·1	56·5	94·6	—	—	—	—	—	Aged. Teeth absent and alveolar rim much absorbed. Obliteration of coronal suture
70	1300	172	173	135	100	131	96	304	348	123	127	98	88	491	59	89	124	42	24	40	33	49	52	42	101	35	30	27	30	120	78·0	75·7	103·1	66·3	57·1	82·5	80·8	—	--	—	—	Ossicle of lambda 28 mm. wide and 16 mm. high. Others in lambdoid and R. and L. masto-parietal sutures. Frontal processes of squamous. (Fronto-squamosal sutures 18 mm. long R. and L.) Marked bregmatic eminence
72	1200	165	166	128	90	120	87	290	335	125	110	100	85	464	51	77	113	37	21	36	31	40	44	31	87	36	30	14	26	107	77·1	72·3	106·7	66·2	56·8	86·1	70·5	–	71	22	43	Young. Basilar synchondrosis. Second molar just appearing
74	1220	169	169	130	95	126	91	292	355	120	120	115	89	478	59	87	118	45	22	39	31	44	48	37	89	31	27	26	30	113	76·9	74·6	103·2	67·8	48·9	79·5	77·1	—	—	—	—	About 30 years. Metopic. Slight post-coronal depression. Sylvian depression. Ossicles in lambdoid suture
75	1035	172	173	130	96	128	92	300	348	122	130	96	89	476	59	92	117	44	26	40	34	45	48	—	92	34	24	26	–	111	75·1	74·0	101·6	64·1	59·1	85·0	—	109	84	—	40	Aged. Teeth absent. Alveolar margin absorbed. Lower ends of coronal suture obliterated. Frontal processes of squamous R. and L. Posterior portion of sagittal suture irregular and joins lambdoid suture at a point 40 mm. to L. of median line. (See figure in text.)
78	1295	178	179	130	95	130	96	308	355	125	120	110	97	498	67	98	126	50	26	40	34	47	52	37	99	39	35	23	31	108	72·6	72·6	100·0	68·4	52·0	85·0	71·2	115	87	33	47	Young adult. Male? Flattening of obelion. Mandible relatively prominent. Subnasal prognathism. Frontal process of squamous on L.
81	1257	174	176	138	101	131	96	303	375	130	125	120	101	495	54	100	132	40	24	43	32	46	49	41	103	35	27	26	29	113	78·4	74·4	105·3	54·0	60·0	74·4	83·7	119	84	30	44	Coronal suture entirely and sagittal largely obliterated. Formation of nose as in No. 11 a male of this series. Antero-inferior angle of parietal synostosed with sphenoid and frontal
82	1262	169	170	135	96	126	95	300	347	111	119	114	98	483	—	91	122	—	-	41	33	-	—	—	—	31	29	24	—	110	79·4	74·1	107·1	—	—	80·5	—	—	—	—	—	Aged. Frontal processes of squamous R. and L. Small ossicles in masto-parietal and masto-occipital sutures. Cleft of palate with complete absence of praemaxilla, but anterior to the socket of the R. canine is the socket of a rudimentary tooth—not seen on L. The posterior portions of the palate are united in the median line. (See Plate accompanying this paper.)
84	1398	174	175	137	91	137	100	320	360	120	124	116	103	493	66	—	—	47	24	37	33	50	53	41	106	36	30	22	30	103	78·3	78·3	100·0	—	51·1	89·2	77·4	114	90	28	41	Sylvian depression and inferior frontal protuberance
86	1033	171	172	132	94	140	101	285	343	120	113	110	96	473	64	90	124	44	24	39	30	44	48	34	108	32	29	26	28	111	76·7	81·4	94·3	71·1	54·6	76·9	70·8	106	93	32	45	30—40. Good teeth, well ground. Front incisors slightly filed. Flattening of obelion. Flat nose. Long pointed mastoid processes. Frontal processes of squamous R. and L. Slight metopic crest rising to a very marked bregmatic eminence (as in No. 38)
88	1305	179	179	130	94	140	99	307	370	129	121	126	107	499	66	91	121	47	28	40	34	49	—	40	97	37	28	26	29	112	72·6	78·2	92·9	72·5	59·6	85·0	—	--	—	33	41	Frontal processes of squamous (Fronto-squamosal suture 22 mm. long on R., 17 mm. long on L.)
91	1430	177	179	131	92	134	98	304	365	130	120	115	96	493	62	99	118	44	26	39	33	47	50	42	104	34	31	21	33	113	73·2	74·9	97·8	62·6	59·1	84·6	84·0	--	—	—	—	Young adult. Third molar just appearing. Small epipteric on R. and ossicles at R. and L. asterion
90	930	157	158	124	87	117	89	272	320	110	110	100	85	457	67	88	116	46	24	39	34	53	57	38	99	27	23	20	30	99	78·5	74·1	106·0	76·1	52·2	87·2	66·7	111	93	23	43	About 39—40 years. Sutures open. Very prognathous

BRITISH MUSEUM OF NATURAL HISTORY. 1880 SERIES. CRANIA FROM FERNAND VAZ, GABOON.

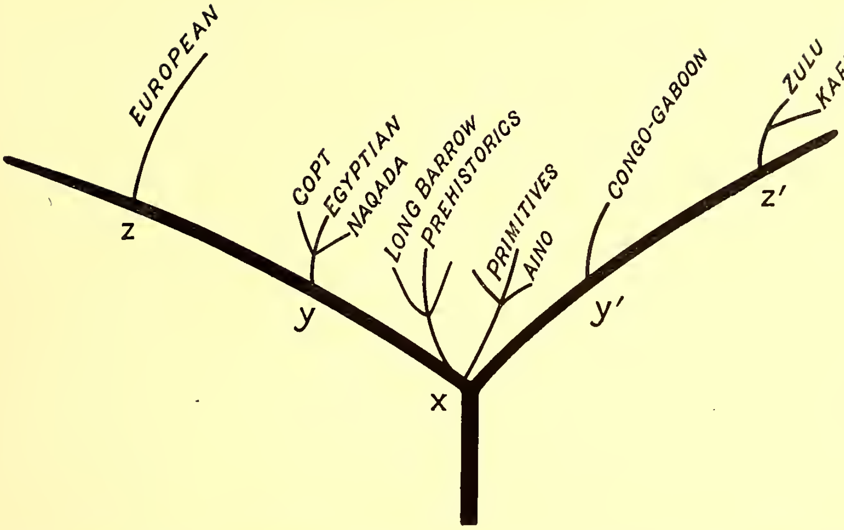
TABLE IV.

MALES.

No.	C	F	L	B	B'	H	LB	Q	S	S_1	S_2	S_3	S_3'	U	$G'H$	GB	J	NH	NB	O_1	O_2	G_1'	G_1	G_2	GL	fml	fmb	Dacry	B_2	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1	G_2/G_1	W_1	W_2	h_1	j	Remarks
1	1513	185	186	137	98	134	105	306	371	137	120	108	96	520	69	95	—	48	26	42	35	47	—	44	103	37	32	27	30	118	73·7	72·0	102·2	72·6	54·2	83·3	—	121	96	35	47	About 50 years old. Obliteration of sagittal suture. Flattening of obelion
3	1622	186	188	141	104	142	103	315	387	149	123	115	102	525	66	99	139	48	29	45	38	47	—	41	100	38	31	27	35	123	75·0	75·5	99·3	66·7	60·4	84·4	—	129	110	30	40	About 50 years old. Obliteration of sagittal suture. Inion well marked. Slight post-coronal depression. Pterion in K
4	1448	178	180	139	103	140	103	318	360	126	127	107	94	510	75	106	142	47	28	45	35	50	56	43	109	42	33	30	33	117	77·2	77·8	99·3	70·8	59·6	77·8	76·8	125	102	27	42	Ossicles in lambdoid suture. Posterior sagittal ossicle. Subnasal prognathism
5	1453	183	184	140	106	141	106	308	377	130	130	117	98	513	69	92	134	50	29	41	34	57	63	39	110	35	33	26	30	116	76·1	76·6	99·3	75·0	58·0	82·9	61·9	123	95	33	50	Well marked inion and mastoids. Ossicles in lambdoid suture. Coronal and sagittal sutures nearly obliterated. Marked temporal ridges. Frontal portion of orbits slender. R. frontal process of squamous 7 mm. long
14	1270	174	174	131	93	128	93	330	360	125	125	110	107	488	67	94	125	46	27	42	34	51	—	37	100	34	25	26	24	111	75·3	73·6	102·3	71·3	58·7	81·0	—	113	93	35	50	20—30 years old. Flattening of obelion. Epipteric bones, 28 mm. R., 19 mm. L. Very small mastoids. Ossicle in lambdoid suture
22	1330	177	179	134	94	128	100	305	365	125	120	115	102	498	-	98	126	45	25	42	36	—	—	—	101	32	26	22	—	105	74·9	71·5	104·7	—	55·6	85·7	—	-	—	—	—	Sutures distinct, but teeth absent and alveolar margin absorbed. Lambdoid ossicles
23	1440	177	179	143	101	139	103	317	369	125	127	117	102	512	68	91	131	50	27	41	31	50	—	40	100	33	30	28	32	118	79·9	77·7	102·9	74·7	54·0	75·6	—	120	114	30	48	Very small mastoids and inion. Ossicles in lambdoid suture and at asterion. Sylvian depression
24	1335	178	181	132	100	136	106	298	357	120	122	115	101	492	-	99	136	51	29	42	40	50	52	32	—	34	31	22	29	106	72·9	75·1	97·1	—	56·9	95·2	61·5	115	83	32	47	Obliteration of sagittal suture. Sylvian depression. Slight bathrocephaly
26	1340	181	182	131	96	141	107	—	370	123	123	124	109	—	73	105	132	52	26	43	34	50	59	45	105	36	30	23	30	117	72·0	77·5	92·9	69·5	50·0	79·1	76·3	—	—	—	—	Flattening of obelion. Temporal ridges marked
27	1520	180	182	148	98	143	103	315	382	143	120	119	100	518	55	95	136	44	27	44	35	45	—	—	92	34	28	23	—	123	81·3	78·6	103·5	57·9	61·4	79·6	—	—	—	—	—	Sutures distinct, teeth absent, alveolar margin absorbed. Ossicles in lambdoid
28	1343	168	175	137	91	132	95	305	358	122	125	111	98	480	67	103	135	45	27	42	34	52	55	44	105	37	30	23	37	107	78·3	75·4	103·8	65·1	60·0	81·0	80·0	-	—	—	—	Very retreating forehead. Nasal bones depressed at root. Prominent overhanging frontal. Subnasal prognathism
29	1640	185	186	143	94	138	101	320	382	130	139	113	101	524	60	98	132	47	27	40	38	44	—	—	93	37	34	25	—	121	76·9	74·2	103·6	61·2	57·5	95·0	—	—	—	—	—	40 years old?
30	1470	186	187	135	95	135	102	315	370	134	136	100	89	512	71	95	128	52	31	41	33	54	57	40	110	39	33	28	31	119	72·2	72·2	100·0	74·7	59·6	80·5	70·2	—	—	—	—	Flattening of obelion and pterion. Both orbits sliced
31	1350	175	176	145	101	131	98	312	361	119	119	123	102	509	70	93	127	47	25	42	37	50	—	43	102	32	25	30	32	122	82·4	74·4	110·7	75·3	53·2	88·1	—	—	—	—	—	Flattening of obelion and pterion. R. pterion in K. Flat nasal bones
32	1485	187	191	143	101	142	106	310	375	132	128	115	100	524	—	105	140	49	27	42	32	—	—	—	—	35	29	—	30	116	74·9	74·4	100·7	—	55·1	76·2	—	—	—	—	—	Squamous suture straight and nearly horizontal. Inion and mastoids immature. Frontal process of squamous extends beyond sphenoid R. and L. Central incisors drawn
35	1600	190	190	134	101	142	107	302	385	130	138	117	103	518	71	101	133	51	27	43	35	54	59	40	110	35	30	27	35	111	70·5	74·7	94·4	70·3	52·9	81·4	67·8	—	—	—	—	Wisdom tooth just appearing. Slight obliteration of coronal suture. Subnasal prognathism
12a	-	175	176	132	96	129	95			—	—	—	—	—	68	—	125	44	23	41	35	-	—		101	—	—	22	—	—	75·0	73·3	102·3	—	52·3	85·4	—	—	—	—	—	Flattening of obelion. ♂? Aged about 30
20	--	184	185	143	99	139	101			—		—	—	—	60	100	132	47	27	39	37		—	—	—	—	—	25	—	—	76·9	74·7	102·9	60·0	57·5	94·9	—	—	—	—	—	Edentulous. Flattening of obelion. Sutures closed. Sylvian depression. Slight bathrocephaly

FEMALES.

No.	C	F	L	B	B'	H	LB	Q	S	S_1	S_2	S_3	S_3'	U	$G'H$	GB	J	NH	NB	O_1	O_2	G_1'	G_1	G_2	GL	fml	fmb	Dacry	B_2	St	B/L	H/L	B/H	$G'H/GB$	NB/NH	O_2/O_1	G_2/G_1	W_1	W_2	h_1	j	Remarks
2	1195	168	168	126	89	133	100	293	315	120	112	113	101	468	67	—	—	45	25	38	33	45	51	36	100	36	31	20	30	90	75·0	79·2	94·7	—	55·6	86·8	70·6	104	85	30	46	Weatherworn. Young adult female
6	1130	160	161	134	89	131	94	294	330	119	116	95	82	465	64	93	—	47	25	41	36	—	—	37	96	36	31	22	27	103	83·2	81·4	102·3	68·8	53·2	87·8	—	102	93	26	46	Middle aged
7	1190	166	168	132	89	130	94	285	345	115	130	100	90	473	57	89	118	46	27	39	32	—	—	40	82	34	28	23	37	102	78·6	77·4	101·5	64·0	58·7	82·1	—	—	—	-	—	Squamous and coronal sutures obliterated. Aged. Four incisors drawn
8	—	162	162	122	91	—	—	282	333	113	114	106	81	452	55	82	106	35	21	35	32	41	45	39	—	—	—	23	31	99	75·3	—	—	67·1	60·0	91·4	86·7	90	82	30	45	About 14 years old. Sagittal suture obliterated. Large interparietal, 86 mm. broad, 43 mm. long
9	1165	167	167	127	89	121	91	296	345	125	117	103	90	468	56	81	—	41	34	38	32	37	40	31	90	28	24	21	27	103	76·1	72·5	105·0	69·1	82·9	84·2	77·5	-	—	—	—	Under 14. Flattening of obelion. Very marked metopic eminence
10	1134	165	169	123	90	129	96	293	333	115	115	103	89	463	56	84	114	43	24	40	34	45	—	—	95	35	27	22	28	103	72·8	76·3	95·4	66·7	55·8	85·0	—	107	80	22	47	Edentulous. Aged. Flattening of obelion. Ossicle in lambdoid. Sylvian depression. Epipteric on R.
12b	1095	166	167	130	97	128	93	298	347	121	115	111	99	475	65	91	124	44	24	41	34	47	52	—	102	30	25	23	—	113	77·8	76·7	101·6	71·4	54·6	82·9	—	109	91	27	56	Sutures open, except end of coronal at pterion. Subnasal prognathism
15	1308	188	190	129	95	139	105	310	335	115	115	105	99	515	71	92	120	52	27	41	37	52	56	35	109	31	30	23	32	108	67·9	73·2	92·8	77·2	51·9	90·2	62·5	—	—	—	—	Coronal much and sagittal entirely obliterated. Edentulous. Aged. Marked
17	1150	161	164	124	88	131	93	285	343	120	121	102	92	462	60	93	118	41	24	39	33	50	56	40	100	36	28	26	31	109	75·6	79·9	94·7	64·5	58·5	84·6	71·4	116	99	32	45	Middle aged. Flattening of obelion [frontal eminences
18	—	174	175	131	92	126	96	302	355	122	123	110	93	493	70	-	—	—	—	40	37	—	—	—	102	41	30	23	—	101	74·9	72·0	104·0	—	—	92·5	—	—	—	—	—	Middle aged. Sutures open [10 mm. long
19	1255	180	181	130	89	130	101	292	360	120	120	120	100	499	64	87	120	46	25	40	36	46	51	36	100	36	28	20	29	97	71·8	71·8	100·0	73·6	54·4	90·0	70·6	—	—	—	—	Aged. Sagittal and coronal much obliterated; frontal process of squamous
21	1195	176	176	132	97	137	102	308	358	120	135	103	91	490	71	96	127	47	27	42	37	49	51	42	106	34	32	25	26	109	75·0	77·8	96·4	74·0	57·5	88·1	82·4	—	—	—	—	Middle aged. Sutures distinct. Metopic eminence. Subnasal prognathism
33	1295	178	178	129	101	137	102	300	360	125	120	115	103	495	51	91	118	47	27	42	35	—	—	—	89	32	27	24	—	113	72·5	77·0	94·2	56·0	57·5	83·3	—	—	—	—	—	Edentulous. Aged. Pterion in K
34	1268	163	163	133	98	129	96	300	342	115	123	106	96	480	65	86	123	48	25	44	34	47	—	43	96	35	30	23	29	108	81·6	79·1	103·1	75·6	52·1	77·3	—	—	—	—	—	Middle aged. Sutures open. External angular process *gnawed* or filed
36	1338	166	167	137	94	132	95	300	360	124	131	105	94	482	65	97	127	50	25	39	34	44	52	36	96	34	31	22	29	106	82·0	79·0	103·8	67·0	50·0	87·2	69·2	—	—	—	—	Middle aged. Medium incisors drawn. Flattened obelion
37	1444	176	179	141	91	141	102	320	370	123	140	107	101	500	64	93	122	46	27	41	32	52	58	33	109	33	29	20	30	112	78·8	78·8	100·0	68·8	58·7	78·1	56·9	112	98	27	47	Old. Sagittal nearly obliterated. Lateral post-coronal depression. Frontal process of squamous R. and L.
38	1315	177	177	138	100	133	100	312	365	137	121	107	93	503	57	100	131	47	28	43	31	—	—	—	97	32	28	23	—	110	78·0	75·1	103·8	57·0	59·6	72·1	—	113	91	21	44	Edentulous. Old. Flattening of obelion. Flat nose
13	1235	167	167	131	97	134	98	300	347	126	112	109	97	478	68	96	123	47	25	39	35	—	—	—	99	30	28	25	—	111	78·4	80·2	97·8	70·8	53·2	89·7	—	104	92	38	51	Middle aged. Flattening of obelion
39	1165	168	169	130	92	126	95	295	343	126	114	103	91	472	61	90	114	44	22	40	34	52	57	—	99	34	26	23	—	104	76·9	74·6	103·2	67·8	50·0	85·0	—	—	—	—	—	Four incisors drawn. Only one molar left on jaw. Sutures open. Sylvian depression. Ossicle in asterion, and on R. lambdoid. Very small mastoids. Aged female



Ancestral races on the main stems, as at x, y z, y' z', may be at present unknown. There may be many more branches to the main stems, and there are undoubtedly several other main stems, but the Congo-Gaboon crania serve a useful purpose in definitely linking up Egyptian and negroid crania, and thus in ultimately associating the Negro races with the European. If European and Egyptian admit of a common ancestry, the differentiation of Egyptian and Congo-Gaboon is no greater, and from the latter to the Kaffir-Zulu is in no sense a wider step; thus we no longer need to question a common ancestry for Negro and Caucasian, or to suppose a double origin for man. If in some respects the Kaffir-Zulu is closer to the European than the Congo-Gaboon group, this may be attributed to the development from y to z being to some extent similar to that from y' to z'. European and Kaffir may be far apart horizontally on the tree, but vertically their divergence may be less. In mere length of head the Zulu-Kaffir is closer to the European than the Congoese, but no one who studies carefully the tables given in this paper will believe the link lies otherwise than from Zulu-Kaffir to Congo-Gaboon, and from Congo-Gaboon to Egyptian, and Egyptian to European.

The main purpose of this memoir was stated to be one of suggestion; to suggest how much more extensive material is needed and how crying is the need for standardisation in the measurement of it when procured. If the hypothesis of this concluding section be contradicted by the careful measurement of larger series, this memoir will still have served most effectively its purpose for it will have led to a recognition of the need for longer series. And no more ambitious purpose ever occurred to the mind of Dr Crewdson Benington himself when he started at my suggestion the measurement of the few available crania from the Gaboon and Congo, than this one of indicating how suggestive the study of the negro skull is, and how incomplete it must remain, until more ample material is available.

Table to pass from measured index $\beta = 100$ *(arc − chord)/chord of a curve to the index and may be closely represented by a common catenary. Suggested use: to pass*

Values of α for given values of β as argument.

β	*·0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
13	23·1	23·2	23·2	23·3	23·4	23·5	23·6	23·7	23·8	23·9
14	24·0	24·1	24·2	24·3	24·4	24·5	24·6	24·7	24·7	24·8
15	24·9	25·0	25·1	25·2	25·3	25·4	25·5	25·6	25·6	25·7
16	25·8	25·9	26·0	26·1	26·2	26·3	26·4	26·4	26·5	26·6
17	26·7	26·8	26·9	27·0	27·0	27·1	27·2	27·3	27·4	27·5
18	27·6	27·7	27·7	27·8	27·9	28·0	28·1	28·2	28·3	28·3
19	28·4	28·5	28·6	28·7	28·7	28·8	28·9	29·0	29·1	29·2
20	29·2	29·3	29·4	29·5	29·6	29·6	29·7	29·8	29·9	30·0
21	30·0	30·1	30·2	30·3	30·4	30·4	30·5	30·6	30·7	30·8
22	30·8	30·9	31·0	31·1	31·2	31·2	31·3	31·4	31·5	31·6
23	31·6	31·7	31·8	31·9	31·9	32·0	32·1	32·2	32·3	32·3
24	32·4	32·5	32·6	32·6	32·7	32·8	32·9	32·9	33·0	33·1
25	33·2	33·3	33·3	33·4	33·5	33·6	33·6	33·7	33·8	33·9
26	33·9	34·0	34·1	34·2	34·2	34·3	34·4	34·5	34·5	34·6
27	34·7	34·8	34·8	34·9	35·0	35·1	35·1	35·2	35·3	35·3
28	35·4	35·5	35·6	35·6	35·7	35·8	35·9	35·9	36·0	36·1
29	36·2	36·2	36·3	36·4	36·4	36·5	36·6	36·7	36·7	36·8
30	36·9	36·9	37·0	37·1	37·2	37·2	37·3	37·4	37·5	37·5
31	37·6	37·7	37·7	37·8	37·9	38·0	38·0	38·1	38·2	38·2
32	38·3	38·4	38·4	38·5	38·6	38·7	38·7	38·8	38·9	38·9
33	39·0	39·1	39·2	39·2	39·3	39·4	39·4	39·5	39·6	39·6
34	39·7	39·8	39·8	39·9	40·0	40·1	40·1	40·2	40·3	40·3
35	40·4	40·5	40·5	40·6	40·7	40·7	40·8	40·9	41·0	41·0
36	41·1	41·2	41·2	41·3	41·4	41·4	41·5	41·6	41·6	41·7
37	41·8	41·8	41·9	42·0	42·0	42·1	42·2	42·2	42·3	42·4
38	42·4	42·5	42·6	42·6	42·7	42·8	42·9	42·9	43·0	43·1
39	43·1	43·2	43·3	43·3	43·4	43·5	43·5	43·6	43·7	43·7
40	43·8	43·9	43·9	44·0	44·1	44·1	44·2	44·3	44·3	44·4
41	44·5	44·5	44·6	44·6	44·7	44·8	44·8	44·9	45·0	45·0
42	45·1	45·2	45·2	45·3	45·4	45·4	45·5	45·6	45·6	45·7
43	45·8	45·8	45·9	46·0	46·0	46·1	46·2	46·2	46·3	46·4
44	46·4	46·5	46·5	46·6	46·7	46·7	46·8	46·9	46·9	47·0
45	47·1	47·1	47·2	47·3	47·3	47·4	47·5	47·5	47·6	47·6
46	47·7	47·8	47·8	47·9	48·0	48·0	48·1	48·2	48·2	48·3
47	48·4	48·4	48·5	48·5	48·6	48·7	48·7	48·8	48·9	48·9
48	49·0	49·1	49·1	49·2	49·2	49·3	49·4	49·4	49·5	49·6
49	49·6	49·7	49·8	49·8	49·9	49·9	50·0	50·1	50·1	50·2
50	50·3	50·3	50·4	50·5	50·5	50·6	50·6	50·7	50·8	50·8
51	50·9	51·0	51·0	51·1	51·1	51·2	51·3	51·3	51·4	51·5
52	51·5	51·6	51·6	51·7	51·8	51·8	51·9	52·0	52·0	52·1
53	52·1	52·2	52·3	52·3	52·4	52·5	52·5	52·6	52·6	52·7
54	52·8	52·8	52·9	53·0	53·0	53·1	53·1	53·2	53·3	53·3
55	53·4	53·4	53·5	53·6	53·6	53·7	53·8	53·8	53·9	53·9
56	54·0	54·1	54·1	54·2	54·3	54·3	54·4	54·4	54·5	54·6
57	54·6	54·7	54·7	54·8	54·9	54·9	55·0	55·0	55·1	55·2
58	55·2	55·3	55·4	55·4	55·5	55·5	55·6	55·7	55·7	55·8
59	55·8	55·9	56·0	56·0	56·1	56·1	56·2	56·3	56·3	56·4
60	56·5	56·5	56·6	56·6	56·7	56·8	56·8	56·9	56·9	57·0
61	57·1	57·1	57 2	57·2	57·3	57·4	57·4	57·5	57·5	57·6
62	57·7	57·7	57·8	57·8	57·9	58·0	58·0	58·1	58·1	58·2
63	58·3	58·3	58·4	58·4	58·5	58·6	58·6	58·7	58·7	58·8
64	58·9	58·9	59·0	59·0	59·1	59·2	59·2	59·3	59·3	59·4

$\alpha = 100$ *subtense/chord, on the assumption that the curve is symmetrical about the subtense from callipers and tape measurements of the nasal bridge to the ratio of "rise" to "span."*

Values of α for given values of β as argument.

β	*·0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
65	59·5	59·5	59·6	59·6	59·7	59·8	59·8	59·9	59·9	60·0
66	60·1	60·1	60·2	60·2	60·3	60·4	60·4	60·5	60·5	60·6
67	60·7	60·7	60·8	60·8	60·9	61·0	61·0	61·1	61·1	61·2
68	61·3	61·3	61·4	61·4	61·5	61·6	61·6	61·7	61·7	61·8
69	61·9	61·9	62·0	62·0	62·1	62·1	62·2	62·3	62·3	62·4
70	62·4	62·5	62·6	62·6	62·7	62·7	62·8	62·9	62·9	63·0
71	63·0	63·1	63·1	63·2	63·3	63·3	63·4	63·4	63·5	63·6
72	63·6	63·7	63·7	63·8	63·9	63·9	64·0	64·0	64·1	64·1
73	64·2	64·3	64·3	64·4	64·4	64·5	64·6	64·6	64·7	64·7
74	64·8	64·9	64·9	65·0	65·0	65·1	65·1	65·2	65·3	65·3
75	65·4	65·4	65·5	65·6	65·6	65·7	65·7	65·8	65·8	65·9
76	66·0	66·0	66·1	66·2	66·2	66·3	66·3	66·4	66·4	66·5
77	66·5	66·6	66·7	66·7	66·8	66·8	66·9	66·9	67·0	67·1
78	67·1	67·2	67·2	67·3	67·4	67·4	67·5	67·5	67·6	67·6
79	67·7	67·8	67·8	67·9	67·9	68·0	68·0	68·1	68·2	68·2
80	68·3	68·3	68·4	68·5	68·5	68·6	68·6	68·7	68·7	68·8
81	68·9	68·9	69·0	69·0	69·1	69·1	69·2	69·3	69·3	69·4
82	69·4	69·5	69·5	69·6	69·7	69·7	69·8	69·8	69·9	70·0
83	70·0	70·1	70·1	70·2	70·2	70·3	70·4	70·4	70·5	70·5
84	70·6	70·6	70·7	70·8	70·8	70·9	70·9	71·0	71·0	71·1
85	71·2	71·2	71·3	71·3	71·4	71·4	71·5	71·6	71·6	71·7
86	71·7	71·8	71·8	71·9	72·0	72·0	72·1	72·1	72·2	72·2
87	72·3	72·4	72·4	72·5	72·5	72·6	72·6	72·7	72·8	72·8
88	72·9	72·9	73·0	73·0	73·1	73·2	73·2	73·3	73·3	73·4
89	73·4	73·5	73·6	73·6	73·7	73·7	73·8	73·8	73·9	73·9
90	74·0	74·1	74·1	74·2	74·2	74·3	74·3	74·4	74·5	74·5
91	74·6	74·6	74·7	74·7	74·8	74·9	74·9	75·0	75·0	75·1
92	75·1	75·2	75·3	75·3	75·4	75·4	75·5	75·5	75·6	75·6
93	75·7	75·8	75·8	75·9	75·9	76·0	76·0	76·1	76·2	76·2
94	76·3	76·3	76·4	76·4	76·5	76·6	76·6	76·7	76·7	76·8
95	76·8	76·9	76·9	77·0	77·1	77·1	77·2	77·2	77·3	77·3
96	77·4	77·5	77·5	77·6	77·6	77·7	77·7	77·8	77·8	77·9
97	78·0	78·0	78·1	78·1	78·2	78·2	78·3	78·3	78·4	78·5
98	78·5	78·6	78·6	78·7	78·7	78·8	78·9	78·9	79·0	79·0
99	79·1	79·1	79·2	79·2	79·3	79·4	79·4	79·5	79·5	79·6
100	79·6	79·7	79·8	79·8	79·9	79·9	80·0	80·0	80·1	80·1
101	80·2	80·3	80·3	80·4	80·4	80·5	80·5	80·6	80·6	80·7
102	80·8	80·8	80·9	80·9	81·0	81·0	81·1	81·1	81·2	81·3
103	81·3	81·4	81·4	81·5	81·5	81·6	81·6	81·7	81·8	81·8
104	81·9	81·9	82·0	82·0	82·1	82·1	82·2	82·3	82·3	82·4
105	82·4	82·5	82·5	82·6	82·6	82·7	82·8	82·8	82·9	82·9
106	83·0	83·0	83·1	83·1	83·2	83·3	83·3	83·4	83·4	83·5
107	83·5	83·6	83·6	83·7	83·8	83·8	83·9	83·9	84·0	84·0
108	84·1	84·1	84·2	84·3	84·3	84·4	84·4	84·5	84·5	84·6
109	84·6	84·7	84·8	84·8	84·9	84·9	85·0	85·0	85·1	85·1
110	85·2	85·3	85·3	85·4	85·4	85·5	85·5	85·6	85·6	85·7
111	85·8	85·8	85·9	85·9	86·0	86·0	86·1	86·1	86·2	86·2
112	86·3	86·4	86·4	86·5	86·5	86·6	86·6	86·7	86·7	86·8
113	86·9	86·9	87·0	87·0	87·1	87·1	87·2	87·2	87·3	87·4
114	87·4	87·5	87·5	87·6	87·6	87·7	87·7	87·8	87·8	87·9
115	88·0	88·0	88·1	88·1	88·2	88·2	88·3	88·3	88·4	88·5
116	88·5	88·6	88·6	88·7	88·7	88·8	88·8	88·9	88·9	89·0

ON THE RELATION OF STATURE AND WEIGHT TO PIGMENTATION.

By ETHEL M. ELDERTON, Galton Eugenics Laboratory.

It is well-known that pigmentation differs widely from race to race. Further stature certainly and weight probably are racial characters. We might therefore anticipate that in investigations of stature and weight we should find a differentiation in these characters associated with the different pigmentation classes within the population of any town. This would arise not only because the chief classes of pigmentation are supposed to have originated from separate racial types, but because in many large towns there are considerable foreign elements not yet blended with the native population, e.g. in England and Scotland, we find Italian, Irish, Jewish and even Polish groups. In many cases these non-native elements form a considerable percentage of the population, and if, as often happens, they are the poorer section, then we may discover a relation between physique and environment which is not causal but racial in origin. Little attention seems to have been paid to this point in considering, for example, the effects of overcrowding on the weight and stature of children. The children who dwell in four-roomed homes are not necessarily of the same race as those who live in one or two-roomed homes. In a town like Glasgow, the Irish and Italian contingents are possibly more likely to be in one or two-roomed homes than the native Scottish, and a similar state of affairs probably occurs in those London school districts which contain large foreign contingents.

Before we have investigated the racial homogeneity of a population or before we have shown that racial differentiation does not indicate any economic differentiation, we ought not to lay much stress on the association of physique—measured by stature and weight—with environment. There is little doubt that a comparative study of native and foreign elements in our big towns, from the standpoints of both physique and economic condition would be of much service. Meanwhile it occurred to me that if an investigation were made of the relation of stature and weight to pigmentation a positive result would indicate the general importance of the problem. A negative result would show that within a homogeneous population pigmentation had little or no relation to stature and weight.

It would not demonstrate that a large foreign element might not modify the conclusion, unless we were absolutely certain that in the material considered we were really dealing with a heterogeneous population.

In the present inquiry I have worked with Glasgow data, which I selected because I thought that I should include considerable Italian and Irish elements. For the schools, however, where the stature and weight were known as well as the pigmentation, the surnames do not suggest that the foreign element is extensive. Hence I think my inquiry is concerned only with the problem of whether in a fairly homogeneous population* pigmentation is related to stature and weight.

Owing to the kindness of Sir John Struthers the Educational Board of Glasgow placed at our disposal schedules containing particulars of 72,857 Glasgow children attending the public schools of that town in 1905. Among the information given were the height and weight of these children. Now two years previously (1903) Mr Tocher obtained in his great pigmentation survey of the school children of Scotland the hair and eye colour of the Glasgow children (see *Biometrika*, Vol. VI. p. 131 *et seq.*). It occurred to me therefore that it might be feasible to combine these two records if we could identify the children in the two surveys. The colour of the eye and hair would not have changed much in the interval. Mr Tocher most kindly placed his material at my disposal.

The main difficulty was the identification of the children in the two surveys. In the 1905 records the ages of the children were given to the nearest year, but in the 1903 survey this does not appear to have been the case, the year of age being stated. Further, all the schools were not necessarily examined at the same time and the age was therefore only the roughest guide to identity. It was finally decided that only those children should be included whose initials or Christian names were given in the pigmentation inquiry†, and the first part of the work was to place the hair and eye records from the 1903 survey on the 1905 schedules containing the physical characters. Of the schools that sent returns of pigmentation in 1903 and physical measurements in 1905, only five of the earlier survey‡ gave the Christian names or initials of the children and my results are based on those schools only; the schools are Grove Street, Provanside, Alexandra Parade, Abbotsford and Wolseley Street.

Under the school board of Glasgow the City of Glasgow is divided into ten *educational districts* and the schools we are considering are included in the following districts:

---

* By homogeneous population I understand one which *may* be genetically a blend of several races, but has not unmixed foreign elements.

† In many schools only the child's surname had been written down, and even in the case of rare names, the age data were not close enough to admit of absolutely certain identification.

‡ The surnames were taken in the 1903 record with a view to some estimate of foreign parentage, and not for the purpose of identifying the child. It was naturally not foreseen how useful the full names might be at a later date and for a different inquiry.

Grove Street in Milton District; Provanside in St Rollox; Alexandra Parade in Dennistown; Abbotsford in Gorbals and Wolseley Street in Hutchesontown.

Hair was classified in five shades:

(1) all shades of red; light red, bright red, and sandy red, &c.

(2) all shades of fair; flaxen, white, and golden yellow.

(3) chestnut brown, dull brown, and all shades not red, fair, or dark.

(4) dark includes very dark brown (looking black at a moderate distance) and black.

(5) jet black.

Eyes were classified in four shades:

(1) deep blue or pure blue.

(2) light blue and light grey eyes.

(3) not blue, light grey, or brown; they are called medium eyes and include grey, green, orange and other mixed shades.

(4) hazel-brown, dark brown, and dark eyes generally.

Table I. gives the distribution of hair and eye colour in (*a*) Scotland, (*b*) Glasgow City, (*c*) the selected schools.

TABLE I. *Percentages.*

		Hair Colour					Eye Colour			
		Red	Fair	Medium	Dark	Black	Blue	Light	Medium	Dark
a	Boys ...	5·5	24·9	43·3	25·0	1·2	14·7	30·3	32·7	22·3
	Girls ...	5·1	27·4	40·9	25·4	1·2	14·9	30·3	32·1	22·8
b	Boys ...	5·3	21·8	45·7	26·1	1·1	10·7	30·7	34·1	24·5
	Girls ...	4·8	21·3	44·1	28·6	1·2	11·7	29·9	34·0	24·4
c	Boys ...	5·2	22·2	47·2	24·5	·9	11·0	31·0	37·2	20·8
	Girls ...	5·2	20·5	49·1	24·2	1·1	10·7	31·4	36·2	21·7

It will be noticed, if a comparison between the selected schools and the whole of Scotland be made, that the selected schools show a defect in the percentage of fair-haired children, which is greater in girls than in boys, and an excess in the percentage of medium-haired children also greater in girls than in boys; and further they show a defect of pure blue-eyed and an excess of medium coloured eyed children.

If we compare the selected schools with the schools in Glasgow, we find an excess of medium-haired and a defect of dark-haired children in the selected schools, and an excess of medium-coloured eyes, and a defect of dark-eyed children.

The number of children* with jet-black hair is too few to be considered separately.

The fact that the number of dark-haired children in the schools we have selected are not in excess of the number of dark-haired children in Scotland as a whole, leads me to think that the selected schools are schools without a large foreign element and consequently not the most satisfactory schools for a discussion of the influence of race, as measured by hair and eye colour on height and weight. It has been pointed out by Mr Tocher that excess of medium or brown hair, of medium eyes and dark eyes is associated with more thickly populated regions, which in turn are also regions of excess of foreigners.

TABLE II. *Means.*

FREQUENCY		Hair Colour	BOYS			GIRLS			BOYS		GIRLS	
Boys	Girls		Inches of Height	Pounds of Weight	Age	Inches of Height	Pounds of Weight	Age	Height corrected for Age	Weight corrected for Age	Height corrected for Age	Weight corrected for Age
95	94	Red ...	51·11	62·9	10·44	50·50	59·34	10·41	51·35	63·7	50·67	59·8
410	367	Fair ...	50·85	61·8	10·36	50·84	61·34	10·36	51·24	63·0	51·08	62·0
869	880	Brown ...	51·25	62·5	10·51	50·97	61·58	10·45	51·37	62·9	51·10	61·9
451	434	Dark ...	51·95	65·3	10·90	51·57	63·66	10·91	51·38	63·5	51·09	62·3
18	19	Jet Black	52·93	65·2	11·50	51·32	61·86	10·74	51·29	60·1	51·06	61·1
		Eye Colour										
202	192	Blue ...	51·90	64·6	10·80	51·14	62·18	10·58	51·50	63·4	51·09	62·1
572	563	Light ...	51·21	62·7	10·51	50·84	61·42	10·42	51·33	63·1	51·01	61·9
685	650	Medium	51·36	63·3	10·56	51·11	61·77	10·55	51·39	63·4	51·10	61·8
384	389	Dark ...	51·22	62·4	10·59	51·28	62·02	10·70	51·20	62·3	51·07	61·4

The hair scale has been arranged roughly in order of pigmentation; red comes first since it contains fewer pigment granules, and black last since the darker the hair the more pigment granules, and it will be noticed that pigmentation increases slightly with age. Differences in height and weight are slight and an examination of the correlation coefficients in Table III. will show that when corrected, for age, the association between hair or eye colour and height and weight is insignificant.

A rough correction for age can be made on the average height and weight for each hair and eye colour. The deviation of the mean age of each eye and hair colour group from the mean age of the total is obtained and is multiplied by the differences in height and weight of children of 10 and 11, and the result so obtained is added or subtracted to the mean height or weight already found for each age group—the results so found are given in Table II.

---

* Only children of 7 to 14 years inclusive are considered.

TABLE III. *Correlations.*

Hair	Boys	Girls
Hair Colour and Height	·10	·07
Hair Colour and Weight	·11	·08
Hair Colour and Age	·11	·11
Height and Age	·76	·78
Weight and Age	·73	·75
Hair Colour and Height for Constant Age	·03 ± ·02	– ·03 ± ·02
Hair Colour and Weight for Constant Age	·04 ± ·02	– ·00 ± ·02
Eyes		
Eye Colour and Height	·05	·04
Eye Colour and Weight	·05	·03
Eye Colour and Age	·04	·06
Eye Colour and Height for Constant Age	·03 ± ·02	– ·01 ± ·02
Eye Colour and Weight for Constant Age	·03 ± ·02	– ·02 ± ·02

The differences are not significant as can be seen in the correlation coefficients[*], except that boys with jet-black hair have decidedly less weight than other boys, but means based on 18 and 19 cases are of little value.

It was thought that the combination of hair and eye colour might prove a better criterion of racial differences than either separately, but a much larger number of cases would be required to test the point. For this purpose we grouped red and fair hair (1 and 2) together and dark and jet black (4 and 5), and the mean for each group is given below.

TABLE IV. *Corrected Means.*

Frequency			Boys					Girls					Frequency
			Mean			Corrected for Age		Mean			Corrected for Age		
	Hair	Eye	Height	Weight	Age	Height	Weight	Height	Weight	Age	Height	Weight	
97	1 and 2	with 1	51·16	65·8	10·49	51·31	66·3	50·61	60·8	10·15	51·14	62·3	76
221	1 and 2	,, 2	50·83	62·0	10·45	51·05	62·7	50·94	61·4	10·47	51·04	61·7	200
142	1 and 2	,, 3	50·97	62·4	10·32	51·42	63·8	50·64	60·5	10·39	50·85	61·1	147
45	1 and 2	,, 4	50·39	58·9	9·91	51·58	62·6	50·74	60·8	10·26	51·12	61·9	38
67	3	,, 1	52·07	65·3	10·94	51·42	63·1	51·23	62·0	10·75	50·95	61·2	84
265	3	,, 2	51·27	62·1	10·42	51·56	63·0	50·74	61·3	10·34	51·01	62·1	288
397	3	,, 3	51·39	63·2	10·55	51·43	63·3	51·23	62·5	10·52	51·27	62·6	352
140	3	,, 4	50·45	59·9	10·37	50·82	61·1	50·65	59·9	10·33	50·93	60·7	156
38	4 and 5	,, 1	53·50	67·7	11·34	52·14	63·5	52·18	66·0	11·16	51·36	63·7	32
86	4 and 5	,, 2	51·99	66·2	10·97	51·30	64·1	50·94	61·9	10·56	50·92	61·8	75
146	4 and 5	,, 3	51·65	64·5	10·82	51·24	63·2	51·30	63·5	10·78	50·98	62·6	151
199	4 and 5	,, 4	51·94	65·0	10·89	51·38	63·2	51·89	64·0	11·09	51·16	62·0	195

[*] Correlation ratio calculated when correlation coefficient not possible.

The only divergencies in this table which are of any, if slight, significance are in the group of dark-haired children with blue eyes (4 and 5, with 1), and in the group of brown-haired children with dark eyes (3, with 4). In the first case boys and girls with this colouring are distinctly above the average in each instance when corrected for age and in three instances out of four are more above the average than any other group; the exception is in the weights of boys. In the second case the lowest point in three out of four instances, corrected for age, is reached by the group of brown-haired dark-eyed children (3, with 4); the one exception is in the height of girls, fair-haired children with medium-coloured eyes (1 and 2, with 3) being the shortest.

As far as this material goes, we find that types of hair and eye colour are not associated to any substantially *significant* extent with divergencies in height and weight in children between the ages of seven and fourteen inclusive.

Mr Tocher has pointed out that an examination of two schools in Glasgow, Gorbals and Adelphi Terrace, where "about 500 children had distinctly foreign, mostly Jewish, surnames," shows that there is a decided excess of dark and jet black-haired and of dark-eyed children, and a corresponding defect in the other classes, but no such excess occurs in our selected schools, and it seems probable that the number of foreigners in these selected schools is small. At the same time if hair and eye colour were a satisfactory measure of racial descent, and height and weight differ in different races, we should *a priori* have expected to find significant differences in this material. That they do not occur suggests that in blended races there exists little correlation between pigmentation and weight or stature.

I A. *Eye Colour and Height of Girls.*

Eye Colour.

Height	*1*	*2*	*3*	*4*	Totals
38	—	—	1	—	1
39	—	3	—	—	3
40	—	1	—	2	3
41	2	—	2	1	5
42	1	4	5	5	15
43	6	16	13	7	42
44	3	22	20	10	55
45	8	33	20	18	79
46	14	32	43	13	102
47	14	37	50	27	128
48	20	59	51	29	159
49	18	42	62	41	163
50	20	53	67	43	183
51	14	50	59	33	156
52	8	42	59	31	140
53	9	27	34	26	96
54	11	34	38	21	104
55	7	29	33	26	95
56	8	18	24	15	65
57	10	18	20	11	59
58	7	11	18	7	43
59	4	15	11	8	38
60	5	8	8	4	25
61	2	3	4	4	13
62	1	1	5	4	11
63	—	3	2	1	6
64	—	1	1	2	4
65	—	—	—	—	—
66	—	1	—	—	1
Totals	192	563	650	389	1794

II A. *Hair Colour and Height of Girls.*

Hair Colour.

Height	*1*	*2*	*3*	*4*	*5*	Totals
38	—	—	1	—	—	1
39	1	—	1	1	—	3
40	—	—	2	1	—	3
41	1	—	3	1	—	5
42	4	—	6	5	—	15
43	11	4	17	10	—	42
44	8	1	31	14	1	55
45	19	7	41	12	—	79
46	25	3	55	18	1	102
47	31	5	59	32	1	128
48	35	12	80	30	2	159
49	28	12	84	37	2	163
50	41	14	89	39	—	183
51	30	8	83	33	2	156
52	27	6	62	40	5	140
53	18	3	44	29	2	96
54	14	3	59	27	1	104
55	18	4	39	34	—	95
56	12	6	27	19	1	65
57	17	2	26	14	—	59
58	7	3	24	9	—	43
59	7	1	19	10	1	38
60	8	—	10	7	—	25
61	1	—	9	3	—	13
62	2	—	4	5	—	11
63	1	—	3	2	—	6
64	1	—	1	2	—	4
65	—	—	—	—	—	—
66	—	—	1	—	—	1
Totals	367	94	880	434	19	1794

III A. *Eye Colour and Age of Girls.*

Eye Colour.

Age	*1*	*2*	*3*	*4*	Totals
7	10	26	14	9	59
8	28	94	81	40	243
9	33	75	122	63	293
10	35	97	128	71	331
11	17	91	88	60	256
12	18	83	86	77	264
13	27	70	101	46	244
14	24	27	30	23	104
Totals	192	563	650	389	1794

IV A. *Hair Colour and Age of Girls.*

Hair Colour.

Age	*1*	*2*	*3*	*4*	*5*	Totals
7	17	2	26	14	—	59
8	53	16	138	33	3	243
9	62	15	153	63	—	293
10	76	19	152	79	5	331
11	44	17	127	64	4	256
12	52	8	124	74	6	264
13	48	9	113	74	—	244
14	15	8	47	33	1	104
Totals	367	94	880	434	19	1794

I B. *Eye Colour and Height of Boys.*

Eye Colour.

Height	*1*	*2*	*3*	*4*	Totals
36	—	1	1	—	2
37	—	—	—	1	1
38	—	—	—	1	1
39	—	1	—	—	1
40	—	2	1	—	3
41	2	2	3	—	7
42	1	4	3	3	11
43	1	8	6	4	19
44	2	11	16	8	37
45	6	23	24	15	68
46	9	28	38	25	100
47	8	29	53	17	107
48	10	57	65	33	165
49	16	56	64	31	167
50	21	55	59	45	180
51	21	49	51	39	160
52	27	48	67	31	173
53	21	54	50	37	162
54	14	41	35	24	114
55	13	31	42	22	108
56	9	19	30	21	79
57	11	19	25	10	65
58	5	11	22	7	45
59	3	17	13	2	35
60	1	2	9	4	16
61	—	3	3	3	9
62	—	—	4	1	5
63	—	1	1	—	2
64	1	—	—	—	1
Totals	202	572	685	384	1843

II B *Hair Colour and Height of Boys.*

Hair Colour.

Height	*1*	*2*	*3*	*4*	*5*	Totals
36	1	1	—	—	—	2
37	—	—	—	1	—	1
38	—	—	1	—	—	1
39	—	—	1	—	—	1
40	2	—	1	—	—	3
41	2	—	3	2	—	7
42	3	—	7	1	—	11
43	8	2	7	2	—	19
44	10	4	15	8	—	37
45	22	2	33	11	—	68
46	23	7	46	24	—	100
47	20	7	48	31	1	107
48	43	8	89	24	1	165
49	42	6	81	38	—	167
50	46	8	79	44	3	180
51	31	9	76	42	2	160
52	28	9	91	44	1	173
53	30	7	81	40	4	162
54	25	9	52	27	1	114
55	22	2	50	32	2	108
56	18	3	33	24	1	79
57	16	2	26	20	1	65
58	6	6	20	12	1	45
59	5	2	18	10	—	35
60	3	—	8	5	—	16
61	1	1	1	6	—	9
62	2	—	2	1	—	5
63	1	—	—	1	—	2
64	—	—	—	1	—	1
Totals	410	95	869	451	18	1843

III B. *Eye Colour and Age of Boys.*

Eye Colour.

Age	*1*	*2*	*3*	*4*	Totals
7	8	26	29	16	79
8	19	87	91	49	246
9	26	88	112	55	281
10	34	73	114	73	294
11	36	101	107	52	296
12	36	91	87	63	277
13	30	74	95	55	254
14	13	32	50	21	116
Totals	202	572	685	384	1843

IV B. *Hair Colour and Age of Boys.*

Hair Colour.

Age	*1*	*2*	*3*	*4*	*5*	Totals
7	24	3	35	16	1	79
8	59	19	122	45	1	246
9	69	11	135	65	1	281
10	74	16	134	68	2	294
11	56	18	158	62	2	296
12	54	11	133	76	3	277
13	53	8	103	83	7	254
14	21	9	49	36	1	116
Totals	410	95	869	451	18	1843

V A. *Eye Colour and Weight of Girls.*

Eye Colour.

Weight	*1*	*2*	*3*	*4*	Totals
29—	—	—	—	1	1
33—	—	2	1	—	3
37—	3	5	3	2	13
41—	7	25	26	10	68
45—	20	53	50	32	155
49—	31	78	77	47	233
53—	22	87	94	57	260
57—	20	61	95	63	239
61—	19	59	65	44	187
65—	11	42	69	41	163
69—	16	50	52	30	148
73—	9	29	24	9	71
77—	8	20	27	17	72
81—	10	9	18	9	46
85—	3	13	14	7	37
89—	6	13	13	7	39
93—	4	5	9	2	20
97—	1	1	7	3	12
101—	1	3	—	2	6
105—	—	3	3	3	9
109—	1	4	1	—	6
113—	—	—	—	1	1
117—	—	—	2	1	3
121—	—	—	—	—	—
125—	—	1	—	1	2
Totals	192	563	650	389	1794

VI A. *Hair Colour and Weight of Girls.*

Hair Colour.

Weight	*1*	*2*	*3*	*4*	*5*	Totals
29—	—	—	1	—	—	1
33—	1	—	2	—	—	3
37—	3	—	7	3	—	13
41—	15	5	37	11	—	68
45—	37	12	73	31	2	155
49—	48	11	130	42	2	233
53—	56	13	123	66	2	260
57—	43	17	117	59	3	239
61—	40	9	86	48	4	187
65—	31	9	74	48	1	163
69—	30	8	69	38	3	148
73—	15	1	35	20	—	71
77—	12	3	41	16	—	72
81—	10	3	20	12	1	46
85—	4	1	22	9	1	37
89—	10	2	16	11	—	39
93—	5	—	9	6	—	20
97—	1	—	5	6	—	12
101—	3	—	2	1	—	6
105—	1	—	5	3	—	9
109—	2	—	4	—	—	6
113—	—	—	1	—	—	1
117—	—	—	—	3	—	3
121—	—	—	—	—	—	—
125—	—	—	1	1	—	2
Totals	367	94	880	434	19	1794

VII A. *Height and Age of Girls.*

Age of Girls.

Height	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
38	—	1	—	—	—	—	—	—	1
39	1	1	—	1	—	—	—	—	3
40	1	1	—	1	—	—	—	—	3
41	1	3	—	—	1	—	—	—	5
42	3	4	5	2	—	1	—	—	15
43	13	20	4	4	1	—	—	—	42
44	15	19	12	5	3	1	—	—	55
45	5	34	28	7	3	2	—	—	79
46	10	39	36	13	2	1	1	—	102
47	4	43	35	28	12	4	1	1	128
48	5	36	50	40	20	5	2	1	159
49	—	27	43	49	27	11	6	—	163
50	—	11	39	57	44	24	6	2	183
51	1	3	27	51	34	32	6	2	156
52	—	—	7	36	41	30	22	4	140
53	—	—	5	16	22	28	20	5	96
54	—	—	1	12	20	41	25	5	104
55	—	—	—	3	15	43	24	10	95
56	—	1	—	4	7	16	26	11	65
57	—	—	—	1	3	10	36	9	59
58	—	—	—	—	—	8	26	9	43
59	—	—	1	—	1	2	20	14	38
60	—	—	—	1	—	3	10	11	25
61	—	—	—	—	—	1	4	8	13
62	—	—	—	—	—	1	7	3	11
63	—	—	—	—	—	—	2	4	6
64	—	—	—	—	—	—	—	4	4
65	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	—	—	1	1
Totals	59	243	293	331	256	264	244	104	1794

VIII A. *Weight and Age of Girls.*

Age of Girls.

Weight	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
29	1	—	—	—	—	—	—	—	1
33	—	2	—	1	—	—	—	—	3
37	3	8	1	1	—	—	—	—	13
41	20	28	16	3	1	—	—	—	68
45	19	63	39	26	7	1	—	—	155
49	9	61	80	48	25	6	3	1	233
53	6	49	80	63	33	26	2	1	260
57	1	24	44	71	55	27	14	3	239
61	—	7	16	57	41	44	15	7	187
65	—	—	13	36	42	42	28	2	163
69	—	1	2	18	33	51	34	9	148
73	—	—	—	4	9	25	27	6	71
77	—	—	2	1	7	18	29	15	72
81	—	—	—	1	2	9	25	9	46
85	—	—	—	1	—	5	20	11	37
89	—	—	—	—	1	5	19	14	39
93	—	—	—	—	—	3	11	6	20
97	—	—	—	—	—	1	6	5	12
101	—	—	—	—	—	—	3	3	6
105	—	—	—	—	—	1	3	5	9
109	—	—	—	—	—	—	3	3	6
113	—	—	—	—	—	—	—	1	1
117	—	—	—	—	—	—	1	2	3
121	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	1	1	2
Totals	59	243	293	331	256	264	244	104	1794

V B. *Eye Colour and Weight of Boys.*

Eye Colour.

Weight	*1*	*2*	*3*	*4*	Totals
31	—	—	1	—	1
35	—	1	1	1	3
39	1	7	9	5	22
43	5	25	25	19	74
47	14	48	57	26	145
51	17	61	83	45	206
55	26	93	111	66	296
59	34	75	81	49	239
63	20	70	91	55	236
67	27	61	54	31	173
71	24	45	45	26	140
75	8	33	38	31	110
79	16	19	27	9	71
83	2	14	23	8	47
87	2	10	14	4	30
91	3	3	10	4	20
95	3	1	7	3	14
99	—	2	3	—	5
103	—	2	3	—	5
107	—	—	1	1	2
111	—	1	1	1	3
115	—	1	—	—	1
Totals	202	572	685	384	1843

VI B. *Hair Colour and Weight of Boys.*

Hair Colour.

Weight	*1*	*2*	*3*	*4*	*5*	Totals
31	—	—	1	—	—	1
35	—	—	3	—	—	3
39	7	—	11	4	—	22
43	22	5	34	13	—	74
47	49	10	65	21	—	145
51	39	14	106	45	2	206
55	64	15	144	69	4	296
59	56	9	115	58	1	239
63	43	10	115	63	5	236
67	35	6	90	40	2	173
71	32	7	55	45	1	140
75	24	6	46	33	1	110
79	18	5	29	19	—	71
83	11	4	19	12	1	47
87	4	—	16	9	1	30
91	1	2	10	7	—	20
95	4	1	4	5	—	14
99	—	1	2	2	—	5
103	—	—	2	3	—	5
107	—	—	1	1	—	2
111	—	—	1	2	—	3
115	1	—	—	—	—	1
Totals	410	95	869	451	18	1843

VII B. *Height and Age of Boys.*

Age of Boys.

Height	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
36	—	1	—	1	—	—	—	—	2
37	—	—	—	1	—	—	—	—	1
38	1	—	—	—	—	—	—	—	1
39	1	—	—	—	—	—	—	—	1
40	2	1	—	—	—	—	—	—	3
41	2	4	1	—	—	—	—	—	7
42	3	5	1	1	—	—	1	—	11
43	9	7	1	1	—	—	—	1	19
44	16	13	6	—	1	—	1	—	37
45	15	35	11	7	—	—	—	—	68
46	10	37	28	12	5	4	2	2	100
47	11	34	35	13	9	2	3	—	107
48	5	49	58	27	13	5	7	1	165
49	3	26	47	48	21	14	8	—	167
50	1	18	46	47	37	26	5	—	180
51	—	12	27	47	37	22	13	2	160
52	—	1	11	44	53	44	17	3	173
53	—	1	4	29	43	39	39	7	162
54	—	1	3	9	28	41	19	13	114
55	—	1	1	2	26	31	30	17	108
56	—	—	—	2	12	22	35	8	79
57	—	—	—	1	5	16	26	17	65
58	—	—	—	—	5	8	23	9	45
59	—	—	1	1	1	2	16	14	35
60	—	—	—	—	—	1	5	10	16
61	—	—	—	—	—	—	2	7	9
62	—	—	—	1	—	—	2	2	5
63	—	—	—	—	—	—	—	2	2
64	—	—	—	—	—	—	—	1	1
Totals	79	246	281	294	296	277	254	116	1843

VIII B. *Weight and Age of Boys.*

Age of Boys.

Weight	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
31	—	1	—	—	—	—	—	—	1
35	2	—	—	—	—	—	1	—	3
39	8	10	3	—	1	—	—	—	22
43	22	33	11	8	—	—	—	—	74
47	26	53	39	17	5	2	3	—	145
51	12	66	62	33	17	8	7	1	206
55	9	58	87	78	33	18	12	1	296
59	—	15	46	57	62	43	11	5	239
63	—	8	27	60	61	41	31	8	236
67	—	1	3	25	51	57	27	9	173
71	—	1	1	11	28	46	40	13	140
75	—	—	2	3	16	28	44	17	110
79	—	—	—	1	13	15	29	13	71
83	—	—	—	1	5	13	17	11	47
87	—	—	—	—	2	3	18	7	30
91	—	—	—	—	1	3	7	9	20
95	—	—	—	—	1	—	5	8	14
99	—	—	—	—	—	—	—	5	5
103	—	—	—	—	—	—	1	4	5
107	—	—	—	—	—	—	1	1	2
111	—	—	—	—	—	—	—	3	3
115	—	—	—	—	—	—	—	1	1
Totals	79	246	281	294	296	277	254	116	1843

IX A. *Hair and Eye Colour and Height of Girls.*

Hair and Eye Colour.

Height	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *4*	Totals
38	—	—	—	—	—	—	1	—	—	—	—	—	1
39	—	1	—	—	—	1	—	—	—	1	—	—	3
40	—	—	—	—	—	1	—	1	—	—	—	1	3
41	1	—	—	—	1	—	2	—	—	—	—	1	5
42	1	2	1	—	—	1	3	2	—	1	1	3	15
43	3	8	3	1	1	6	6	4	2	2	4	2	42
44	—	5	4	—	3	13	10	5	—	4	6	5	55
45	6	11	6	3	2	19	11	9	—	3	3	6	79
46	7	11	8	2	4	20	24	7	3	1	11	4	102
47	3	17	14	2	10	13	26	10	1	7	10	15	128
48	10	20	14	3	7	32	24	17	3	7	13	9	159
49	6	12	19	3	9	24	34	17	3	6	9	21	163
50	12	20	15	8	8	25	39	17	—	8	13	18	183
51	4	15	13	6	7	29	31	16	3	6	15	11	156
52	1	17	15	—	3	22	28	9	4	3	16	22	140
53	4	7	8	2	3	15	14	12	2	5	12	12	96
54	2	9	5	1	8	20	25	6	1	5	8	14	104
55	3	12	3	4	1	13	20	5	3	4	10	17	95
56	1	8	8	1	5	5	12	5	2	5	4	9	65
57	3	11	5	—	6	5	11	4	1	2	4	7	59
58	5	3	1	1	2	7	12	3	—	1	5	3	43
59	—	5	2	1	2	8	8	1	2	2	1	6	38
60	3	3	2	—	2	4	4	—	—	1	2	4	25
61	1	—	—	—	—	3	3	3	1	—	1	1	13
62	—	1	1	—	—	—	3	1	1	—	1	3	11
63	—	1	—	—	—	1	1	1	—	1	1	—	6
64	—	1	—	—	—	—	—	1	—	—	1	1	4
65	—	—	—	—	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

IX B. *Hair and Eye Colour and Height of Boys.*

Hair and Eye Colour.

Height	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *4*	Totals
36	—	1	1	—	—	—	—	—	—	—	—	—	2
37	—	—	—	—	—	—	—	—	—	—	—	1	1
38	—	—	—	—	—	—	—	1	—	—	—	—	1
39	—	—	—	—	—	1	—	—	—	—	—	—	1
40	—	2	—	—	—	—	1	—	—	—	—	—	3
41	1	1	—	—	1	—	2	—	—	1	1	—	7
42	1	2	—	—	—	2	3	2	—	—	—	1	11
43	1	6	1	2	—	2	4	1	—	—	1	1	19
44	2	5	6	1	—	6	7	2	—	—	3	5	37
45	4	11	7	2	2	7	15	9	—	5	2	4	68
46	6	13	9	2	1	14	20	11	2	1	9	12	100
47	3	11	13	—	2	12	26	8	3	6	14	9	107
48	4	21	18	8	6	25	39	19	—	11	8	6	165
49	11	18	15	4	5	28	37	11	—	10	12	16	167
50	15	21	10	8	5	28	31	15	1	6	18	22	180
51	7	21	7	5	10	20	33	13	4	8	11	21	160
52	10	14	9	4	11	26	40	14	6	8	18	13	173
53	8	21	7	1	8	30	34	9	5	3	9	27	162
54	7	18	5	4	2	23	21	6	5	—	9	14	114
55	5	10	8	1	4	15	24	7	4	6	10	14	108
56	5	4	11	1	3	10	16	4	1	5	3	16	79
57	5	9	4	—	3	4	16	3	3	6	5	7	65
58	2	4	4	2	—	5	13	2	3	2	5	3	45
59	—	4	3	—	3	7	8	—	—	6	2	2	35
60	—	2	1	—	1	—	5	2	—	—	3	2	16
61	—	1	1	—	—	—	1	—	—	2	1	3	9
62	—	—	2	—	—	—	1	1	—	—	1	—	5
63	—	1	—	—	—	—	—	—	—	—	1	—	2
64	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

X A. *Hair and Eye Colour and Weight of Girls.*

Hair and Eye Colour.

Weight	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 4	Totals
29	—	—	—	—	—	—	—	1	—	—	—	—	1
33	—	1	—	—	—	1	1	—	—	—	—	—	3
37	2	—	1	—	1	4	1	1	—	1	1	1	13
41	3	9	6	2	3	13	16	5	1	3	4	3	68
45	10	20	16	3	7	26	26	14	3	7	8	15	155
49	8	27	22	2	18	43	40	29	5	8	15	16	233
53	11	33	20	5	7	41	48	27	4	13	26	25	260
57	9	20	20	11	10	30	56	21	1	11	19	31	239
61	10	21	16	2	7	33	29	17	2	5	20	25	187
65	5	15	15	5	4	23	36	11	2	5	17	25	163
69	6	19	9	4	6	24	28	11	4	7	15	15	148
73	2	10	4	—	5	13	16	1	2	5	5	8	71
77	1	7	5	2	6	12	18	5	1	1	4	10	72
81	4	3	5	1	5	3	10	2	1	3	3	6	46
85	1	3	1	—	2	7	9	4	—	3	4	3	37
89	2	6	3	1	—	7	6	3	4	—	4	3	39
93	—	3	2	—	2	1	6	—	2	1	1	2	20
97	—	—	1	—	1	—	4	—	—	1	2	3	12
101	1	2	—	—	—	—	—	2	—	1	—	—	6
105	—	—	1	—	—	3	1	1	—	—	1	2	9
109	1	1	—	—	—	3	1	—	—	—	—	—	6
113	—	—	—	—	—	—	—	1	—	—	—	—	1
117	—	—	—	—	—	—	—	—	—	—	2	1	3
121	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	1	—	—	—	—	—	1	2
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

XI A. *Hair and Eye Colour and Age of Girls.*

Hair and Eye Colour.

Age	1 and 2 with 1	1 and 2 with 2	1 and 2 with 3	1 and 2 with 4	3 with 1	3 with 2	3 with 3	3 with 4	4 and 5 with 1	4 and 5 with 2	4 and 5 with 3	4 and 5 with 5	Totals
7	7	9	2	1	1	13	7	5	2	4	5	3	59
8	14	34	18	3	11	51	53	23	3	9	10	14	243
9	10	28	29	10	18	37	68	30	5	10	25	23	293
10	17	30	37	11	13	52	58	29	5	15	33	31	331
11	7	30	20	4	10	49	45	23	—	12	23	33	256
12	7	31	19	3	8	42	48	26	3	10	19	48	264
13	7	27	18	5	12	33	56	12	8	10	27	29	244
14	7	11	4	1	11	11	17	8	6	5	9	14	104
Totals	76	200	147	38	84	288	352	156	32	75	151	195	1794

X B. *Hair and Eye Colour and Weight of Boys.*

Hair and Eye Colour.

Weight	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *4*	Totals
31	—	—	—	—	—	—	1	—	—	—	—	—	1
35	—	—	—	—	—	1	1	1	—	—	—	—	3
39	1	6	—	—	—	1	7	3	—	—	2	2	22
43	4	12	7	4	1	11	14	8	—	2	4	7	74
47	11	25	19	4	2	19	30	14	1	4	8	8	145
51	7	23	17	6	7	30	45	24	3	8	21	15	206
55	14	29	27	9	9	46	66	23	3	18	18	34	296
59	14	29	13	9	15	33	48	19	5	13	20	21	239
63	9	25	13	6	5	39	54	17	6	6	24	32	236
67	10	21	9	1	11	34	37	8	6	6	8	22	173
71	10	16	10	3	6	22	20	7	8	7	15	16	140
75	6	15	7	2	2	14	23	7	—	4	8	22	110
79	9	6	8	—	4	6	16	3	3	7	3	6	71
83	1	6	7	1	1	5	12	1	—	3	4	6	47
87	—	3	1	—	—	3	10	3	2	4	3	1	30
91	1	2	—	—	2	—	7	1	—	1	3	3	20
95	—	1	4	—	2	—	2	—	1	—	1	3	14
99	—	1	—	—	—	—	2	—	—	1	1	—	5
103	—	—	—	—	—	—	2	—	—	2	1	—	5
107	—	—	—	—	—	—	—	1	—	—	1	—	2
111	—	—	—	—	—	1	—	—	—	—	1	1	3
115	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

XI B. *Hair and Eye Colour and Age of Boys.*

Hair and Eye Colour.

Age	*1* and *2* with *1*	*1* and *2* with *2*	*1* and *2* with *3*	*1* and *2* with *4*	*3* with *1*	*3* with *2*	*3* with *3*	*3* with *4*	*4* and *5* with *1*	*4* and *5* with *2*	*4* and *5* with *3*	*4* and *5* with *4*	Totals
7	4	13	9	1	4	11	15	5	—	2	5	10	79
8	13	35	23	7	3	42	52	25	3	10	16	17	246
9	13	31	26	10	9	42	60	24	4	15	26	21	281
10	20	33	22	15	10	28	76	20	4	12	16	38	294
11	16	33	20	5	12	60	64	22	8	8	23	25	296
12	16	34	11	4	13	46	55	19	7	11	21	40	277
13	9	29	21	2	12	27	45	19	9	18	29	34	254
14	6	13	10	1	4	9	30	6	3	10	10	14	116
Totals	97	221	142	45	67	265	397	140	38	86	146	199	1843

PIGMENTATION IN RELATION TO SELECTION AND TO ANTHROPOMETRIC CHARACTERS.

By A. M. CARR SAUNDERS, M.A.

I. Pigmentation and Disease.

Introduction. The problem of the relation between pigmentation and disease has in recent years commanded a certain amount of attention; but, as yet, very little work has been done which has helped materially to elucidate the problem. In a recent number of *Biometrika* a memoir by Macdonald* deals with this question; the introduction to the memoir gives a summary of previous work in this subject, and the theories which have been put forward with regard to it. Readers of this summary will see how much of what is actually laid down about the relation between pigmentation and disease consists of little more than casual observations and vague suggestions. One theory that has gained currency is that the dark haired type is becoming more common in towns, and that this is due to the elimination of the lighter stocks owing to their greater susceptibility to disease. It was with the hope of bringing forward evidence to test such rather vague theories that the present work was undertaken. The problem is part of the larger subject—the action of natural selection on the human race. About this we know at present lamentably little. It has been shown that the great majority of deaths are selective. But we are almost entirely ignorant of the precise mode in which selection is influencing the race.

There exist some further contributions to the subject which are not mentioned by Macdonald. Shrubsall† in a memoir of 1909 finds that tuberculosis is more frequent in dark types, while the onset is earlier among the blondes. In another paper‡ he comes to the conclusion that blonde traits are associated with rheumatism, heart-disease, tonsilitis and osteo-arthritis, and that brunette traits are associated with nervous diseases, tuberculosis and malignant disease. Pfitzner§ found a marked increase of black hair with increasing age; the increase seems to be too

---

* D. Macdonald, *Biometrika*, Vol. VIII. p. 13.

† F. C. Shrubsall, *Proc. Roy. Soc. Med.* 1909, Vol. II.

‡ F. C. Shrubsall, *St Bartholomew Hospital Reports*, Vol. XXXIX.

§ Pfitzner, *Schwalbe's Zeitschrift für Morphologie*, Bd. I.

great to be accounted for by the normal darkening of the hair, and the most probable explanation is that the black haired type has been less subject to elimination*. But the most interesting result obtained is due to Pearson, which I am allowed to quote, but which has not yet been published elsewhere. He found a correlation of ·19 between health and hair colour, and of ·07 between health and eye colour in the case of data relating to 2317 boys. A very similar result was obtained from data relating to a nearly equal number of girls. These results of Pearson and Pfitzner tend to confirm the theory referred to above, that the darker types are less subject to disease. The data used by Macdonald are in some respects comparable with those employed in this paper, and they bear on very similar problems in both cases. All discussion and comparison must be reserved, however, until the results of the present investigation have been set forth.

Data and Methods. The data used in this work are derived from the information collected as the result of the Medical Survey of School Children carried out to meet the requirements of the Act of 1907. The data come entirely from Birmingham, and the fact that they were obtained for use in the Biometric Laboratory, is due to the kindness of Dr Auden, medical superintendent of the Birmingham Education Committee. The information with regard to each child is contained in a separate schedule, and includes a large number of particulars. The facts which have been of use in the present enquiry are those collected with regard to the hair and eye colour of the children, and to certain selected diseases. The hair colour was classified under six headings: red, fair, light brown, medium brown, dark brown and black, and the eye colour under four headings: blue, light, neutral and dark. The examination of the children was actually carried out by a number of doctors working with the help of nurses, all of whom Dr Auden had trained to estimate the pigmentation with the aid of a colour-meter. The classification of hair and eye colour always presents great difficulties; in the case of hair colour the most satisfactory method is to construct a scale with specimens of actual hair, and to compare the hair of each individual examined with the specimens on the scale. The colour-meter employed by Dr Auden consisted of different shades of coloured glass; and those responsible for the filling up of the schedules gained by its use a general idea of the categories of hair colour, but did not directly compare the hair of the children with their standard scale. Such a method of estimation is necessarily somewhat rough; but the results obtained are no doubt of sufficient exactness to enable broad conclusions to be drawn from them.

The six specified diseases are measles, whooping cough, scarlet fever, diphtheria, mumps and chicken pox. The schedules also contain information about other diseases, but this is not employed here.

Table I on the following page exhibits some of the weaknesses of the scheme of hair-classification employed. Taking the children of 13 years of age, it is clear

---

* See *Biometrika*, Vol. III. 1904, p. 463.

that a method of classification which divides the data into groups varying from 1·6 per cent. to 32·2 per cent. of the total is not entirely satisfactory.

TABLE I.

Percentage of Hair Colour Groups to the Total.

Hair Colour	Boys. Age 13		Girls. Age 13		Boys. Age 3—7		Girls. Age 3—7	
	Numbers	Percentage	Numbers	Percentage	Numbers	Percentage	Numbers	Percentage
Red	62	3·1	74	3·5	21	4·5	7	1·9
Fair	253	12·7	237	11·4	52	11·1	34	9·4
Light Brown ...	443	22·2	406	19·5	128	27·2	114	31·7
Medium Brown	626	31·4	652	31·2	189	40·2	144	40·0
Dark Brown ...	579	29·0	672	32·2	77	16·4	60	16·7
Black	31	1·6	46	2·2	3	·6	1	·3
Totals	1994	100·0	2087	100·0	470	100·0	360	100·0

But apart from this it is interesting to compare the percentage of the various hair colour groups so obtained in Birmingham with those obtained elsewhere. Table II is copied from Tocher's "Pigmentation Survey of School Children in Scotland*." The comparison between the hair colour schemes cannot be made in detail because the classification is not identical. The percentage of red haired children is less in Birmingham, where it does not exceed 3·5 per cent., while in Scotland it is not less than 5 per cent.; the black haired class, however, would seem to be present in Birmingham in larger numbers. The two classes denominated Fair clearly do not correspond; many of the Scottish children placed as Fair go in the Birmingham arrangement under the head of Light Brown.

TABLE II.

Colour Distribution of Scottish Children.

		Hair					Eyes				
		Red	Fair	Medium	Dark	Black	Blue	Light	Medium	Dark	Totals
Boys.	Numbers	14162	64312	111569	64511	3212	37788	78140	84334	57504	257766
,,	Percentage	5·5	24·9	43·3	25·0	1·3	14·7	30·3	32·7	22·3	—
Girls.	Numbers	12435	67036	99873	62073	2972	36347	74068	78357	55617	244389
,,	Percentage	5·1	27·4	40·9	25·4	1·2	14·8	30·3	32·1	22·8	—

---

* *Biometrika*, Vol. vi. p. 147.

The comparison of the hair colour percentages of children aged 13 and those aged 3—7 shows no very considerable change. There is a certain darkening of the hair, but it is not very pronounced*. The percentage of fair haired children is actually higher among those of 13 years of age than among the younger children. The chief change is a gain of the Dark Brown class at the expense of the Light and Medium Brown classes.

TABLE III.

Percentage of Eye Colour Groups to the Total.

Eye Colour	BOYS. AGE 13		GIRLS. AGE 13		BOYS. AGE 3—7		GIRLS. AGE 3—7	
	Numbers	Percentage	Numbers	Percentage	Numbers	Percentage	Numbers	Percentage
Blue ...	354	17·76	385	18·45	208	44·26	173	48·06
Light ...	568	28·49	577	27·65	58	12·34	48	13·33
Neutral	544	27·28	564	27·02	89	18·94	64	17·78
Dark ...	528	26·47	561	26·88	115	24·46	75	20·83
Totals	1994	100·0	2087	100·0	470	100·0	360	100·0

Table III shows that for the older children the classification adopted for eye colour provides a fairly equal division into the four categories. A direct comparison with the Scottish figures is here possible since presumably the medium class in one case is identical with the neutral class in the other. The percentage of blue eyed children is distinctly higher in our data; and this is remarkable since the darker type of eye is more frequent in Scotland; and, further, Tocher's data give a higher percentage of blue eyes than is usually found. The suggestion is that at Birmingham the classification of blue eyes has not been very strict, and that a number of light blue eyes have been admitted into the blue eye category when they should have been placed in the light category. This idea is supported by the figures given for the children aged 3—7. In this case the blue eyed children amount to nearly 50 per cent. of the total. This is almost certainly too high a total, if the term blue eye is to be used in its usual meaning. Among the younger children the percentage of light eyes falls rather low; and probably a considerable number of the blue eyes should have been included in the light eyed class. The table is noteworthy as marking a very considerable change with age to the darker eye classes.

Table IV shows the average number of diseases per child in each hair category. The hair categories have also been combined into two larger groups—roughly representing light and dark hair—and the average number of diseases for these groups has been worked out. Further figures are also given to show the average number of diseases per boy and girl in the different age groups.

---

* For estimates of the correlation between hair colour and age, see *Biometrika*, Vol. III. p. 462.

If we consider first the average number of diseases to the different hair colour groups, we find no evidence of any significant connection between one hair colour and disease. The differences in the averages are not only slight, but occur apparently at random; we do not find one hair colour associated in both boys and girls with a distinctive average whether greater or less than the normal. Further it does not appear that dark hair shows any significant difference from light hair in this respect. Red, fair and light brown have been classed together as light hair, and the remaining categories as dark hair. In two cases the average number of diseases is exactly equal in the different groups; in the other two cases, the light haired group is in excess on one occasion, and the dark haired group in the other. So far, therefore, as this table goes, there is no reason for thinking that liability to disease is connected with any particular hair colour.

TABLE IV. (See Tables XV and XVI, pp. 378—379.)

Incidence of Disease in Hair Colour Groups.

Hair Colour	Boys. Age 13				Girls. Age 13			
	Cases of Disease		Average		Cases of Disease		Average	
Red	125	1317	2·01	1·74	148	1520	2·0	2·12
Fair	434		1·71		483		2·04	
Light Brown ...	758		1·71		889		2·19	
Medium Brown	1183	2323	1·88	1·88	1395	2908	2·13	2·12
Dark Brown ...	1071		1·84		1416		2·11	
Black	69		2·22		97		2·1	
Totals	3640		—		4428		—	

Average number of diseases to each boy 1·83. Average number of diseases to each girl 2·12.

Hair Colour	Boys. Age 3—7				Girls. Age 3—7			
	Cases of Disease		Average		Cases of Disease		Average	
Red	36	313	1·71	1·56	11	260	1·58	1·68
Fair	93		1·79		51		1·5	
Light Brown ...	184		1·44		198		1·74	
Medium Brown	296	414	1·57	1·54	217	286	1·5	1·40
Dark Brown ...	115		1·49		68		1·13	
Black	3		1·0		1		1·0	
Totals	727		—		546		—	

Average number of diseases to each boy 1·55. Average number of diseases to each girl 1·52.

When we turn to consider the average number of diseases per head in the two sexes, we do find a significant difference, but only in the children of the older group. Boys from 3—7 and girls of the same age have almost exactly the same number of diseases per head. The difference only occurs between the boys and girls aged 13, and between them it is marked, the boys having an average of 1·83 and the girls one of 2·12 diseases per child.

Now, the source of this fact is by no means clear; there are two possible explanations which may be at once put aside as not applicable here. In the first place the difference cannot be due to bias. For if there were any reason why the diseases of the girls should be remembered more vividly than those of the boys, then that reason ought to work in the case of the younger children as well. But it does not seem possible to find any such reason. Dr Auden has, indeed, pointed out to me that a parent is more often present during the examination of girls than of boys. Taking about 1500 of each sex, he calculated that a parent was present when 65 per cent. of the boys and 71 per cent. of the girls were being examined. But I agree with him in thinking that this could not possibly account for the difference in the average number of diseases per child in the two sexes. In the second place it is not due to an epidemic among the girls, since the girls are found to have had a higher average number of all the six diseases with which we are dealing.

The first explanation of this anomaly which promised to be at all sufficient is connected with the fact that, although more boys are born than girls, yet in a few years the numbers of the two sexes become equal, and later the girls outnumber the boys. So far these facts are a matter of common knowledge, and the manner in which it was at first thought that they might explain the above figures was as follows. If we suppose that the incidence rate for the special diseases with which we are concerned is of equal intensity for the two sexes, and that more boys die from these diseases than girls, then if a number of boys and girls aged 13 were examined, we might expect to find more girls who had diseases than boys; for supposing an equal number to have been attacked from each sex, and a greater percentage of boys to have died, there will clearly be present at a later age more girls living who have had diseases. Now, this explanation does not as a fact hold good, as will appear; but it was while examining data bearing on these points that certain facts were found which do help to elucidate the question.

The above reasoning rests on the assumptions that (1) the incidence rate is equal or at least not very dissimilar in the two cases, and (2) the death rate is higher among boys than girls. Firstly as to the incidence rate:—the age distributions of the cases of scarlet fever and diphtheria notified to the Metropolitan Asylums Board as given in the Board Report for 1900 were extracted. These figures alone do not give an accurate measure of the incidence, since an allowance must be made for the fact that the numbers of the two sexes are not equal at the different ages. The total numbers of the two sexes for the various age groups were taken from the census of 1900 as given for London, and a calculation was then made on the basis of the number of cases notified among boys relative to the population totals thus obtained to find the expected number of cases among the girls relative to the population of that sex. The following figures give the cases actual and expected obtained in this way:

Scarlet Fever.

Age of cases notified			*0—5*	*5—10*	*10—15*
Number of cases.	Boys,	actual	2,040	2,312	1,116
,, ,,	Girls,	actual	2,009	2,803	1,285
,, ,,	,,	expected	2,038	2,311	1,138

Diphtheria.

Age of cases notified			*0—5*	*5—10*	*10—15*
Number of cases.	Boys,	actual	2,322	1,782	607
,, ,,	Girls,	actual	2,218	2,075	759
,, ,,	,,	expected	2,320	1,800	618

The incidence rate, therefore, in the case of these two diseases is not the same in the two sexes. Up to five years of age the incidence is nearly equal; if anything, the boys are attacked rather more often. But in the older age groups, the difference is most marked. The girls have both diseases far more often than the boys.

Secondly as to the death rate;—the following figures are taken from the Registrar General's Annual Report:

Annual Death Rate per Million living from 1861—1900.

Diphtheria.

Ages ...	*0—5*	*5—10*	*10—15*
Boys ...	827	396	94
Girls ...	821	499	131

Scarlet Fever.

Boys ...	2,734	1,225	254
Girls ...	2,596	1,190	278

Whooping Cough.

Boys ...	3,145	102	4
Girls ...	3,805	154	7

Measles.

Boys ...	3,092	225	21
Girls ...	2,890	247	24

Before we discuss the bearing of these figures on the problem, it will be of assistance to look at the following Table:

TABLE V.

Percentages of Boys and Girls who have had the following Diseases.

Age 13.

Measles,	Boys	75·2	Whooping Cough,	Boys	37·1	Scarlet Fever,	Boys	16·3
„	Girls	85·4	„ „	Girls	44·9	„ „	Girls	18·4
Diphtheria,	Boys	3·9	Mumps,	Boys	21·4	Chicken Pox,	Boys	28·6
„	Girls	4·7	„	Girls	23·2	„ „	Girls	35·5

This gives the percentages of boys and girls who have had the various diseases. It will be seen that not only do the older girls have, as before shown, a higher average number of diseases, but in each case a higher percentage of girls have had the disease than of boys; and further, the differences between the percentages as a fraction of the whole are roughly the same between boys and girls. The difference is rather greater in the case of diphtheria and rather less in the case of mumps; but on the whole there seems to be a consistently higher average among the girls for each disease. If we now turn back to the incidence and mortality rates, it is evident that they show the first explanation suggested to be impossible. The incidence rates are not the same for both sexes, and the mortality rates do not show that marked and persistent mortality among the younger boys which the explanation supposes. In the case of whooping cough there is a far higher death rate among girls, and here the explanation breaks down entirely; for as Table V shows, the difference between the two sexes is as marked as in any other disease. It must, of course, be remembered that we are here only dealing with six specified diseases; the elimination of boys, which we know to occur, must take place through the agency of other diseases.

The figures for the incidence rate suggest a further explanation. Unfortunately they are only obtainable for scarlet fever and diphtheria. But in these two cases the girls from 5 years of age and upwards are more liable to attack. Now, if the death rate were the same for both boys and girls, then this would be an explanation, in so far as the bare fact that girls are as a matter of fact more susceptible to disease than boys is an explanation. If we examine the death rates for diphtheria and scarlet fever, we see that they are not the same; the general tendency is for the rate among the younger boys to be higher than that among the younger girls, and the opposite way about for the older children. On the whole, however, up to the age of 15 diphtheria is a more fatal disease for girls than boys, while the case is reversed in scarlet fever. In such cases as that of diphtheria, in which the death rate is higher among girls, there must be a tendency to counteract the results of the higher incidence rate among girls, and in the girls of 13, as shown in Table V, we might expect to find such a large difference between the sexes as in the other cases. But, as already pointed out, the difference between the two sexes in the case of diphtheria is rather larger than for other diseases. The explanation is, it must be admitted, not very satisfactory; but granted the existence of the higher incidence rate among girls, we must suppose that the differences in the death rate are not such as to seriously counteract the former factor. It would seem that the death rate can have very little influence on the figures in question, since the death rates for four out of the six specified diseases show no similarity. In whooping cough, for instance, the death rate is always higher for girls; while in scarlet fever it is almost always higher for boys. The only explanation, therefore, is that girls at the older ages suffer more from these specified diseases than boys; this is no real explanation, but a physiological fact. It does, however, rule out of account any explanation through selection by means of the six specified diseases which seemed at first the most probable

It is well known that the correlation between hair colour and eye colour is fairly high; it seems, in fact, to be something under ·4*. Now Table VI is constructed on exactly similar lines to Table IV, but with eye colour in place of hair colour. There is no reason to suppose *a priori* that, owing to the correlation between hair and eye colour, this table must show a similar absence of connection between eye colour and disease. If there be three characters *A*, *B* and *C* of which

TABLE VI. (See Tables XV and XVI, pp. 378—379.)

Incidence of Disease in Eye Colour Groups.

Eye Colour	Boys. Age 13				Girls. Age 13			
	Cases of Disease		Average		Cases of Disease		Average	
Blue... ...	623	1635	1·75	1·77	804	2027	2·08	2·11
Light ...	1012		1·78		1223		2·16	
Neutral ...	997	2005	1·83	1·87	1226	2401	2·17	2·21
Dark ...	1008		1·90		1175		2·19	
Totals	3640		—		4428		—	

Eye Colour	Boys. Age 3—7				Girls. Age 3—7			
	Cases of Disease		Average		Cases of Disease		Average	
Blue... ...	325	425	1·6	1·60	256	341	1·5	1·54
Light ...	100		1·7		85		1·8	
Neutral ...	144	302	1·7	1·48	101	205	1·6	1·48
Dark ...	158		1·4		104		1·4	
Totals	727		—		546		—	

---

* *Biometrika*, Vol. III. p. 459.

A and B are correlated and A and C are correlated, there must be a correlation of over ·75 between A and B, and between A and C in order that B and C should be necessarily correlated. This table, however, does show a very similar result to the preceding table; and we do not find that any particular eye colour is associated with a greater or less average number of diseases; nor do we find, if we divide the classes of eye into light and dark, that there is any significant difference between them. In one case it might appear at first sight that there was some significant difference. The average number of diseases to both dark eyed boys and girls aged 3—7 falls somewhat below the general average. Calculations were made to test for random sampling according to the method given by Elderton*. P was calculated in each case. For the boys aged 3—7, P worked out at ·33 and for the girls at ·37, or in other words a less regular sample would occur more than 30 times in a hundred chances. There is, therefore, no importance whatsoever to be attributed to the drop in the average for the younger dark eyed boys and girls.

TABLE VII.

Incidence of Disease in Hair Colour Groups. Percentages.

Boys and Girls. Age 13.

Hair Colour	MEASLES		WHOOPING COUGH		SCARLET FEVER		DIPHTHERIA		MUMPS		CHICKEN POX	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Red	79·0	87·8	38·7	40·5	17·7	9·5	1·6	4·0	27·6	28·4	37·1	29·7
Fair	73·1	87·4	31·7	44·4	14·0	16·2	2·8	5·5	20·2	21·1	30·1	29·6
Light Brown	71·9	86·0	34·2	44·7	14·5	21·5	3·2	4·4	19·8	26·9	27·9	35·7
Medium Brown	77·6	85·8	39·9	44·9	17·4	19·7	4·7	5·2	20·9	22·9	28·4	38·8
Dark Brown	75·5	84·5	37·8	44·2	17·7	17·4	4·2	4·5	23·3	22·3	27·2	38·8
Black ...	79·5	84·8	52·0	47·9	22·6	19·6	0·0	2·2	22·6	13·1	48·4	43·5

TABLE VIII.

Incidence of Disease in Eye Colour Groups. Percentages.

Boys and Girls. Age 13.

Eye Colour	MEASLES		WHOOPING COUGH		SCARLET FEVER		DIPHTHERIA		MUMPS		CHICKEN POX	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Blue ...	73·7	85·6	36·7	41·3	15·6	22·2	4·9	3·1	19·5	21·3	25·8	35·7
Light...	75·2	85·9	36·8	45·9	15·0	18·6	2·5	6·9	18·3	22·3	30·4	32·8
Neutral	73·5	85·9	36·4	44·7	17·5	18·6	4·4	4·6	25·0	27·5	26·7	35·9
Dark ...	77·9	84·3	38·3	45·0	17·5	15·8	4·2	3·7	22·5	21·4	30·7	39·4

---

* *Biometrika*, Vol. I. p. 155.

These tables refer one to hair and the other to eye colour. They are both constructed on the same lines, and give the percentage out of the total of the various hair and eye colour categories which have had the diseases in question. They include only the older children. If any one hair or eye colour was definitely connected with a special liability to disease, these tables ought to show it. If, for example, dark hair was less liable to disease, we ought to find some indication of it here. But the variations that do occur appear wholly irregular, and give no indication of any systematic correlation between one shade of pigmentation and disease—whether we look at diseases as a whole or at any one disease alone. There are a few deviations from the general course of averages which appear at first sight to be somewhat remarkable; thus, 52 per cent. of the black haired boys had whooping cough, and only 13 per cent. of the black haired girls had mumps. But such cases always occur where the probable error is very large, due in this case to the fact that the number of black haired children out of the total is less than 2 per cent. In Table VIII, which deals with eye colour, the deviations are not so remarkable since the eye colour groups form more equal divisions of the total number than the hair colour groups do; in this table, where large deviations are found, it is due to the fact that in certain diseases, such as diphtheria, the total number of cases is very small.

In discussing the fact that girls were found to have a larger average number of diseases than boys, it was stated that this was not due to a special outbreak of one disease among the girls. These tables provide the justification for this statement; it can be seen that the girls in all six diseases have a larger average number than the boys.

Tables IX and X are constructed on exactly similar lines to Tables VII and VIII, but refer to the younger children aged 3—7. They show the percentage of each hair and eye colour group that have had each of the six specified diseases. It is again equally evident that no general coincidence can be traced—such as the

TABLE IX.

Incidence of Disease in Hair Colour Groups. Percentages.

Boys and Girls. Age 3—7.

Hair Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Red	71·4	85·7	38·6	42·9	9·5	14·3	4·8	0·0	23·8	0·0	23·8	14·3
Fair	78·9	64·7	36·5	44·1	5·7	8·8	0·0	0·0	17·3	0·0	40·4	32·4
Light Brown	63·3	64·0	31·2	43·0	7·0	6·1	0·8	2·6	10·2	18·4	30·5	39·5
Medium Brown	57·7	57·6	38·1	38·2	7·4	9·7	2·1	2·1	17·5	18·8	33·9	24·3
Dark Brown	59·7	45·0	41·6	33·3	5·2	5·0	1·3	0·0	13·0	13·3	28·6	16·7
Black ...	33·3	100·0	33·3	100·0	0·0	0·0	0·0	0·0	0·0	0·0	33·3	100·0

TABLE X.

Incidence of Disease in Eye Colour Groups. Percentages.

Boys and Girls. Age 3—7.

Eye Colour	MEASLES		WHOOPING COUGH		SCARLET FEVER		DIPHTHERIA		MUMPS		CHICKEN POX	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Blue ...	62·0	58·4	37·0	38·2	8·2	5·8	1·5	1·2	14·9	17·3	31·7	27·2
Light...	72·4	68·8	41·4	52·1	5·2	10·4	1·7	4·2	10·3	8·3	41·4	33·3
Neutral	65·2	57·8	39·3	39·1	4·5	10·9	1·1	1·6	19·1	20·3	34·8	28·1
Dark ...	65·7	54·7	32·2	34·7	7·0	8·0	1·7	1·3	21·8	12·0	27·0	28·0

alliance of one hair colour with a general excess of disease or liability to one disease. There is a greater variation and irregularity in these percentages, due to the smaller number of children, and to the consequently larger probable errors.

TABLE XI.

Values of P.

		Measles	Whooping Cough	Scarlet Fever	Diphtheria	Mumps	Chicken Pox
Hair	Boys, Age 13	·298	·075	·506	·346	·611	·098
	" " 3—7	·171	·612	·661	·969	·522	·264
	Girls, " 13	·787	·990	·123	·899	·164	·115
	" " 3—7	·097	·756	·416	·770	·082	·019
Eye	Boys, " 13	·304	·909	·596	·238	·039	·215
	" " 3—7	·079	·825	·963	·651	·356	·679
	Girls, " 13	·833	·571	·102	·023	·044	·143
	" " 3—7	·477	·256	·513	·543	·248	·858

In the above table all the data used in calculating all the preceding tables of percentages are employed in a different way. Percentages alone give no clear or accurate measure of such a point as we are investigating; we want to obtain some definite indication of the value and importance of the fluctuations which the percentages showed. This can be obtained by using Elderton's tables to test for random sampling. It involves calculating χ^2 for the various distributions; this is done by the use of Pearson's formula given in a recent number of *Biometrika**. The actual procedure is as follows: we are dealing with forty-eight groups and we may consider any one of them. We may, for instance, take the girls aged 13 arranged according to the eye colour classification, when the cases of measles among them are given. There are therefore two populations—an infected and a non-infected population; and we have two independent distributions of the frequency of eye colour, namely in an infected and in a non-infected population.

---

* *Biometrika*, 1911, Vol. VII. p. 186.

The method then enables us to test whether both are random samples of a general population. This is done in each of the forty-eight cases, and the χ^2 is found. Having calculated the χ^2, it is then possible to read off the P from Elderton's tables. P gives a measure of the probability of the occurrence of such a distribution. When P is found to be ·1 or more, it cannot be said that the distributions are significantly different. This indicates a probability of 1 in 10. ·01 would mean 1 in 100, and this would show that the distribution was almost certainly not a random sample. Any values of P between ·1 and ·01 which occur in the table are worth noticing. There are eight values of P which work out at less than ·1, but none of them indicative of probability of less than 1 in 50, which is in itself hardly significant in 48 trials. When, therefore, we examine the table, and find that these low values of P occur entirely at random, we cannot draw any conclusions from it. It would be reasonable to expect to find one particular age or one disease distinguished by a low value of P. In the former case it would indicate that children of that age group were subject to selection from disease; or in the latter case there would be reason to think that the disease in question at all ages exercised a selective influence. On looking at the columns giving the values of P for the six different diseases, it is clear that one column does not tend to show a lower average value of P than the others. As regards ages, however, there are some indications which might at first seem to show that the distribution of cases was not equally random between the older and younger children. In examining the measles column, it is clear that the values of P for the younger children are always less than those which we find for the older children. If the values of P were low enough to be really significant in the case of the younger children, we should have some grounds for thinking that the younger children were being selected by measles; or, in other words, that some type represented in one or more of the pigmentation classes was in process of elimination owing to a greater susceptibility to measles. But, as regards measles, the figures do not warrant any conclusion of the kind; for, although P is always larger for the older children, the difference in the value of P between the two age groups is sometimes only slight, as between ·298 and ·171; and further the lowest value of P is only ·079, which indicates a probability of 1 in 13, by no means an especially low value. All that can be said is that these figures for measles are possibly suggestive.

When we turn to the other diseases, we cannot discover any regular alternation of probabilities between older and younger children, such as there is an indication of in the case of measles. In the case of whooping cough, what we discovered in measles shows some signs of appearing again; but the series is spoilt by the fact that the value of P for boys 13 years old is considerably less than for boys 3—7 years old. The other columns do not even show an approach to a regular series. Therefore, from this method of regarding the problem, we are justified in saying that the distributions of the various diseases appear with these frequencies to be entirely random; and that this is equivalent to saying that there is no correlation between any of the hair or eye colour classes and disease in general or any one disease in particular.

There is, however, another point of view from which we may look at this table. It shows no orderly system from the standpoint of age, sex and disease. But in another way the table seems to be improbable. In 48 cases we have enquired whether the samples are so divergent that they ought to occur. Therefore there ought to be one case that would only occur once on the average in 48 trials, or in other words with a probability of ·02. Now, there are actually two such cases ·023 and ·019. In the same way there should be five cases which occur once in ten trials—or with a probability roughly of ·1. Actually there are nine cases instead of five below ·1. There should be between nine and ten cases below ·2; actually there are fifteen. Thus it would seem that the whole table is somewhat improbable as a result of random sampling; but there are no systematic differences to be made out of it. Is the general irregularity a result of personal equation?

General Conclusions.

The data used in this section of my work consist of random samples of boys and girls at two different ages. We possess a classification of their pigmentation, and figures giving the number of cases of six specified diseases. An analysis of this data has shown that no obvious systematic connection can be discovered between any one category of pigmentation and disease. In other words, taking any child in our population we should not, starting from our knowledge of its pigmentation, be able to say that there was any probability of the child having had any more or less diseases than the average, or that there was a greater or less probability of the child having had one particular disease. Are we justified in going a step further and saying that in this population pigmentation is not a factor in natural selection? Suppose, for example, that the fair haired children were in process of elimination through selection, owing to their greater susceptibility to disease, would these figures show it? Now it is impossible to compare the proportions of the different categories of pigmentation, in the two age groups, and thus to estimate if these change as the children grow older, because the change in pigmentation cannot be corrected for with sufficient accuracy; and thus we cannot say how much of any change in the relative proportions of the classes of pigmentation is due to increasing age and how much to selection. We merely have the fact that boys and girls in this population, aged 3—7 and 13, do not show any connection between pigmentation and disease. Now, we argued before that if boys were being selected owing to their liability to disease, the girls should show a larger average number of diseases per child. In the same way we may argue that if the lighter haired children were being selected, the darker haired should show a greater average number of diseases per child. But we do not find anything of the kind. This argument, however, is by no means conclusive; it depends upon the assumption that, if one of two classes is in process of elimination through disease, that class differs from the other class simply in the fact that the mortality is greater. Each class or category is, according to this argument, supposed to have the same number of diseases, but one class to show less resistance and to suffer a larger mortality; and, therefore, when the two classes are examined at a later period, one is found to have more individuals who have suffered from disease. But this assumption may be false; that class which has the higher mortality may very likely have also a larger number of cases of disease; for a higher mortality might indicate a smaller power, not only to recover from disease, but also to resist the attacks. Thus that class which had the higher mortality would also have the larger number of cases. Therefore when considering the two classes, we might expect to find an equal number of individuals in both categories who had had disease. If this reasoning be correct, such results, as those we have reached in this memoir, may fail entirely to show whether selection is taking place or not. It can only be said that these results make it unlikely that pigmentation is a basis of selection; but definite results can only be obtained from data which include not merely the surviving members of the population, but those which have died as well.

In his recent memoir Macdonald* deals with the same problem. His figures come from Glasgow; and in comparing his results with those reached in this paper, it must first be remembered that the populations in Glasgow and Birmingham are by no means of similar composition. The Birmingham population is probably as homogeneous as that of any large English city. The foreign element is extremely small; children with foreign names in the schools are found very rarely. In Glasgow, on the contrary, there are large Irish and foreign elements; and this fact may be of great importance in comparing the data derived from these two cities. Macdonald obtained figures from Fever Hospitals giving the pigmentation of children who were admitted for scarlet fever, diphtheria, measles and whooping cough. The pigmentation was measured by Tocher's method†. The children in the hospitals were then compared with Tocher's results from his "Pigmentation Survey of Scottish School Children." Differences in the proportions of various classes of pigmentation between these two populations were attributed to the greater liability of those classes of pigmentation, which were found to be in excess in the hospitals, to contract disease. Before examining Macdonald's results, it may be pointed out that the fact that the pigmentations of the general and fever populations were not measured by the same persons in the same survey makes it possible that the difference in the pigmentation classes in the two populations was due to personal equation. It is difficult to avoid such sources of error when the determination of such characters as hair and eye colour is not made by some very strict scale.

Putting aside the question whether this fact could affect the value of Macdonald's results, his conclusions are at variance with those we have reached. He finds that the medium haired and medium eyed child is more liable to be

---

* Macdonald, *Biometrika*, Vol. VIII. p. 13 *et seq.*

† Tocher, *Biometrika*, Vol. VI. p. 147.

infected than a child belonging to any other category. The dark type is the least liable to infection. It is unnecessary to quote his conclusions in detail; he has made out a number of similar points. The Birmingham and Glasgow data thus seem to come into conflict; but a possible explanation may be found in the fact above noted of the different composition of the two populations. The different racial groups in Glasgow may possibly represent types whose liability to disease is of unequal intensity, while no such racial differences exist in Birmingham. At this point we are approaching the question of local races in man, a subject about which, as yet, very little is known. It is quite uncertain whether we can justly use the term local races, and we are entirely in the dark as to how far those characteristics which are supposed to determine local races are to be attributed to the influence of heredity or of the environment.

Certain of Macdonald's results refer to the mortality in the hospitals and the power of the children to recuperate. These results, which are based on data collected in the hospitals alone, cannot be affected by the objection on the ground of personal equation which might be brought against his comparison of the hospital data with those collected for the school children outside. This latter series of results confirms his general conclusion that pigmentation is a factor in selection, since he finds that ability to resist disease when once contracted is connected with hair and eye colour.

To sum up: The conclusions reached in the first section of this paper would seem to indicate that pigmentation is not a factor in selection; but the data at our disposal do not enable any very decisive answer to be given. The apparent opposition between these results and those reached by Macdonald may possibly be explained by the fact that the data used in the two inquiries are derived from very differently composed populations.

II. The Relation of Pigmentation to Height and Weight.

Introduction. In the introduction to the first section of this paper attention was called to the small quantity and vague character of the work which had been done upon the problem with which that section was concerned. But in connection with this second problem we find an almost complete absence of exact investigation. The interest of the question in relation to local races and similar biological subjects is obvious; nor is its importance from other points of view less great. Until we have some exact knowledge of the relative value of hereditary and environmental influences, we are not justified in attributing greater importance to the one than to the other. In spite of this absence of knowledge it is the fashion at the present day to attribute all low averages of such characters as height and weight, when found in connection with poor social conditions, to those conditions no pause is made to enquire whether the low average of the physical measurements might not be racial characters which are developed without, or at least with only slight, reference to the environment. Now, it is a matter of common knowledge that races are distinguished by differences in pigmentation; if, therefore, a relation between pigmentation and height and weight were established, there would no longer be any justification for attributing these characters entirely to the environment. In order to elucidate this question the ideal material to examine would be data collected from a city where there were known to be different racial elements. All the individuals examined should be of the same age, and preferably should have been born in the district, or at least have lived a long time there. But such material is not easy to get; the examination of school children, however, does provide useful data. It is better that the individuals should be of the same age, since in that case no correction is required for the change in pigmentation owing to increasing age. Although such a correction can be made, it introduces a disturbing factor which does not tend to increase the accuracy of the results. Birth and not only residence in the same district is a desirable feature for all the individuals, since the results are otherwise open to the criticism that the influence of an earlier environment has not been taken into account. If, with such material as is here indicated, a statistical examination were to show that pigmentation as a racial character was correlated with other physical characters, then we should have obtained an important result which would bear on the correct degree of influence to be attributed to social conditions.

The data which have been employed in the present work are for our immediate purposes unfortunately far from ideal. They are derived from two sources. Firstly, there is the medical survey of Birmingham School Children, which has been used in the first section of this paper. The one advantage which it offers is the fact that all the children are the same age; and therefore there is no need to correct for a change of pigmentation. But we do not know how long the children have been in Birmingham or where they were born. This, however, is not so serious a defect as the fact that the Birmingham population is a singularly homogeneous one. Foreign names are extremely rare among the children. This means that, if in such a population a connection between pigmentation and stature is found, it must be a very important feature indeed. The failure to find it here would not be surprising, and it might well exist in a more heterogeneous population. The Birmingham data give both height and weight for every child. Now, as has been explained in the first part of the paper, if two characters A and B are correlated and A is also correlated with another character C, then there is no reason to expect a correlation between B and C unless the correlation between A and B is very high. In other words, although we know that height and weight are correlated, if we find a correlation between pigmentation and height, there is no reason to expect a correlation between pigmentation and weight, unless the correlation between weight and height is a very high one. Therefore the investigations of height and weight and their relation to pigmentation are treated as entirely separate.

Secondly, there are the data collected by Tocher in his "Survey of the Pigmentation of the Scottish Insane*." In this case we have height and pigmentation

---

* *Biometrika*, Vol. v. p. 298.

only. Although many correlations were worked out and various points investigated by Tocher from his data, he did not use his material to throw light on this point. The Scottish data are more numerous than those from Birmingham, and the population more heterogeneous. But the environmental conditions are by no means the same, and moreover the population is a selected one. Therefore any positive results from such data, though interesting, would be far from conclusive, and would demand further investigation of the subject.

Birmingham Data. Height and Pigmentation. The method of pigmentation classification used at Birmingham has already been described. The height was taken to the nearest millimetre. This latter fact is of importance in calculating the means. In order to form a table the numbers of each hair colour group were classed into groups of two centimetres each. Now, owing to the fact that the height is taken to the nearest millimetre, the middle point of the group 150—152 centimetres will not be 151 but 150·95. The data were first treated by calculating the means and probable error for the total number of boys and girls and thus for each of the six hair colour groups. The result is given in the following table.

TABLE XII. (See Tables XVII and XVIII, pp. 380—381.)

Hair Colour and Height.

Mean Height in Centimetres.

Hair Colour	Boys. Age 13	Girls. Age 13
Red	142·86 ± ·64	144·15 ± ·51
Fair...	142·47 ± ·28	144·72 ± ·34
Light Brown ...	142·39 ± ·22	143·71 ± ·24
Medium Brown ...	142·41 ± ·19	144·69 ± ·19
Dark Brown ...	143·69 ± ·21	145·41 ± ·20
Black	142·69 ± ·93	144·56 ± ·60
General Mean ...	142·80 ± ·10	144·67 ± ·12

It is there seen that at this age girls are substantially taller than boys, i.e. by about two centimetres. When we look at the means of the hair colour groups, it is at once evident that none of them are going to show any marked deviation from the general mean. There are, however, some cases which demand closer attention; these are the dark brown hair groups in both cases, and the light brown hair group in the case of girls. In order to show this more clearly two figures have been constructed—one for the boys and the other for the girls.

The method adopted in both these cases is the same. A straight line shows the mean of the whole number of boys or girls as the case may be. The means of the various hair colour groups have been marked and joined together. Two other lines have been introduced and tabled as indicating twice the probable error. The probable error here referred to was obtained in the following way. The formula for the probable error of the mean $67449 \frac{\sigma}{\sqrt{N}}$ was used; σ was taken in each case to represent the standard deviation of the mean of the whole, while N was the total of each hair colour group. Thus instead of calculating the probable error ot each mean in the usual manner, the probable error of each mean with reference to the mean of the whole is obtained. This gives the easiest method of estimating the value of the separate means from the mean of the whole. Lines have been drawn in each table to mark twice the probable error thus calculated. On looking at Figure I A it is clear that the only group which need occupy our attention is that of the dark brown hair. All the other means fall well within the line representing twice the probable error; but the mean of the dark brown hair group stands outside it. A line representing four times the probable error would include it, and, therefore, the deviation is not a very serious one; on this evidence alone it

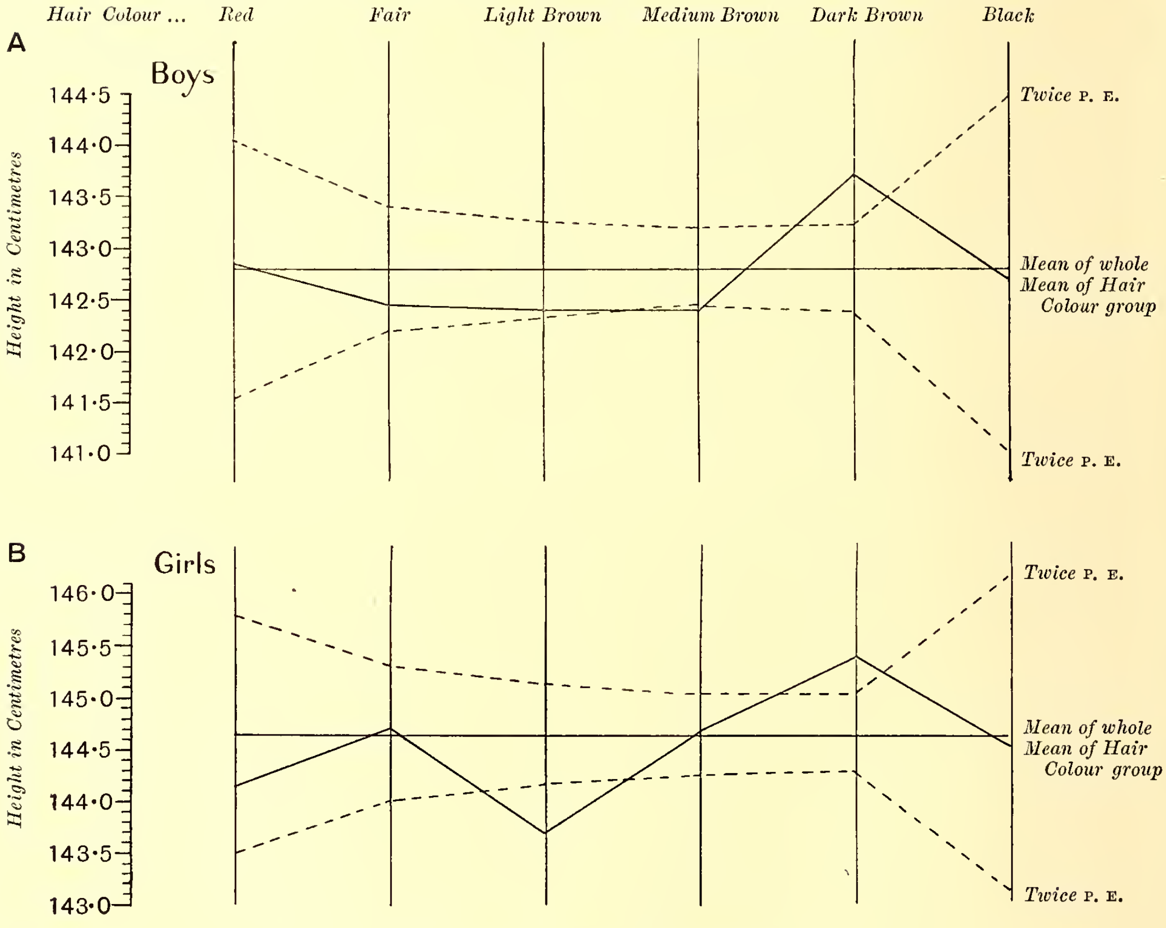


FIG. I. Age 13. Hair Colour and Height.

cannot be called a really significant difference. But Figure I B which is constructed in exactly the same manner for the girls gives reason for thinking that there may be some significance attaching to the greater height of the dark brown group; for in this case again the dark brown group shows a distinctly greater mean height. It is not quite, though it is nearly, as great a deviation from the general mean as in the case of the boys. One other hair colour group also calls for attention in Figure I B. The mean height of the light brown hair group is considerably less than the general mean; it would be included by a line drawn to represent about three and a half times the probable error. It seems unlikely that boys and girls would differ in this respect, and that the same pigmentation class should be associated with a significant difference in stature in one sex and not in the other. But apart from the improbability of this occurrence the deviation is not such as would be accounted very markedly significant in any case.

Scottish Hair Data. Height and Pigmentation. As has been already mentioned, the data collected by Tocher in his survey of the Scottish insane have also been employed to investigate the problem of the relation between pigmentation and

TABLE XIII. (See Tables XIX and XX, p. 382.)

Pigmentation and Height. Scottish Insane.

Mean Height in Feet.

		Men	Women
Hair Colour:			
Red		5 feet 7 inches ·31 ± ·27	5 feet 1 inches ·92 ± ·21
Fair		5 ,, 6 ,, ·42 ± ·1	5 ,, 1 ,, ·49 ± ·24
Medium		5 ,, 6 ,, ·33 ± ·04	5 ,, 1 ,, ·51 ± ·04
Dark		5 ,, 6 ,, ·28 ± ·05	5 ,, 1 ,, ·72 ± ·05
Eye Colour:			
Light		5 ,, 6 ,, ·30 ± ·05	5 ,, 1 ,, ·53 ± ·08
Medium		5 ,, 6 ,, ·46 ± ·05	5 ,, 1 ,, ·70 ± ·05
Dark		5 ,, 6 ,, ·28 ± ·07	5 ,, 1 ,, ·60 ± ·06
Hair and Eye Colour:			
Red	Light	5 ,, 7 ,, ·10 ± ·31	5 ,, 0 ,, ·76 ± ·39
	Medium	5 ,, 7 ,, ·63 ± ·74	5 ,, 2 ,, ·95 ± ·24
	Dark	5 ,, 7 ,, ·60 ± ·76	5 ,, 0 ,, ·71 ± ·56
Fair	Light	5 ,, 6 ,, ·51 ± ·18	5 ,, 1 ,, ·27 ± ·31
	Medium	5 ,, 6 ,, ·58 ± ·21	5 ,, 2 ,, ·02 ± ·39
	Dark	5 ,, 7 ,, ·08 ± ·47	5 ,, 1 ,, ·75 ± ·66
Medium	Light	5 ,, 6 ,, ·27 ± ·15	5 ,, 2 ,, ·35 ± ·17
	Medium	5 ,, 6 ,, ·49 ± ·07	5 ,, 1 ,, ·41 ± ·08
	Dark	5 ,, 6 ,, ·12 ± ·1	5 ,, 1 ,, ·60 ± ·11
Dark	Light	5 ,, 6 ,, ·21 ± ·11	5 ,, 1 ,, ·47 ± ·10
	Medium	5 ,, 6 ,, ·34 ± ·09	5 ,, 1 ,, ·89 ± ·08
	Dark	5 ,, 6 ,, ·29 ± ·09	5 ,, 1 ,, ·69 ± ·07
General Mean		5 ,, 6 ,, ·32 ± ·03	5 ,, 1 ,, ·6 ± ·03

height. Both the methods used and the colour categories adopted to classify pigmentation are in this case different from those employed at Birmingham. There are four hair colour groups—red, fair, medium and dark, and three eye colour groups, light, medium and dark. In this case again the usefulness of the data suffers from lack of precision in classification. The height was taken to the nearest inch.

The data have been treated in the same way as those derived from Birmingham. Table XIII corresponds to Table XII; but the mean height and probable error have here been found not only for the hair colour groups, but also for the eye colour groups, and every combination of hair and eye colour. An inspection of this Table shows how little deviation there is from the general mean. When we look to see whether there is any indication of the same deviations from the general mean commented upon in connection with the Birmingham data, we fail to find any correspondence whatever. In this case, on the whole, there is less deviation than at Birmingham, and there is nothing to call for attention.

In order to make this more clear, figures have been constructed on the same principle as that followed to form Figures I A and B. Here again in Figure II A, B, C and D the mean of the whole is represented together with the means of the various groups and twice the probable error calculated with reference to the mean of the whole.

Figure II A and B gives the height and hair colour shown according to this method. In this case we have the only mean which exceeds twice the probable error. The mean of the red haired men is considerably in excess of the general mean, but not so much as to be of itself very markedly significant; and further, when it is found that among the women in Figure II B no such very marked deviation is indicated, then we are probably justified in thinking that it does not represent any true connection between red hair and stature in excess of the mean. A glance at Figure II is enough to show that the differences are, except for red-haired men, absolutely insignificant, and that there is no indication whatever to be found of those deviations which we observed in the Birmingham data.

Height and Pigmentation. Summary. The data, therefore, which have been examined in this paper do not afford any grounds for the belief that pigmentation and height are correlated. As has been pointed out, there are some vague indications of possible association between a certain pigmentation class, namely dark brown, and a stature in excess of the general mean. But this scarcely does more than to provide hints for future research; and the main essential for such research is that pigmentation should be measured with greater accuracy. With data such as have been at our disposal, which in the first place are derived from populations that would not be deliberately chosen for such an investigation, and which are lacking in the accuracy of pigmentation classification that is desirable, it would be surprising to reach any markedly definite result if the relation be really small; the failure to do so, therefore, does not make it less probable than before that,

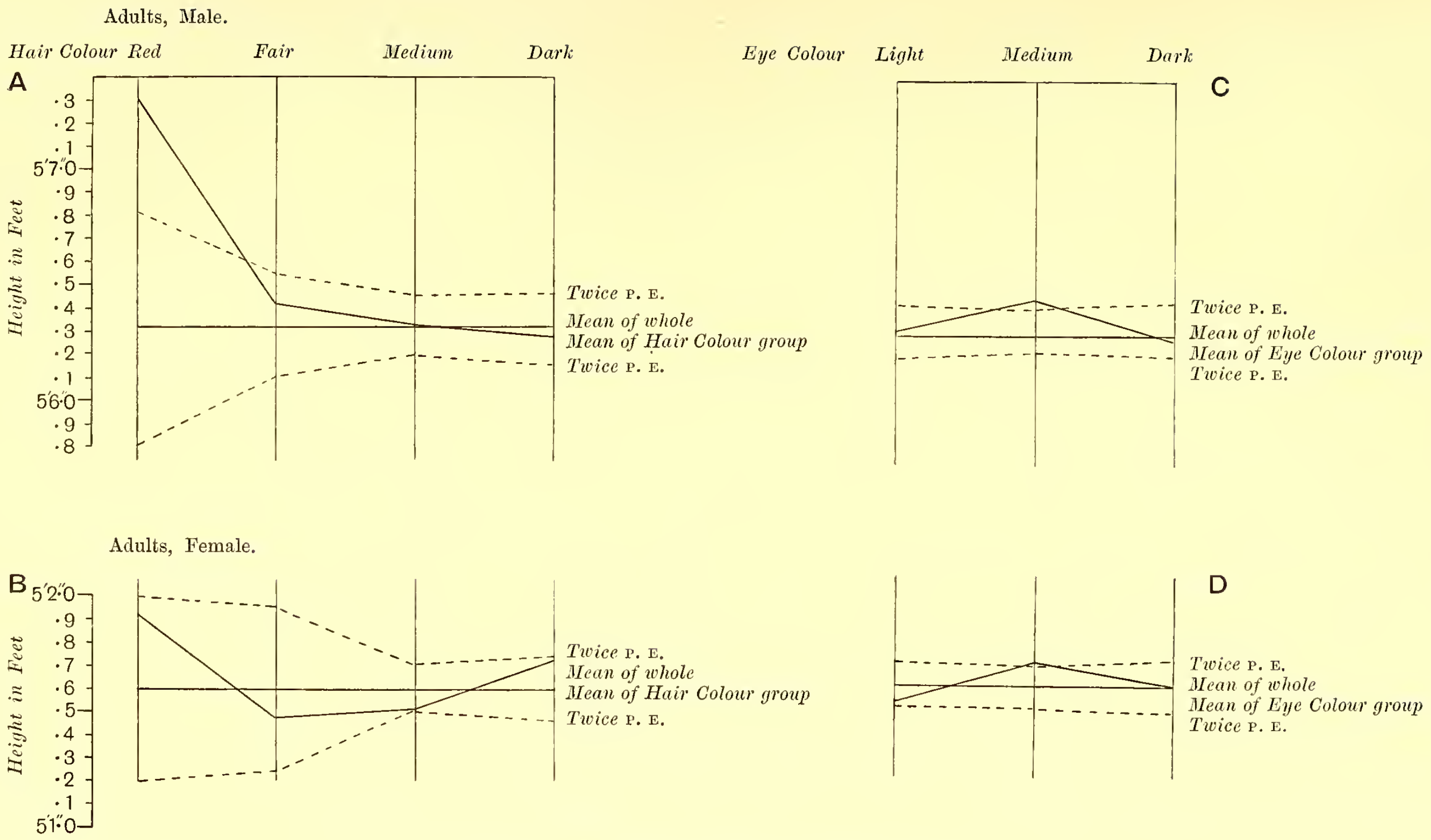


Fig. II. Pigmentation and Height. Scottish Insane.

with more suitable means, some *not very marked* connection between pigmentation and height may ultimately be found.

Birmingham Data. Pigmentation and Weight. The Birmingham data also give the weight of the children, and therefore afford an opportunity of investigating the relation between pigmentation and weight. As has been explained, this is a problem separate from that of the relation between pigmentation and height, and there might possibly be a correlation between pigmentation and weight although there were none between pigmentation and height. The hair and eye colour method of classification has been described; the weight was measured in kilogs.

TABLE XIV. (See Tables XXI and XXII, pp. 383—384.)

Hair Colour and Weight. Birmingham.

Mean Weight in Kilogs.

Hair Colour	Boys. Age 13	Girls. Age 13
Red	33·69 ± ·38	36·22 ± ·43
Fair	34·28 ± ·21	35·71 ± ·26
Light Brown ...	34·40 ± ·14	35·59 ± ·19
Medium Brown ...	34·15 ± ·12	36·28 ± ·15
Dark Brown ...	35·08 ± ·15	36·62 ± ·15
Black	34·95 ± ·57	35·92 ± ·49
General Mean ...	34·39 ± ·04	36·22 ± ·04

The data have been treated in exactly the same way as in the previous section of the paper. Table XIV gives the mean weight for the total number of both boys and girls and their probable errors; the means and probable errors are also given for each hair colour group. Again we notice no striking deviation from the mean; but certain groups differ from the remainder enough to require special attention.

Figure III is constructed in the same manner as before in the case of the pigmentation and height data. With regard to the boys, the only group which differs in any remarkable way is the dark brown group. The deviation is not very large, and would be covered by four times the probable error. It is interesting to find that among the girls the same deviation is apparent, though to a much smaller extent. In the latter case the mean of the group only falls just above the line representing twice the probable error. These deviations are similar to, but not quite so marked as, those which we have observed in the pigmentation and height table for the dark brown group. The coincidence is certainly interesting and suggestive, though it cannot fairly be called more. The coincidence is, however, further increased by the fact shown in Figure III B that the light brown group have a lower mean weight than the mean of the whole. It is impossible to estimate the true value, if there be any, of these results; further research alone can show whether there is anything in these possible connections between pigmentation and height and weight. The general value of these latter figures with reference to pigmentation and weight is depreciated by just the same circumstances as those already discussed in connection with the data as a whole, and it is not necessary to repeat them here.

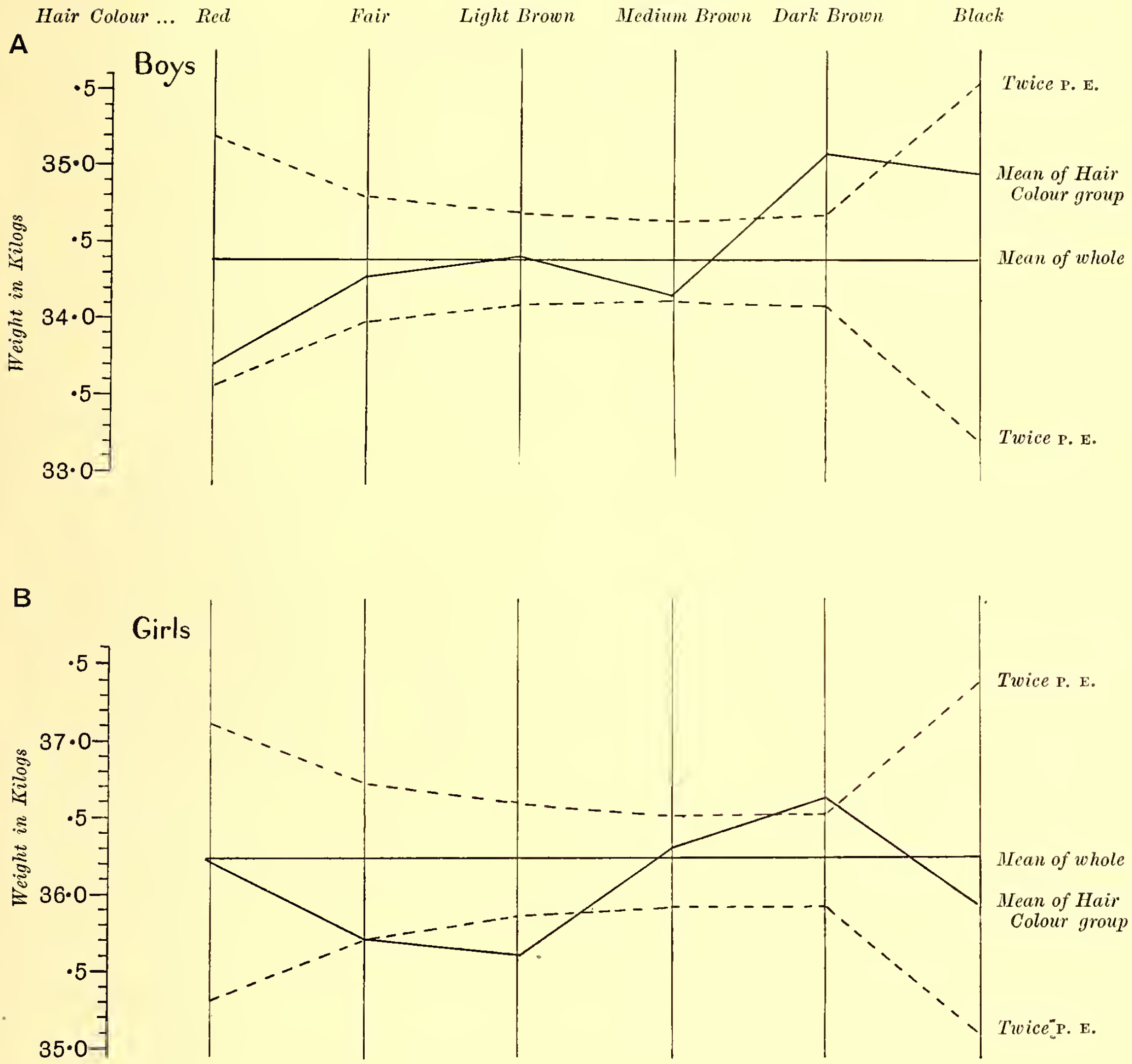


Fig. III. Age 13. Hair Colour and Weight.

TABLE XV.

Boys. *Age* 13.

Hair Colour	Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox		Totals	Totals of Hair Colour
		n. a.	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*		
Red	Blue ...	5	2	1	6	—	7	—	7	1	6	2	5	7	62
,, ...	Light ...	17	4	7	14	3	18	—	21	2	19	6	15	21	
,, ...	Neutral	17	4	8	13	7	14	—	21	10	11	9	12	21	
,, ...	Dark ...	10	3	8	5	1	12	1	12	4	9	6	7	13	
Fair	Blue ...	53	14	25	42	12	55	4	63	13	54	25	42	67	253
,, ...	Light ...	75	34	33	76	14	95	—	109	23	86	36	73	109	
,, ...	Neutral	48	19	19	48	8	59	3	64	13	54	13	54	67	
,, ...	Dark ...	9	1	3	7	1	9	—	10	2	8	2	8	10	
Light Brown ...	Blue ...	84	36	43	77	14	106	3	117	21	99	26	94	120	443
,, ...	Light ...	116	42	59	99	25	133	3	155	29	129	44	114	158	
,, ...	Neutral	75	34	33	76	19	90	6	103	26	83	35	74	109	
,, ...	Dark ...	43	13	16	40	6	50	2	54	11	45	19	37	56	
Medium Brown	Bluc ...	88	25	43	70	18	95	5	108	26	87	28	85	113	626
,, ...	Light ...	119	36	65	90	24	131	6	149	30	125	53	102	155	
,, ...	Neutral	152	47	77	122	37	162	9	190	44	155	52	147	199	
,, ...	Dark ...	127	32	65	94	29	130	11	148	31	128	44	115	159	
Dark Brown ...	Blue ...	30	16	18	28	11	35	5	41	8	38	9	37	46	579
,, ...	Light ...	99	25	45	79	19	105	5	119	20	104	33	91	124	
,, ...	Neutral	103	39	58	84	24	118	6	136	41	101	34	108	142	
,, ...	Dark ...	205	62	97	170	48	219	8	259	65	202	80	187	267	
Black	Blue ...	1	—	—	1	—	1	—	1	—	1	1	—	1	31
,, ...	Light ...	1	—	—	1	—	1	—	1	—	1	1	—	1	
,, ...	Neutral	4	2	3	3	—	6	—	6	2	4	2	4	6	
,, ...	Dark ...	18	5	13	10	7	16	—	23	5	18	11	12	23	
Totals ...	...	1499	495	739	1255	327	1667	77	1917	427	1567	571	1423	1994	1994

n. a. = not-attacked; *a.* = attacked.

Totals of Eye Colour

Blue	354
Light	568
Neutral ...	544
Dark	528
Total ...	1994

Totals of Diseases

Measles ...	1499
Whooping Cough	739
Scarlet Fever ...	327
Diphtheria ...	77
Mumps ...	427
Chicken Pox ...	571
Total ...	3640

TABLE XVI.

Girls. Age 13.

Hair Colour	Eye Colour	Measles		Whooping Cough		Scarlet Fever		Diphtheria		Mumps		Chicken Pox		Totals	Totals of Hair Colour
		n. a.	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*	*n. a.*	*a.*		
Red	Blue ...	16	2	7	11	1	17	1	17	4	14	3	15	18	74
"	Light ...	19	2	12	9	3	18	1	20	9	12	8	13	21	
"	Neutral	17	1	7	11	2	16	1	17	5	13	7	11	18	
"	Dark ...	13	4	4	13	1	16	—	17	3	14	4	13	17	
Fair	Blue ...	53	9	26	36	10	52	1	61	11	51	21	41	62	237
"	Light ...	100	16	55	61	18	98	9	107	21	95	30	86	116	
"	Neutral	40	4	17	27	8	36	2	42	13	31	12	32	44	
"	Dark ...	14	1	7	8	2	13	1	14	5	10	7	8	15	
Light Brown ...	Blue ...	77	15	43	49	23	69	2	90	21	71	38	54	92	406
" ...	Light ...	137	23	71	89	34	126	11	149	46	114	47	113	160	
" ...	Neutral	90	11	46	55	20	81	4	97	32	69	41	60	101	
" ...	Dark ...	45	8	21	32	10	43	1	52	10	43	19	34	53	
Medium Brown	Blue ...	118	21	52	87	33	106	5	134	34	105	48	91	139	652
" ...	Light ...	127	15	75	67	27	115	13	129	25	117	54	88	142	
" ...	Neutral	188	35	101	122	40	183	11	212	60	163	77	146	223	
" ...	Dark ...	126	22	64	84	28	120	5	143	30	118	54	94	148	
Dark Brown ...	Blue ...	61	9	29	41	17	53	3	67	12	58	24	46	70	672
" ...	Light ...	111	23	51	83	23	111	6	128	26	108	49	85	134	
" ...	Neutral	140	25	73	92	34	131	8	157	41	124	63	102	165	
" ...	Dark ...	252	51	144	159	42	261	13	290	70	233	124	179	303	
Black	Blue ...	4	—	2	2	1	3	—	4	—	4	3	1	4	46
"	Light ...	2	2	—	4	2	2	—	4	—	4	1	3	4	
"	Neutral	10	3	8	5	1	12	—	13	4	9	3	10	13	
"	Dark ...	23	2	12	13	5	20	1	24	2	23	13	12	25	
Totals ...	...	1783	304	937	1150	385	1702	99	1988	484	1603	740	1347	2087	2087

n. a. = not-attacked; *a.* = attacked.

Totals of Eye Colour

Blue ...	...	385
Light ...	...	577
Neutral	...	564
Dark ...	...	561
Total	...	2087

Totals of Diseases

Measles	...	1783
Whooping Cough		937
Scarlet Fever ...		385
Diphtheria	...	99
Mumps	...	484
Chicken Pox	...	740
Total	...	4428

TABLE XVII.

Boys. Age 13. Birmingham. Hair and Eye Colour with Height.

Height in Centimetres.

Hair and Eye Colour.		*112*	*114*	*116*	*118*	*120*	*122*	*124*	*126*	*128*	*130*	*132*	*134*	*136*	*138*	*140*	*142*	*144*	*146*	*148*	*150*	*152*	*154*	*156*	*158*	*160*	*162*	*164*	*166*	*168*	*170*	*172*	Totals
Red	Blue ...	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	3	1	1	—	—	—	—	—	—	—	—	—	—	—	7
”	Light	—	—	—	—	—	—	—	1	2	—	1	1	2	1	3	1	5	—	1	—	2	—	—	1	—	—	—	—	—	—	—	21
”	Neutral	—	—	—	—	—	—	—	—	—	2	1	2	1	—	3	3	—	—	4	1	2	1	1	—	—	—	—	—	—	—	—	21
”	Dark ...	—	—	—	—	—	—	—	—	—	—	1	—	2	3	—	1	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	13
Fair	Blue ...	—	—	—	—	—	—	—	—	—	5	5	4	6	8	12	3	7	5	3	6	2	—	—	—	1	—	—	—	—	—	—	67
”	Light	—	—	—	—	—	—	—	1	—	3	4	7	11	11	8	17	11	14	10	4	3	4	1	—	—	—	—	—	—	—	—	109
”	Neutral	—	—	—	—	—	—	—	2	1	1	4	1	9	4	6	7	8	8	12	—	1	—	—	1	1	1	—	—	—	—	—	67
”	Dark ...	—	—	—	—	—	—	—	—	1	1	1	—	—	1	2	1	—	—	1	—	1	—	—	—	1	—	—	—	—	—	—	10
Light Brown	Blue ...	—	—	—	—	—	—	—	—	1	4	5	7	19	6	13	15	10	16	10	3	5	2	2	2	—	—	—	—	—	—	—	120
” ...	Light	—	—	—	—	—	—	1	1	3	10	7	10	14	13	16	14	15	20	13	8	3	2	1	4	1	1	—	1	—	—	—	158
” ...	Neutral	—	—	—	—	—	—	—	—	2	3	3	7	16	15	6	16	12	12	7	7	—	2	—	1	—	—	—	—	—	—	—	109
” ...	Dark ...	—	—	—	—	—	—	—	2	1	2	4	5	5	2	5	4	7	5	3	5	4	1	—	1	—	—	—	—	—	—	—	56
Medium Brown	Blue ...	—	—	—	—	—	—	—	1	—	1	5	5	14	14	18	15	7	12	5	9	4	3	—	—	—	—	—	—	—	—	—	113
” ...	Light	—	—	—	—	—	—	—	2	5	5	7	8	12	15	19	18	18	17	10	4	8	3	1	1	2	—	—	—	—	—	—	155
” ...	Neutral	—	—	—	1	—	—	1	2	6	8	6	5	15	18	26	23	32	13	11	13	10	4	2	—	—	1	—	1	—	—	1	199
” ...	Dark ...	—	—	—	—	1	—	1	1	3	4	5	14	18	22	13	22	18	7	9	8	6	1	2	2	—	—	—	1	—	—	1	159
Dark Brown ...	Blue ...	—	—	—	—	—	—	—	—	1	1	1	1	3	5	5	3	7	5	2	2	4	5	1	—	—	—	—	—	—	—	—	46
” ...	Light	1	—	—	—	—	—	2	—	1	4	8	8	9	10	16	10	14	12	6	6	5	4	3	3	2	—	—	—	—	—	—	124
” ...	Neutral	—	—	—	—	—	—	—	—	4	5	7	10	11	15	12	10	16	10	11	11	6	5	3	3	1	1	1	—	—	—	—	142
” ...	Dark ...	1	—	—	—	1	1	1	3	1	2	6	13	18	25	30	34	36	25	18	14	17	4	5	7	5	—	—	—	—	—	—	267
Black	Blue ...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
”	Light	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
”	Neutral	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	1	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	6
”	Dark ...	—	—	—	—	—	—	—	—	—	1	1	2	4	1	3	1	2	3	1	3	1	—	—	—	—	—	—	—	—	—	—	23
Totals ...	...	2	—	—	1	2	1	6	17	32	62	83	111	189	189	217	221	228	189	139	105	85	42	22	26	14	5	1	3	—	—	2	1994

TABLE XVIII.

Girls. Age 13. *Birmingham. Hair and Eye Colour with Height.*

Height in Centimetres.

Hair and Eye Colour.

		106	*112*	*114*	*116*	*118*	*120*	*122*	*124*	*126*	*128*	*130*	*132*	*134*	*136*	*138*	*140*	*142*	*144*	*146*	*148*	*150*	*152*	*154*	*156*	*158*	*160*	*162*	*164*	*166*	*168*	*170*	*172*	*184*	Totals
Red	Blue ...	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3	3	4	1	2	2	2		—	—	—	1	—	—	—	—	—	—	—	19
„	Light	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	2	2	1	4	4	—	2	1	1	—	—	1	—	—	—	—	—	—	21
„	Neutral	—	—	—	—	—	—	—	—	—	—	—	2	1	2	3	2	2	3	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	18
„	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	3	—	3	3	2	1	2	—	—	—	2	1	—	—	—	—	—	—	—	—	—	17
Fair	Blue ...	—	—	—	—	—	1	—	—	2	—	2	3	4	2	6	7	8	7	4	2	8	3	1	1	1	—	—	—	—	—	—	—	—	62
„	Light	—	—	—	—	—	—	—	2	—	4	—	1	6	7	6	9	12	12	10	13	12	6	4	5	1	—	3	1	1	1	—	—	—	116
„	Neutral	—	—	—	—	—	—	—	1	—	—	—	1	1	3	—	6	8	2	6	3	4	3	2	2	2	—	—	—	—	—	—	—	—	44
„	Dark ...	—	—	—	—	—	—	—	—	—	1	1	—	—	2	3	1	2	2	—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	15
Light Brown	Blue ...	—	—	—	—	—	—	—	—	—	1	4	4	6	5	5	12	13	12	7	4	8	6	3	1	—	1	—	—	—	—	—	—	—	92
„ ...	Light	—	—	—	—	—	—	—	1	1	—	6	2	9	10	16	12	23	21	16	11	12	6	8	2	1	1	—	—	—	—	—	—	—	158
„ ...	Neutral	—	—	—	—	1	—	—	—	1	2	2	2	8	6	12	8	10	12	11	6	8	5	3	3	1	—	—	—	—	—	—	—	—	101
„ ...	Dark ...	—	—	—	1	—	—	1	—	2	—	—	1	—	4	3	7	6	3	3	6	3	4	6	1	—	1	—	—	—	—	—	—	—	52
Medium Brown	Blue ...	—	—	—	—	—	—	—	—	1	2	3	5	9	9	9	10	16	14	15	11	10	10	5	5	2	2	1	—	—	—	—	—	—	139
„ ...	Light	—	—	—	—	—	—	—	—	3	1	1	2	10	10	8	10	15	21	13	9	16	10	9	2	2	—	—	—	—	—	—	—	—	142
„ ...	Neutral	—	—	—	—	—	1	—	1	1	2	2	5	14	9	20	13	29	22	25	22	18	10	8	10	3	7	1	—	—	—	—	—	—	223
„ ...	Dark ...	—	1	—	—	1	—	—	1	—	1	—	6	7	12	10	16	18	18	20	14	8	7	3	3	1	2	—	—	—	—	—	—	—	149
Dark Brown	Blue ...	—	—	—	—	—	—	—	—	2	2	2	3	4	5	5	3	5	7	12	8	2	6	3	1	—	—	—	—	—	—	—	—	—	70
„ ...	Light	—	—	—	—	—	—	1	—	2	2	1	4	4	10	12	7	11	14	9	15	11	11	9	5	2	2	—	1	—	—	1	—	—	134
„ ...	Neutral	—	—	—	—	—	—	—	—	1	1	1	3	7	4	9	19	14	13	25	20	10	17	5	9	5	2	—	—	—	—	—	—	—	165
„ ...	Dark ...	1	—	1	1	—	—	1	—	1	3	3	9	15	12	25	23	29	26	31	41	30	20	15	4	3	3	2	2	—	—	—	—	1	302
Black	Blue ...	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Light	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Neutral	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	2	4	3	—	1	—	—	—	—	—	—	—	—	—	—	13
„	Dark ...	—	—	—	—	—	—	—	—	—	—	1	2	—	2	1	4	1	6	3	2	2	—	—	1	—	—	—	—	—	—	—	—	—	25
Totals ...	...	1	1	1	2	2	2	3	6	18	22	29	57	109	114	161	180	230	221	221	202	168	127	89	57	24	22	8	4	1	1	1	—	1	2085

TABLE XIX. *Men. Scottish Insane.*

Height in feet.

Hair and Eye Colour.		FOUR FEET												FIVE FEET												SIX FEET				Totals
		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	
Red ...	Light	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	3	—	3	5	9	4	3	4	1	1	—	—	—	36
" ...	Medium	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	2	3	3	1	2	—	1	1	—	—	16
" ...	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	—	1	1	—	—	2	2	—	—	—	—	10
Fair ...	Light	—	—	—	—	—	—	—	—	—	—	—	2	—	4	5	16	21	20	26	26	26	9	8	1	4	—	—	—	168
" ...	Medium	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	7	14	11	11	7	5	7	5	3	3	—	—	—	78
" ...	Dark ...	—	—	—	—	—	—	—	—	—	1	1	—	—	2	2	1	3	2	5	—	6	—	1	—	—	—	—	—	24
Medium	Light	—	1	—	1	—	—	1	1	1	6	11	6	20	40	73	99	146	180	215	172	148	96	62	36	15	3	4	—	1337
" ...	Medium	—	—	1	—	—	—	1	—	2	1	3	9	7	24	39	40	74	113	110	124	95	54	34	22	13	4	1	1	772
" ...	Dark ...	—	—	1	—	—	—	—	—	1	1	2	2	6	12	29	31	44	51	60	44	32	30	15	14	5	1	—	—	381
Dark ...	Light	—	1	1	—	—	—	—	—	—	1	—	2	9	13	19	24	41	46	51	48	30	21	21	10	3	4	—	—	345
" ...	Medium	—	—	—	—	—	—	—	—	2	2	1	5	8	11	23	37	49	68	76	70	52	27	28	7	8	3	1	—	478
" ...	Dark ...	1	—	—	—	—	—	1	1	—	1	4	4	9	11	28	39	54	73	78	78	52	33	20	7	10	6	1	—	511
Totals	...	1	2	3	1	—	—	3	2	6	13	23	31	61	120	222	299	448	567	640	582	453	281	202	103	63	22	7	1	4156

TABLE XX. *Women. Scottish Insane.*

Height in Feet.

Hair and Eye Colour.		THREE FEET				FOUR FEET												FIVE FEET												SIX FEET		Totals
		8	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*0*	*1*	
Red ...	Light	—	—	—	—	—	1	—	—	—	—	—	—	—	2	2	4	6	1	6	2	2	1	—	—	—	—	—	—	—	—	27
" ...	Medium	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	3	3	9	9	8	6	1	3	2	—	—	1	—	—	—	47
" ...	Dark ...	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2	1	2	—	4	2	—	1	—	—	—	—	—	—	—	—	14
Fair ...	Light	—	—	—	—	1	1	—	2	—	1	—	—	1	2	9	4	11	9	12	6	3	6	2	3	—	—	—	1	—	—	74
" ...	Medium	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	5	1	3	6	3	6	1	—	1	—	—	—	—	—	—	29
" ...	Dark ...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	4
Medium	Light	—	—	—	—	1	—	1	1	1	3	2	11	16	43	62	104	139	178	171	117	71	42	19	13	4	1	—	—	1	1	1002
" ...	Medium	1	—	1	—	—	5	1	1	—	—	1	7	7	30	49	67	88	91	95	76	45	33	11	5	2	2	—	—	—	—	618
" ...	Dark	—	—	—	—	—	—	—	—	1	3	1	3	6	18	25	39	56	65	60	45	23	14	9	4	1	1	1	—	—	—	375
Dark ...	Light	—	—	—	—	1	—	—	—	—	1	—	4	13	13	20	33	63	54	44	44	20	15	9	3	2	—	—	—	—	1	340
" ...	Medium	—	—	—	—	—	—	1	—	2	3	1	3	5	18	40	51	69	89	90	81	33	32	19	10	3	2	—	1	—	—	553
" ...	Dark ...	—	—	—	—	2	1	—	2	1	—	2	8	10	20	46	58	66	98	100	63	49	30	14	11	6	—	—	—	2	—	589
Totals	...	1	—	1	—	5	8	3	6	5	12	8	36	59	147	259	369	504	597	599	448	258	176	86	52	18	6	2	2	3	2	3672

TABLE XXI.

Boys. Age 13. Hair and Eye Colour with Weight.

Weight in Kilogs.

Hair and Eye Colour.		*20*	*22*	*24*	*26*	*28*	*30*	*32*	*34*	*36*	*38*	*40*	*42*	*44*	*46*	*48*	*50*	*52*	*54*	*56*	Totals
Red	Blue ...	—	—	1	—	—	—	3	—	2	—	1	—	—	—	—	—	—	—	—	7
,, ...	Light ...	—	—	2	1	2	—	5	6	2	1	2	—	—	—	—	—	—	—	—	21
,, ...	Neutral	—	—	1	2	1	6	2	2	1	2	2	2	—	—	—	—	—	—	—	21
,, ...	Dark ...	—	—	—	—	1	3	3	4	2	—	—	—	—	—	—	—	—	—	—	13
Fair	Blue ...	—	—	2	2	12	7	17	13	6	3	2	—	1	1	1	—	—	—	—	67
,, ...	Light ...	—	1	1	3	15	15	21	18	12	11	7	3	—	2	—	—	—	—	—	109
,, ...	Neutral	—	—	1	3	6	9	12	11	7	10	3	2	1	—	1	1	—	—	—	67
,, ...	Dark ...	—	—	1	1	2	—	1	1	—	1	1	—	1	—	—	—	—	—	1	10
Light Brown	Blue ...	—	—	1	7	15	17	25	14	20	9	9	—	1	2	—	—	—	—	—	120
,, ...	Light ...	—	—	2	11	18	24	29	24	21	7	10	7	2	2	—	—	—	1	—	158
,, ...	Neutral	—	1	1	6	8	18	32	15	17	3	5	1	—	1	1	—	—	—	—	109
,, ...	Dark ...	—	—	4	4	4	10	5	10	7	7	2	2	1	—	—	—	—	—	—	56
Medium Brown	Blue ...	—	—	2	6	11	12	35	17	11	8	5	5	—	1	—	—	—	—	—	113
,, ...	Light ...	—	—	3	6	23	15	30	35	20	8	9	5	—	1	—	—	—	—	—	155
,, ...	Neutral	—	—	5	10	15	36	39	25	23	23	6	10	5	1	1	—	—	—	—	199
,, ...	Dark ...	1	1	3	8	13	26	27	30	17	13	9	5	2	1	1	—	—	—	1	158
Dark Brown...	Blue ...	—	—	1	2	3	6	5	7	5	6	3	3	1	3	1	—	—	—	—	46
,, ...	Light ...	—	1	—	11	16	16	16	20	16	9	8	6	1	1	1	—	—	1	1	124
,, ...	Neutral	—	—	2	10	15	18	23	20	19	16	5	4	4	—	3	2	—	1	—	142
,, ...	Dark ...	2	—	2	9	17	39	44	49	34	35	14	8	5	5	1	—	2	1	—	267
Black... ...	Blue ...	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
,, ...	Light ...	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
,, ...	Neutral	—	—	—	1	1	1	1	1	—	—	—	—	—	1	—	—	—	—	—	6
,, ...	Dark ...	—	—	—	—	1	7	3	3	2	3	4	—	—	—	—	—	—	—	—	23
Totals	...	3	4	35	103	199	285	378	326	244	175	107	64	25	22	11	3	2	4	3	1993

TABLE XXII.

Girls. Age 13. Hair and Eye Colour with Weight.

Weight in Kilogs.

Hair and Eye Colour.

		20	*22*	*24*	*26*	*28*	*30*	*32*	*34*	*36*	*38*	*40*	*42*	*44*	*46*	*48*	*50*	*52*	*54*	*56*	*58*	*60*	Totals
Red	Blue ...	—	—	—	1	2	2	4	3	3	—	3	1	—	—	—	—	—	—	—	—	—	19
„	Light ...	—	—	—	—	1	1	1	4	3	1	6	2	—	1	—	—	1	—	—	—	—	21
„	Neutral	—	—	—	1	2	2	3	2	3	2	2	—	—	1	—	—	—	—	—	—	—	18
„	Dark ...	—	—	—	1	3	2	3	2	—	1	2	1	1	—	—	1	—	—	—	—	—	17
Fair	Blue ...	—	1	1	4	7	7	14	8	6	2	3	4	4	1	—	—	—	—	—	—	—	62
„	Light ...	—	1	2	7	5	11	10	17	15	17	8	8	5	4	3	1	—	2	—	—	—	116
„	Neutral	—	1	—	1	4	5	10	7	3	3	3	3	2	1	1	—	—	—	—	—	—	44
„	Dark ...	—	—	1	2	—	3	2	—	4	3	—	—	—	—	—	—	—	—	—	—	—	15
Light Brown	Blue ...	—	—	2	10	5	12	13	15	12	9	6	2	3	1	—	1	1	—	—	—	—	92
„ ...	Light ...	—	—	1	5	10	31	20	16	24	22	10	11	2	2	1	2	—	1	—	1	—	159
„ ...	Neutral	1	—	1	8	8	12	15	14	15	8	8	4	4	1	2	—	—	—	—	—	—	101
„ ...	Dark ...	—	—	1	3	4	2	11	6	7	4	3	1	1	4	3	1	1	1	—	—	—	53
Medium Brown	Blue ...	—	1	—	9	5	18	14	23	15	17	10	16	1	3	4	2	—	—	—	—	1	139
„ ...	Light ...	—	—	4	3	13	11	17	27	14	19	10	10	5	4	1	2	2	—	—	—	—	142
„ ...	Neutral	—	1	2	7	20	20	28	25	34	28	17	11	13	10	1	2	—	—	1	—	—	220
„ ...	Dark ...	2	—	2	5	10	15	25	35	14	11	11	7	5	1	4	1	—	1	—	—	—	149
Dark Brown ...	Blue ...	—	—	—	6	7	7	9	7	12	5	10	2	3	2	—	—	—	—	—	—	—	70
„ ...	Light ...	—	—	1	2	11	13	20	21	16	14	14	4	7	2	3	2	—	1	1	1	—	133
„ ...	Neutral	—	—	1	6	9	13	18	23	27	18	18	9	10	7	3	2	1	—	—	—	—	165
„ ...	Dark ...	—	2	3	8	20	24	40	46	41	42	28	17	16	6	3	1	2	—	2	2	—	303
Black ...	Blue ...	—	—	1	—	—	—	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	4
„	Light ...	—	—	—	—	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	4
„	Neutral	—	—	—	1	—	1	1	5	1	3	—	1	—	—	—	—	—	—	—	—	—	13
„	Dark ...	—	—	—	1	2	1	8	2	1	4	2	2	3	—	—	—	—	—	—	—	—	26
Totals ...	...	3	7	23	91	148	213	289	309	271	233	175	116	86	51	29	18	8	6	4	4	1	2085

SUPPLEMENTARY TABLES FOR FINDING THE CORRELATION COEFFICIENT FROM TETRACHORIC GROUPINGS.

By P. F. EVERITT, B.Sc.

Object of the Tables.

These tables have been prepared to facilitate the determination of the correlation coefficient by the method of the fourfold table in those cases where the correlation has a very high value. The usual method involves the solution of an equation in ascending powers of the correlation coefficient r; the numerical coefficients of these powers are the products of the corresponding terms of two series, alternate terms of these series being ultimately absolutely convergent. When r is large, these alternate terms will not in general converge rapidly, so that it usually becomes necessary to take a large number of terms into consideration and the work becomes very long and tedious. The present tables are designed to allow the value of r to be obtained easily and rapidly by a series of simple interpolations, thus avoiding the lengthy and rather intricate calculations formerly necessary and consequently decreasing the opportunities of making slips in the numerical work. It is precisely in the case of those fourfold tables for which r is large that we must go beyond the number of terms provided for in my paper on the tetrachoric functions.

Method of Construction.

The nomenclature used will be that of Pearson's[*] paper describing the method of the fourfold table, and explained at length in the Tables of Tetrachoric Functions for Fourfold Correlation Tables[†].

---

[*] *Phil. Trans.* A, Vol. 195, pp. 1—47.

[†] *Biometrika*, Vol. VII. pp. 437—451, 1910. The notation is given by the scheme:

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

It has been shown by Elderton[*] that

$$\frac{d}{N}=\frac{1}{2\pi\sqrt{1-r^2}}\int_h^\infty\int_k^\infty e^{-\frac{1}{2}\cdot\frac{1}{1-r^2}(x^2+y^2-2rxy)}\,dx\,dy$$

may be transformed into

$$\frac{d}{N}=\frac{1}{2\pi}\int_k^\infty e^{-\frac{1}{2}y^2}\left\{\int_t^\infty e^{-\frac{1}{2}X^2}\,dX\right\}dy,$$

where $t=\frac{h-yr}{\sqrt{1-r^2}}$ and this form is utilised to compute d as a check for the value of r obtained by the usual method of solution. Now $\frac{1}{\sqrt{2\pi}}\int_t^\infty e^{-\frac{1}{2}X^2}\,dX$ and $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}$ are known quantities, being the $\frac{1}{2}(1+\alpha)$ and z of Sheppard's Tables[†]. Putting $\frac{1}{2}(1+\alpha)=I$ for convenience, we now have $\frac{d}{N}=\int_k^\infty zI\,dy$ and this form was used for constructing the tables.

The work was carried out in columns; the first column contained the values of y or k rising from zero by intervals of ·1, while the second column contained the corresponding values of z as found from Sheppard's Tables. These two columns remained constant during the whole of the work. The remaining columns were grouped in threes and contained the values of t, I and zI, the headings having suffixes attached showing the value of h to which they applied. The values of $\int_k^\infty zI\,dy$ were now obtained by a quadrature formula, first using all the values of zI in the column and then striking out each in turn from the top of the column downwards. Considerable difficulty was found in choosing a suitable quadrature formula and after many trials Weddle's Rule was adopted as being the most suitable[‡]. The table for $r=1$ was formed directly from Sheppard's Tables.

Checking.

Since
$$\frac{d}{N}=\frac{1}{2\pi\sqrt{1-r^2}}\int_h^\infty\int_k^\infty e^{-\frac{1}{2}\cdot\frac{1}{1-r^2}(x^2+y^2-2rxy)}\,dx\,dy$$

is perfectly symmetrical in x and y, it is obvious that the value of d/N is unaltered when the values of h and k are interchanged. Consequently (excluding the case $h=k$) each value of d/N in the tables occurs twice and by the above method of construction each of the two values is obtained independently; it will also be noticed that in the final quadrature, the number of ordinates employed will also

---

[*] *Frequency Curves and Correlation.* Layton. London.

[†] New Tables of the Probability Integral. W. F. Sheppard. *Biometrika*, Vol. II. pp. 174 ff.

[‡] It was always possible to make the ordinates number a multiple of six, as required by Weddle's Rule, by the simple process of adding sensibly zero ordinates at the asymptotic tail of the curve.

differ, so that the checking not only applies to the actual numerical work but also tests the accuracy of the quadrature formula. The whole of the work was carried out using five places of decimals, and the resulting values of d/N taken to five places gave a maximum discrepancy of one unit in the last place; this discrepancy could always be traced to the effect of cutting off at the fifth place in the formation of the product zI and only occurred when, as must occasionally happen, a number of values grouped together by the quadrature formula happened to be rounded off in the same direction.

In the special case $h = k$, the value of d/N only occurs once in each table and these cases were checked by differences.

Method of using the Tables.

In using the Tables to find the value of r the quantities $(b + d)/N$, $(c + d)/N$ and d/N are first found by division and then the values of h and k taken out from Sheppard's Tables. Using these values of h and k as arguments and comparing the values of d/N given in the Tables with the actual value of d/N in the particular case, the two values of r, between which the required value probably lies, can be readily found by inspection.

For each of these two values of r, the value of d/N is next found by interpolation, using the actual values of h and k as arguments; for each d/N there will be two such interpolations, one for h and one for k, and it is preferable as a general rule to perform that interpolation first for which the correction to d/N is the smaller. Having now obtained the value of d/N for the actual h and k for the two assumed values of r, the actual value of r is easily found by one more simple interpolation. The process may, at first sight, seem rather complicated but in actual practice the result is obtained with ease in a very few minutes.

Example.

Consider the fourfold table given below.

608	45	653
9	48	57
617	93	710

By division $\frac{b+d}{N} = \cdot 13099$, $\frac{c+d}{N} = \cdot 08028$, $\frac{d}{N} = \cdot 0676$,

whence from Sheppard's Tables $h = 1 \cdot 1218$, $k = 1 \cdot 4032$.

The values between which r will probably lie are next found by inspection of the tables, and it will at once be evident that the value of r lies between ·9 and ·95.

We accordingly proceed to find d/N for these values of r by interpolating in these two tables for the above values of h and k; in accordance with the general rule previously given, the interpolation is first carried out for k and afterwards for h.

From the tables we have

		$h=1{\cdot}1$	$h=1{\cdot}2$
$r={\cdot}9$	$k=1{\cdot}4$	·0686	·0645
	$k=1{\cdot}5$	·0591	·0562

			$h=1{\cdot}1$	$h=1{\cdot}2$
which gives	$r={\cdot}9$	$k=1{\cdot}4032$	·0683	·0642
and finally	$r={\cdot}9$,	$k=1{\cdot}4032$,	$h=1{\cdot}1218$,	$d/N={\cdot}0674$.

Similarly we obtain

$r={\cdot}95$, $k=1{\cdot}4032$, $h=1{\cdot}1218$, $d/N={\cdot}0745$.

Interpolating for r, bearing in mind that the interval of r from one table to the next is ·05, and using $d/N={\cdot}0676$ as argument, we obtain the result $r={\cdot}903$.

Accuracy of the Result.

In order to test the accuracy of the result, the value of r in the above example was calculated by the usual method, using the Tables of Tetrachoric Functions and including terms up to the twelfth power of r.

The equation obtained was

$$\begin{aligned}{\cdot}057090 = {} & {\cdot}010516r + {\cdot}024948r^2 + {\cdot}01322r^3 + {\cdot}003733r^4 + {\cdot}003868r^5 - {\cdot}000225r^6 \\ & + {\cdot}002923r^7 - {\cdot}000121r^8 + {\cdot}001597r^9 + {\cdot}000442r^{10} + {\cdot}000630r^{11} + {\cdot}000017r^{12},\end{aligned}$$

whence solving by Newton's rule the value of r is found to be ·904.

It is not suggested that this remarkably close agreement between the two values ·903 and ·904 is always to be expected; but the difference between the two values will always, I think, be found very much less than the probable error of r and will therefore be without any significance.

Further Note on the Tables.

If D_{hk} be the tabulated value of d/N for arguments h and k for any one of the values of r for which the Tables are constructed, then the volume of the frequency solid on the area bounded by the lines corresponding to the values h_1, h_2, k_1, k_2, is given by $D_{h_1k_1} + D_{h_2k_2} - D_{h_1k_2} - D_{h_2k_1}$. Consequently within the limits $r={\cdot}8$ to $r=1$ the distribution of the frequency within the one quadrant, for which the Tables are constructed, may be readily found.

Cases where the Method of the Fourfold Table fails.

A careful examination of the Tables shows that, when the values of h and k differ widely and r is large, then the corresponding values of d/N differ very little from one value of r to the next; in such a case the probable error of r will be large. It may also quite easily happen in such cases, that the value of d/N does not vary for values of r between ·8 and 1 and in such a case the method of the fourfold table fails, as the magnitude of the probable error would render the determination by the ordinary method quite idle as regards a quantitative result although it would generally be possible to say whether the correlation was large or small. Unfortunately it is not generally possible to classify the data, on which the fourfold table has to be based, in more than one way, or else by a suitable choice of classification it might be possible to avoid such cases.

In conclusion I desire to express my gratitude to Prof. Pearson for many valuable suggestions, especially in connection with the choice of a suitable quadrature formula.

Supplementary Tables for determining High

$r = {\cdot}80.$

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·3976	·3766	·3538	·3294	·3039	·2778	·2515	·2254	·2001	·1759	·1531	·1320	·1127
0·1	·3766	·3583	·3380	·3162	·2930	·2689	·2445	·2200	·1960	·1728	·1509	·1304	·1116
0·2	·3538	·3380	·3204	·3011	·2804	·2586	·2361	·2134	·1909	·1690	·1481	·1284	·1102
0·3	·3294	·3162	·3011	·2843	·2661	·2466	·2263	·2056	·1848	·1643	·1446	·1258	·1083
0·4	·3039	·2930	·2804	·2661	·2503	·2332	·2152	·1965	·1775	·1587	·1402	·1226	·1060
0·5	·2778	·2689	·2586	·2466	·2332	·2186	·2028	·1862	·1692	·1520	·1351	·1187	·1031
0·6	·2515	·2445	·2361	·2263	·2152	·2028	·1893	·1748	·1598	·1444	·1291	·1140	·0995
0·7	·2254	·2200	·2134	·2056	·1965	·1862	·1748	·1625	·1494	·1359	·1222	·1086	·0954
0·8	·2001	·1960	·1909	·1848	·1775	·1692	·1598	·1494	·1383	·1266	·1146	·1025	·0906
0·9	·1759	·1728	·1690	·1643	·1587	·1520	·1444	·1359	·1266	·1167	·1064	·0958	·0852
1·0	·1531	·1509	·1481	·1446	·1402	·1351	·1291	·1222	·1146	·1064	·0976	·0886	·0794
1·1	·1320	·1304	·1284	·1258	·1226	·1187	·1140	·1086	·1025	·0958	·0886	·0809	·0731
1·2	·1127	·1116	·1102	·1083	·1060	·1031	·0995	·0954	·0906	·0852	·0794	·0731	·0665
1·3	·0953	·0946	·0936	·0923	·0906	·0885	·0859	·0828	·0791	·0749	·0702	·0652	·0597
1·4	·0798	·0793	·0787	·0778	·0766	·0751	·0733	·0710	·0682	·0650	·0614	·0574	·0530
1·5	·0662	·0659	·0655	·0649	·0641	·0631	·0618	·0601	·0581	·0557	·0529	·0498	·0464
1·6	·0545	·0543	·0540	·0536	·0531	·0524	·0515	·0503	·0489	·0471	·0451	·0427	·0401
1·7	·0444	·0443	·0441	·0438	·0435	·0430	·0424	·0416	·0406	·0394	·0379	·0362	·0342
1·8	·0358	·0357	·0357	·0355	·0353	·0350	·0346	·0341	·0334	·0325	·0315	·0302	·0287
1·9	·0287	·0286	·0286	·0285	·0283	·0281	·0279	·0275	·0271	·0265	·0258	·0249	·0238
2·0	·0227	·0227	·0227	·0226	·0225	·0224	·0223	·0220	·0217	·0213	·0209	·0202	·0195
2·1	·0178	·0178	·0178	·0178	·0177	·0177	·0176	·0174	·0172	·0170	·0167	·0163	·0158
2·2	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0137	·0135	·0134	·0132	·0129	·0126
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104	·0103	·0101	·0099
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0080	·0079	·0078	·0077
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061	·0061	·0060	·0059
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046	·0046	·0045	·0045

$r = {\cdot}85.$

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4117	·3905	·3670	·3417	·3149	·2873	·2595	·2319	·2052	·1798	·1560	·1341	·1141
0·1	·3905	·3723	·3518	·3292	·3050	·2796	·2537	·2277	·2022	·1777	·1546	·1332	·1136
0·2	·3670	·3518	·3342	·3145	·2930	·2702	·2464	·2222	·1983	·1749	·1527	·1319	·1127
0·3	·3417	·3292	·3145	·2978	·2791	·2588	·2374	·2154	·1931	·1712	·1501	·1301	·1116
0·4	·3149	·3050	·2930	·2791	·2632	·2457	·2268	·2070	·1867	·1665	·1467	·1277	·1099
0·5	·2873	·2796	·2702	·2588	·2457	·2309	·2146	·1972	·1790	·1606	·1423	·1246	·1078
0·6	·2595	·2537	·2464	·2374	·2268	·2146	·2008	·1859	·1700	·1535	·1370	·1206	·1049
0·7	·2319	·2277	·2222	·2154	·2070	·1972	·1859	·1733	·1597	·1453	·1306	·1158	·1014
0·8	·2052	·2022	·1983	·1931	·1867	·1790	·1700	·1597	·1483	·1360	·1232	·1101	·0971
0·9	·1798	·1777	·1749	·1712	·1665	·1606	·1535	·1453	·1360	·1258	·1149	·1035	·0920
1·0	·1560	·1546	·1527	·1501	·1467	·1423	·1370	·1306	·1232	·1149	·1058	·0962	·0862
1·1	·1341	·1332	·1319	·1301	·1277	·1246	·1206	·1158	·1101	·1035	·0962	·0882	·0798
1·2	·1141	·1136	·1127	·1116	·1099	·1078	·1049	·1014	·0971	·0920	·0862	·0798	·0729
1·3	·0963	·0959	·0954	·0947	·0936	·0921	·0901	·0876	·0845	·0807	·0763	·0712	·0656
1·4	·0805	·0803	·0800	·0795	·0788	·0778	·0765	·0748	·0725	·0698	·0665	·0626	·0583
1·5	·0666	·0665	·0664	·0661	·0656	·0650	·0642	·0630	·0615	·0595	·0571	·0543	·0510
1·6	·0547	·0547	·0546	·0544	·0541	·0538	·0532	·0525	·0514	·0501	·0484	·0464	·0439
1·7	·0445	·0445	·0444	·0443	·0442	·0440	·0436	·0432	·0425	·0416	·0405	·0390	·0373
1·8	·0359	·0359	·0359	·0358	·0357	·0356	·0354	·0351	·0347	·0341	·0334	·0324	·0312
1·9	·0287	·0287	·0287	·0287	·0286	·0285	·0284	·0283	·0280	·0276	·0272	·0265	·0257
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0224	·0221	·0218	·0214	·0209
2·1	·0179	·0179	·0179	·0178	·0178	·0178	·0178	·0177	·0176	·0175	·0173	·0171	·0167
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0136	·0135	·0133
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0081	·0080
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046

Correlations from Tetrachoric Groupings.

$r = \cdot 80$.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0953	·0798	·0662	·0545	·0444	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·1	·0946	·0793	·0659	·0543	·0443	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·2	·0936	·0787	·0655	·0540	·0441	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·3	·0923	·0778	·0649	·0536	·0438	·0355	·0285	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0906	·0766	·0641	·0531	·0435	·0353	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·5	·0885	·0751	·0631	·0524	·0430	·0350	·0281	·0224	·0177	·0138	·0107	·0082	·0062	·0047
0·6	·0859	·0733	·0618	·0515	·0424	·0346	·0279	·0223	·0176	·0137	·0106	·0082	·0062	·0046
0·7	·0828	·0710	·0601	·0503	·0416	·0341	·0275	·0220	·0174	·0137	·0106	·0081	·0062	·0046
0·8	·0791	·0682	·0581	·0489	·0406	·0334	·0271	·0217	·0172	·0135	·0105	·0081	·0061	·0046
0·9	·0749	·0650	·0557	·0471	·0394	·0325	·0265	·0213	·0170	·0134	·0104	·0080	·0061	·0046
1·0	·0702	·0614	·0529	·0451	·0379	·0315	·0258	·0209	·0167	·0132	·0103	·0079	·0061	·0046
1·1	·0652	·0574	·0498	·0427	·0362	·0302	·0249	·0202	·0163	·0129	·0101	·0078	·0060	·0045
1·2	·0597	·0530	·0464	·0401	·0342	·0287	·0238	·0195	·0158	·0126	·0099	·0077	·0059	·0045
1·3	·0541	·0484	·0427	·0372	·0319	·0271	·0226	·0186	·0151	·0121	·0096	·0075	·0058	·0044
1·4	·0484	·0436	·0388	·0341	·0295	·0252	·0212	·0176	·0144	·0117	·0093	·0073	·0057	·0043
1·5	·0427	·0388	·0348	·0309	·0270	·0232	·0197	·0165	·0136	·0111	·0089	·0070	·0055	·0042
1·6	·0372	·0341	·0309	·0276	·0243	·0211	·0181	·0153	·0127	·0104	·0084	·0067	·0053	·0041
1·7	·0319	·0295	·0270	·0243	·0216	·0190	·0164	·0140	·0117	·0097	·0079	·0063	·0050	·0039
1·8	·0271	·0252	·0232	·0211	·0190	·0168	·0146	·0126	·0107	·0089	·0073	·0059	·0047	·0037
1·9	·0226	·0212	·0197	·0181	·0164	·0146	·0129	·0112	·0096	·0081	·0067	·0055	·0044	·0035
2·0	·0186	·0176	·0165	·0153	·0140	·0126	·0112	·0098	·0085	·0072	·0060	·0050	·0040	·0032
2·1	·0151	·0144	·0136	·0127	·0117	·0107	·0096	·0085	·0074	·0064	·0054	·0045	·0037	·0030
2·2	·0121	·0117	·0111	·0104	·0097	·0089	·0081	·0072	·0064	·0055	·0047	·0040	·0033	·0027
2·3	·0096	·0093	·0089	·0084	·0079	·0073	·0067	·0060	·0054	·0047	·0041	·0035	·0029	·0024
2·4	·0075	·0073	·0070	·0067	·0063	·0059	·0055	·0050	·0045	·0040	·0035	·0030	·0025	·0021
2·5	·0058	·0057	·0055	·0053	·0050	·0047	·0044	·0040	·0037	·0033	·0029	·0025	·0022	·0018
2·6	·0044	·0043	·0042	·0041	·0039	·0037	·0035	·0032	·0030	·0027	·0024	·0021	·0018	·0016

$r = \cdot 85$.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0963	·0805	·0666	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0959	·0803	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0954	·0800	·0664	·0546	·0444	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0947	·0795	·0661	·0544	·0443	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0936	·0788	·0656	·0541	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·5	·0921	·0778	·0650	·0538	·0440	·0356	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·6	·0901	·0765	·0642	·0532	·0436	·0354	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·7	·0876	·0748	·0630	·0525	·0432	·0351	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·8	·0845	·0725	·0615	·0514	·0425	·0347	·0280	·0224	·0176	·0138	·0107	·0082	·0062	·0047
0·9	·0807	·0698	·0595	·0501	·0416	·0341	·0276	·0221	·0175	·0137	·0106	·0081	·0062	·0046
1·0	·0763	·0665	·0571	·0484	·0405	·0334	·0272	·0218	·0173	·0136	·0106	·0081	·0062	·0046
1·1	·0712	·0626	·0543	·0464	·0390	·0324	·0265	·0214	·0171	·0135	·0105	·0081	·0061	·0046
1·2	·0656	·0583	·0510	·0439	·0373	·0312	·0257	·0209	·0167	·0133	·0104	·0080	·0061	·0046
1·3	·0597	·0535	·0473	·0411	·0352	·0297	·0247	·0202	·0163	·0130	·0102	·0079	·0060	·0046
1·4	·0535	·0485	·0432	·0380	·0329	·0280	·0234	·0194	·0157	·0126	·0100	·0078	·0060	·0045
1·5	·0473	·0432	·0390	·0346	·0302	·0260	·0220	·0183	·0150	·0121	·0097	·0076	·0058	·0045
1·6	·0411	·0380	·0346	·0311	·0274	·0239	·0204	·0172	·0142	·0116	·0093	·0073	·0057	·0044
1·7	·0352	·0329	·0302	·0274	·0245	·0216	·0186	·0159	·0133	·0109	·0088	·0070	·0055	·0043
1·8	·0297	·0280	·0260	·0239	·0216	·0192	·0168	·0144	·0122	·0102	·0083	·0067	·0053	·0041
1·9	·0247	·0234	·0220	·0204	·0186	·0168	·0149	·0129	·0111	·0093	·0077	·0063	·0050	·0039
2·0	·0202	·0194	·0183	·0172	·0159	·0144	·0129	·0114	·0099	·0084	·0070	·0058	·0047	·0037
2·1	·0163	·0157	·0150	·0142	·0133	·0122	·0111	·0099	·0087	·0075	·0063	·0053	·0043	·0034
2·2	·0130	·0126	·0121	·0116	·0109	·0102	·0093	·0084	·0075	·0065	·0056	·0047	·0039	·0032
2·3	·0102	·0100	·0097	·0093	·0088	·0083	·0077	·0070	·0063	·0056	·0049	·0042	·0035	·0029
2·4	·0079	·0078	·0076	·0073	·0070	·0067	·0063	·0058	·0053	·0047	·0042	·0036	·0031	·0025
2·5	·0060	·0060	·0058	·0057	·0055	·0053	·0050	·0047	·0043	·0039	·0035	·0031	·0026	·0022
2·6	·0046	·0045	·0045	·0044	·0043	·0041	·0039	·0037	·0034	·0032	·0029	·0025	·0022	·0019

Supplementary Tables for determining High Correlations from Tetrachoric Groupings.

$r = \cdot 80.$

h=	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
k=0·0	·3976	·3766	·3538	·3294	·3039	·2778	·2515	·2254	·2001	·1759	·1531	·1320	·1127
0·1	·3766	·3583	·3380	·3162	·2930	·2689	·2445	·2200	·1960	·1728	·1509	·1304	·1116
0·2	·3538	·3380	·3204	·3011	·2804	·2586	·2361	·2134	·1909	·1690	·1481	·1284	·1102
0·3	·3294	·3162	·3011	·2843	·2661	·2466	·2263	·2056	·1848	·1643	·1446	·1258	·1083
0·4	·3039	·2930	·2804	·2661	·2503	·2332	·2152	·1965	·1775	·1587	·1402	·1226	·1060
0·5	·2778	·2689	·2586	·2466	·2332	·2186	·2028	·1862	·1692	·1520	·1351	·1187	·1031
0·6	·2515	·2445	·2361	·2263	·2152	·2028	·1893	·1748	·1598	·1444	·1291	·1140	·0995
0·7	·2254	·2200	·2134	·2056	·1965	·1862	·1748	·1625	·1494	·1359	·1222	·1086	·0954
0·8	·2001	·1960	·1909	·1848	·1775	·1692	·1598	·1494	·1383	·1266	·1146	·1025	·0906
0·9	·1759	·1728	·1690	·1643	·1587	·1520	·1444	·1359	·1266	·1167	·1064	·0958	·0852
1·0	·1531	·1509	·1481	·1446	·1402	·1351	·1291	·1222	·1146	·1064	·0976	·0886	·0794
1·1	·1320	·1304	·1284	·1258	·1226	·1187	·1140	·1086	·1025	·0958	·0886	·0809	·0731
1·2	·1127	·1116	·1102	·1083	·1060	·1031	·0995	·0954	·0906	·0852	·0794	·0731	·0665
1·3	·0953	·0946	·0936	·0923	·0906	·0885	·0859	·0828	·0791	·0749	·0702	·0652	·0597
1·4	·0798	·0793	·0787	·0778	·0766	·0751	·0733	·0710	·0682	·0650	·0614	·0574	·0530
1·5	·0662	·0659	·0655	·0649	·0641	·0631	·0618	·0601	·0581	·0557	·0529	·0498	·0464
1·6	·0545	·0543	·0540	·0536	·0531	·0524	·0515	·0503	·0489	·0471	·0451	·0427	·0401
1·7	·0444	·0443	·0441	·0438	·0435	·0430	·0424	·0416	·0406	·0394	·0379	·0362	·0342
1·8	·0358	·0357	·0357	·0355	·0353	·0350	·0346	·0341	·0334	·0325	·0315	·0302	·0287
1·9	·0287	·0286	·0286	·0285	·0283	·0281	·0279	·0275	·0271	·0265	·0258	·0249	·0238
2·0	·0227	·0227	·0227	·0226	·0225	·0224	·0223	·0220	·0217	·0213	·0209	·0202	·0195
2·1	·0178	·0178	·0178	·0178	·0177	·0177	·0176	·0174	·0172	·0170	·0167	·0163	·0158
2·2	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0137	·0135	·0134	·0132	·0129	·0126
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104	·0103	·0101	·0099
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0080	·0079	·0078	·0077
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061	·0061	·0060	·0059
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046	·0046	·0045	·0045

$r = \cdot 85.$

h=	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2
k=0·0	·4117	·3905	·3670	·3417	·3149	·2873	·2595	·2319	·2052	·1798	·1560	·1341	·1141
0·1	·3905	·3723	·3518	·3292	·3050	·2796	·2537	·2277	·2022	·1777	·1546	·1332	·1136
0·2	·3670	·3518	·3342	·3145	·2930	·2702	·2464	·2222	·1983	·1749	·1527	·1319	·1127
0·3	·3417	·3292	·3145	·2978	·2791	·2588	·2374	·2154	·1931	·1712	·1501	·1301	·1116
0·4	·3149	·3050	·2930	·2791	·2632	·2457	·2268	·2070	·1867	·1665	·1467	·1277	·1099
0·5	·2873	·2796	·2702	·2588	·2457	·2309	·2146	·1972	·1790	·1606	·1423	·1246	·1078
0·6	·2595	·2537	·2464	·2374	·2268	·2146	·2008	·1859	·1700	·1535	·1370	·1206	·1049
0·7	·2319	·2277	·2222	·2154	·2070	·1972	·1859	·1733	·1597	·1453	·1306	·1158	·1014
0·8	·2052	·2022	·1983	·1931	·1867	·1790	·1700	·1597	·1483	·1360	·1232	·1101	·0971
0·9	·1798	·1777	·1749	·1712	·1665	·1606	·1535	·1453	·1360	·1258	·1149	·1035	·0920
1·0	·1560	·1546	·1527	·1501	·1467	·1423	·1370	·1306	·1232	·1149	·1058	·0962	·0862
1·1	·1341	·1332	·1319	·1301	·1277	·1246	·1206	·1158	·1101	·1035	·0962	·0882	·0798
1·2	·1141	·1136	·1127	·1116	·1099	·1078	·1049	·1014	·0971	·0920	·0862	·0798	·0729
1·3	·0963	·0959	·0954	·0947	·0936	·0921	·0901	·0876	·0845	·0807	·0763	·0712	·0656
1·4	·0805	·0803	·0800	·0795	·0788	·0778	·0765	·0748	·0725	·0698	·0665	·0626	·0583
1·5	·0666	·0665	·0664	·0661	·0656	·0650	·0642	·0630	·0615	·0595	·0571	·0543	·0510
1·6	·0547	·0547	·0546	·0544	·0541	·0538	·0532	·0525	·0514	·0501	·0484	·0464	·0439
1·7	·0445	·0445	·0444	·0443	·0442	·0440	·0436	·0432	·0425	·0416	·0405	·0390	·0373
1·8	·0359	·0359	·0359	·0358	·0357	·0356	·0354	·0351	·0347	·0341	·0334	·0324	·0312
1·9	·0287	·0287	·0287	·0287	·0286	·0285	·0284	·0283	·0280	·0276	·0272	·0265	·0257
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0224	·0221	·0218	·0214	·0209
2·1	·0179	·0179	·0179	·0178	·0178	·0178	·0178	·0177	·0176	·0175	·0173	·0171	·0167
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0136	·0135	·0133
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0081	·0080
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046

$r = \cdot 80.$

h=	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
k=0·0	·0953	·0798	·0662	·0545	·0444	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·1	·0946	·0793	·0659	·0543	·0443	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·2	·0936	·0787	·0655	·0540	·0441	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·3	·0923	·0778	·0649	·0536	·0438	·0355	·0285	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0906	·0766	·0641	·0531	·0435	·0353	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·5	·0885	·0751	·0631	·0524	·0430	·0350	·0281	·0224	·0177	·0138	·0107	·0082	·0062	·0047
0·6	·0859	·0733	·0618	·0515	·0424	·0346	·0279	·0223	·0176	·0137	·0106	·0082	·0062	·0046
0·7	·0828	·0710	·0601	·0503	·0416	·0341	·0275	·0220	·0174	·0137	·0106	·0081	·0062	·0046
0·8	·0791	·0682	·0581	·0489	·0406	·0334	·0271	·0217	·0172	·0135	·0105	·0081	·0061	·0046
0·9	·0749	·0650	·0557	·0471	·0394	·0325	·0265	·0213	·0170	·0134	·0104	·0080	·0061	·0046
1·0	·0702	·0614	·0529	·0451	·0379	·0315	·0258	·0209	·0167	·0132	·0103	·0079	·0061	·0046
1·1	·0652	·0574	·0498	·0427	·0362	·0302	·0249	·0202	·0163	·0129	·0101	·0078	·0060	·0045
1·2	·0597	·0530	·0464	·0401	·0342	·0287	·0238	·0195	·0158	·0126	·0099	·0077	·0059	·0045
1·3	·0541	·0484	·0427	·0372	·0319	·0271	·0226	·0186	·0151	·0121	·0096	·0075	·0058	·0044
1·4	·0484	·0436	·0388	·0341	·0295	·0252	·0212	·0176	·0144	·0117	·0093	·0073	·0057	·0043
1·5	·0427	·0388	·0348	·0309	·0270	·0232	·0197	·0165	·0136	·0111	·0089	·0070	·0055	·0042
1·6	·0372	·0341	·0309	·0276	·0243	·0211	·0181	·0153	·0127	·0104	·0084	·0067	·0053	·0041
1·7	·0319	·0295	·0270	·0243	·0216	·0190	·0164	·0140	·0117	·0097	·0079	·0063	·0050	·0039
1·8	·0271	·0252	·0232	·0211	·0190	·0168	·0146	·0126	·0107	·0089	·0073	·0059	·0047	·0037
1·9	·0226	·0212	·0197	·0181	·0164	·0146	·0129	·0112	·0096	·0081	·0067	·0055	·0044	·0035
2·0	·0186	·0176	·0165	·0153	·0140	·0126	·0112	·0098	·0085	·0072	·0060	·0050	·0040	·0032
2·1	·0151	·0144	·0136	·0127	·0117	·0107	·0096	·0085	·0074	·0064	·0054	·0045	·0037	·0030
2·2	·0121	·0117	·0111	·0104	·0097	·0089	·0081	·0072	·0064	·0055	·0047	·0040	·0033	·0027
2·3	·0096	·0093	·0089	·0084	·0079	·0073	·0067	·0060	·0054	·0047	·0041	·0035	·0029	·0024
2·4	·0075	·0073	·0070	·0067	·0063	·0059	·0055	·0050	·0045	·0040	·0035	·0030	·0025	·0021
2·5	·0058	·0057	·0055	·0053	·0050	·0047	·0044	·0040	·0037	·0033	·0029	·0025	·0022	·0018
2·6	·0044	·0043	·0042	·0041	·0039	·0037	·0035	·0032	·0030	·0027	·0024	·0021	·0018	·0016

$r = \cdot 85.$

h=	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
k=0·0	·0963	·0805	·0666	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0959	·0803	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0954	·0800	·0664	·0546	·0444	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0947	·0795	·0661	·0544	·0443	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0936	·0788	·0656	·0541	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·5	·0921	·0778	·0650	·0538	·0440	·0356	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·6	·0901	·0765	·0642	·0532	·0436	·0354	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·7	·0876	·0748	·0630	·0525	·0432	·0351	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·8	·0845	·0725	·0615	·0514	·0425	·0347	·0280	·0224	·0176	·0138	·0107	·0082	·0062	·0047
0·9	·0807	·0698	·0595	·0501	·0416	·0341	·0276	·0221	·0175	·0137	·0106	·0081	·0062	·0046
1·0	·0763	·0665	·0571	·0484	·0405	·0334	·0272	·0218	·0173	·0136	·0106	·0081	·0062	·0046
1·1	·0712	·0626	·0543	·0464	·0390	·0324	·0265	·0214	·0171	·0135	·0105	·0081	·0061	·0046
1·2	·0656	·0583	·0510	·0439	·0373	·0312	·0257	·0209	·0167	·0133	·0104	·0080	·0061	·0046
1·3	·0597	·0535	·0473	·0411	·0352	·0297	·0247	·0202	·0163	·0130	·0102	·0079	·0060	·0046
1·4	·0535	·0485	·0432	·0380	·0329	·0280	·0234	·0194	·0157	·0126	·0100	·0078	·0060	·0045
1·5	·0473	·0432	·0390	·0346	·0302	·0260	·0220	·0183	·0150	·0121	·0097	·0076	·0058	·0045
1·6	·0411	·0380	·0346	·0311	·0274	·0239	·0204	·0172	·0142	·0116	·0093	·0073	·0057	·0044
1·7	·0352	·0329	·0302	·0274	·0245	·0216	·0186	·0159	·0133	·0109	·0088	·0070	·0055	·0043
1·8	·0297	·0280	·0260	·0239	·0216	·0192	·0168	·0144	·0122	·0102	·0083	·0067	·0053	·0041
1·9	·0247	·0234	·0220	·0204	·0186	·0168	·0149	·0129	·0111	·0093	·0077	·0063	·0050	·0039
2·0	·0202	·0194	·0183	·0172	·0159	·0144	·0129	·0114	·0099	·0084	·0070	·0058	·0047	·0037
2·1	·0163	·0157	·0150	·0142	·0133	·0122	·0111	·0099	·0087	·0075	·0063	·0053	·0043	·0034
2·2	·0130	·0126	·0121	·0116	·0109	·0102	·0093	·0084	·0075	·0065	·0056	·0047	·0039	·0032
2·3	·0102	·0100	·0097	·0093	·0088	·0083	·0077	·0070	·0063	·0056	·0049	·0042	·0035	·0029
2·4	·0079	·0078	·0076	·0073	·0070	·0067	·0063	·0058	·0053	·0047	·0042	·0036	·0031	·0025
2·5	·0060	·0060	·0058	·0057	·0055	·0053	·0050	·0047	·0043	·0039	·0035	·0031	·0026	·0022
2·6	·0046	·0045	·0045	·0044	·0043	·0041	·0039	·0037	·0034	·0032	·0029	·0025	·0022	·0019

Supplementary Tables for determining High

$r = \cdot 90$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4282	·4067	·3822	·3552	·3266	·2969	·2670	·2377	·2094	·1827	·1579	·1353	·1149
0·1	·4067	·3887	·3678	·3441	·3183	·2910	·2630	·2350	·2077	·1817	·1574	·1350	·1147
0·2	·3822	·3678	·3504	·3302	·3076	·2830	·2573	·2311	·2052	·1801	·1564	·1345	·1144
0·3	·3552	·3441	·3302	·3135	·2943	·2728	·2498	·2258	·2016	·1778	·1550	·1336	·1140
0·4	·3266	·3183	·3076	·2943	·2784	·2602	·2401	·2187	·1966	·1744	·1528	·1322	·1132
0·5	·2969	·2910	·2830	·2728	·2602	·2453	·2284	·2097	·1900	·1698	·1497	·1302	·1119
0·6	·2670	·2630	·2573	·2498	·2401	·2284	·2145	·1988	·1817	·1637	·1454	·1274	·1101
0·7	·2377	·2350	·2311	·2258	·2187	·2097	·1988	·1860	·1717	·1561	·1399	·1236	·1075
0·8	·2094	·2077	·2052	·2016	·1966	·1900	·1817	·1717	·1600	·1470	·1331	·1186	·1041
0·9	·1827	·1817	·1801	·1778	·1744	·1698	·1637	·1561	·1470	·1365	·1249	·1124	·0997
1·0	·1579	·1574	·1564	·1550	·1528	·1497	·1454	·1399	·1331	·1249	·1155	·1052	·0942
1·1	·1353	·1350	·1345	·1336	·1322	·1302	·1274	·1236	·1186	·1124	·1052	·0969	·0878
1·2	·1149	·1147	·1144	·1140	·1132	·1119	·1101	·1075	·1041	·0997	·0942	·0878	·0806
1·3	·0967	·0966	·0965	·0962	·0958	·0950	·0939	·0923	·0900	·0869	·0830	·0783	·0727
1·4	·0807	·0807	·0806	·0805	·0802	·0798	·0792	·0782	·0767	·0747	·0720	·0686	·0645
1·5	·0668	·0668	·0667	·0667	·0665	·0663	·0660	·0654	·0645	·0632	·0614	·0591	·0562
1·6	·0548	·0548	·0548	·0547	·0547	·0546	·0544	·0540	·0535	·0528	·0516	·0501	·0481
1·7	·0446	·0446	·0446	·0445	·0445	·0445	·0444	·0442	·0439	·0435	·0428	·0418	·0405
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0356	·0353	·0350	·0344	·0338
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0286	·0284	·0282	·0279	·0274
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0223	·0220
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178	·0178	·0178	·0177	·0176	·0175
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

$r = \cdot 95$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4495	·4271	·4005	·3705	·3385	·3055	·2729	·2414	·2116	·1840	·1586	·1357	·1151
0·1	·4271	·4099	·3880	·3622	·3333	·3026	·2713	·2407	·2113	·1839	·1586	·1356	·1151
0·2	·4005	·3880	·3712	·3500	·3252	·2976	·2685	·2392	·2106	·1835	·1585	·1356	·1150
0·3	·3705	·3622	·3500	·3338	·3135	·2898	·2637	·2365	·2092	·1829	·1582	·1355	·1150
0·4	·3385	·3333	·3252	·3135	·2980	·2787	·2564	·2320	·2067	·1816	·1576	·1352	·1149
0·5	·3055	·3026	·2976	·2898	·2787	·2640	·2459	·2250	·2024	·1792	·1563	·1346	·1147
0·6	·2729	·2713	·2685	·2637	·2564	·2459	·2321	·2153	·1960	·1753	·1542	·1335	·1141
0·7	·2414	·2407	·2392	·2365	·2320	·2250	·2153	·2025	·1870	·1694	·1506	·1315	·1131
0·8	·2116	·2113	·2106	·2092	·2067	·2024	·1960	·1870	·1753	·1611	·1452	·1283	·1113
0·9	·1840	·1839	·1835	·1829	·1816	·1792	·1753	·1694	·1611	·1505	·1377	·1234	·1084
1·0	·1586	·1586	·1585	·1582	·1576	·1563	·1542	·1506	·1452	·1377	·1281	·1167	·1041
1·1	·1357	·1356	·1356	·1355	·1352	·1346	·1335	·1315	·1283	·1234	·1167	·1082	·0981
1·2	·1151	·1151	·1150	·1150	·1149	·1147	·1141	·1131	·1113	·1084	·1041	·0981	·0906
1·3	·0968	·0968	·0968	·0968	·0967	·0966	·0964	·0959	·0950	·0934	·0908	·0870	·0818
1·4	·0808	·0808	·0808	·0808	·0807	·0807	·0806	·0804	·0800	·0792	·0778	·0755	·0721
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0667	·0665	·0661	·0654	·0642	·0622
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0547	·0545	·0542	·0536	·0525
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0445	·0445	·0445	·0443	·0440	·0435
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0355
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0285
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = ·90$.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0965	·0806	·0667	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0962	·0805	·0667	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0958	·0802	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0950	·0798	·0663	·0546	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0939	·0792	·0660	·0544	·0444	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0923	·0782	·0654	·0540	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·8	·0900	·0767	·0645	·0535	·0439	·0356	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·9	·0869	·0747	·0632	·0528	·0435	·0353	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·0	·0830	·0720	·0614	·0516	·0428	·0350	·0282	·0225	·0177	·0138	·0107	·0082	·0062	·0047
1·1	·0783	·0686	·0591	·0501	·0418	·0344	·0279	·0223	·0176	·0138	·0107	·0082	·0062	·0047
1·2	·0727	·0645	·0562	·0481	·0405	·0338	·0274	·0220	·0175	·0137	·0106	·0082	·0062	·0047
1·3	·0664	·0596	·0526	·0456	·0388	·0325	·0267	·0216	·0173	·0136	·0106	·0081	·0062	·0046
1·4	·0596	·0543	·0485	·0426	·0367	·0310	·0258	·0211	·0169	·0134	·0105	·0081	·0062	·0046
1·5	·0526	·0485	·0439	·0391	·0341	·0292	·0246	·0203	·0164	·0131	·0103	·0080	·0061	·0046
1·6	·0456	·0426	·0391	·0353	·0312	·0271	·0231	·0193	·0158	·0127	·0101	·0079	·0060	·0046
1·7	·0388	·0367	·0341	·0312	·0281	·0247	·0214	·0181	·0150	·0122	·0098	·0077	·0059	·0045
1·8	·0325	·0310	·0292	·0271	·0247	·0221	·0194	·0167	·0140	·0116	·0094	·0074	·0058	·0044
1·9	·0267	·0258	·0246	·0231	·0214	·0194	·0173	·0151	·0129	·0108	·0088	·0071	·0056	·0043
2·0	·0216	·0211	·0203	·0193	·0181	·0167	·0151	·0134	·0116	·0099	·0082	·0067	·0053	·0042
2·1	·0173	·0169	·0164	·0158	·0150	·0140	·0129	·0116	·0102	·0088	·0075	·0062	·0050	·0040
2·2	·0136	·0134	·0131	·0127	·0122	·0116	·0108	·0099	·0088	·0078	·0067	·0056	·0046	·0037
2·3	·0106	·0105	·0103	·0101	·0098	·0094	·0088	·0082	·0075	·0067	·0058	·0050	·0042	·0034
2·4	·0081	·0081	·0080	·0079	·0077	·0074	·0071	·0067	·0062	·0056	·0050	·0044	·0037	·0031
2·5	·0062	·0062	·0061	·0060	·0059	·0058	·0056	·0053	·0050	·0046	·0042	·0037	·0032	·0027
2·6	·0046	·0046	·0046	·0046	·0045	·0044	·0043	·0042	·0040	·0037	·0034	·0031	·0027	·0024

$r = ·95$.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0964	·0806	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0959	·0804	·0667	·0548	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0950	·0800	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0934	·0792	·0661	·0545	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0908	·0778	·0654	·0542	·0443	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0870	·0755	·0642	·0536	·0440	·0357	·0286	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0818	·0721	·0622	·0525	·0435	·0355	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
1·3	·0752	·0676	·0593	·0508	·0426	·0350	·0283	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·4	·0676	·0619	·0554	·0483	·0411	·0342	·0279	·0224	·0177	·0139	·0107	·0082	·0062	·0047
1·5	·0593	·0554	·0505	·0450	·0390	·0330	·0273	·0221	·0176	·0138	·0107	·0082	·0062	·0047
1·6	·0508	·0483	·0450	·0409	·0362	·0312	·0263	·0215	·0173	·0137	·0106	·0082	·0062	·0047
1·7	·0426	·0411	·0390	·0362	·0328	·0289	·0248	·0207	·0169	·0135	·0105	·0081	·0062	·0047
1·8	·0350	·0342	·0330	·0312	·0289	·0261	·0229	·0195	·0162	·0131	·0104	·0080	·0062	·0046
1·9	·0283	·0279	·0273	·0263	·0248	·0229	·0205	·0179	·0152	·0125	·0101	·0079	·0061	·0046
2·0	·0226	·0224	·0221	·0215	·0207	·0195	·0179	·0160	·0139	·0117	·0096	·0077	·0060	·0046
2·1	·0178	·0177	·0176	·0173	·0169	·0162	·0152	·0139	·0124	·0107	·0090	·0073	·0058	·0045
2·2	·0139	·0139	·0138	·0137	·0135	·0131	·0125	·0117	·0107	·0095	·0082	·0068	·0055	·0043
2·3	·0107	·0107	·0107	·0106	·0105	·0104	·0101	·0096	·0090	·0082	·0072	·0062	·0051	·0041
2·4	·0082	·0082	·0082	·0082	·0081	·0080	·0079	·0077	·0073	·0068	·0062	·0054	·0046	·0038
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0060	·0058	·0055	·0051	·0046	·0040	·0034
2·6	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0045	·0043	·0041	·0038	·0034	·0030

Supplementary Tables for determining High

r = ·90.

h =	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k = *0·0*	·4282	·4067	·3822	·3552	·3266	·2969	·2670	·2377	·2094	·1827	·1579	·1353	·1149
0·1	·4067	·3887	·3678	·3441	·3183	·2910	·2630	·2350	·2077	·1817	·1574	·1350	·1147
0·2	·3822	·3678	·3504	·3302	·3076	·2830	·2573	·2311	·2052	·1801	·1564	·1345	·1144
0·3	·3552	·3441	·3302	·3135	·2943	·2728	·2498	·2258	·2016	·1778	·1550	·1336	·1140
0·4	·3266	·3183	·3076	·2943	·2784	·2602	·2401	·2187	·1966	·1744	·1528	·1322	·1132
0·5	·2969	·2910	·2830	·2728	·2602	·2453	·2284	·2097	·1900	·1698	·1497	·1302	·1119
0·6	·2670	·2630	·2573	·2498	·2401	·2284	·2145	·1988	·1817	·1637	·1454	·1274	·1101
0·7	·2377	·2350	·2311	·2258	·2187	·2097	·1988	·1860	·1717	·1561	·1399	·1236	·1075
0·8	·2094	·2077	·2052	·2016	·1966	·1900	·1817	·1717	·1600	·1470	·1331	·1186	·1041
0·9	·1827	·1817	·1801	·1778	·1744	·1698	·1637	·1561	·1470	·1365	·1249	·1124	·0997
1·0	·1579	·1574	·1564	·1550	·1528	·1497	·1454	·1399	·1331	·1249	·1155	·1052	·0942
1·1	·1353	·1350	·1345	·1336	·1322	·1302	·1274	·1236	·1186	·1124	·1052	·0969	·0878
1·2	·1149	·1147	·1144	·1140	·1132	·1119	·1101	·1075	·1041	·0997	·0942	·0878	·0806
1·3	·0967	·0966	·0965	·0962	·0958	·0950	·0939	·0923	·0900	·0869	·0830	·0783	·0727
1·4	·0807	·0807	·0806	·0805	·0802	·0798	·0792	·0782	·0767	·0747	·0720	·0686	·0645
1·5	·0668	·0668	·0667	·0667	·0665	·0663	·0660	·0654	·0645	·0632	·0614	·0591	·0562
1·6	·0548	·0548	·0548	·0547	·0547	·0546	·0544	·0540	·0535	·0528	·0516	·0501	·0481
1·7	·0446	·0446	·0446	·0445	·0445	·0445	·0444	·0442	·0439	·0435	·0428	·0418	·0405
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0356	·0353	·0350	·0344	·0338
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0286	·0284	·0282	·0279	·0274
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0223	·0220
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178	·0178	·0178	·0177	·0176	·0175
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

r = ·95.

h =	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k = *0·0*	·4495	·4271	·4005	·3705	·3385	·3055	·2729	·2414	·2116	·1840	·1586	·1357	·1151
0·1	·4271	·4099	·3880	·3622	·3333	·3026	·2713	·2407	·2113	·1839	·1586	·1356	·1151
0·2	·4005	·3880	·3712	·3500	·3252	·2976	·2685	·2392	·2106	·1835	·1585	·1356	·1150
0·3	·3705	·3622	·3500	·3338	·3135	·2898	·2637	·2365	·2092	·1829	·1582	·1355	·1150
0·4	·3385	·3333	·3252	·3135	·2980	·2787	·2564	·2320	·2067	·1816	·1576	·1352	·1149
0·5	·3055	·3026	·2976	·2898	·2787	·2640	·2459	·2250	·2024	·1792	·1563	·1346	·1147
0·6	·2729	·2713	·2685	·2637	·2564	·2459	·2321	·2153	·1960	·1753	·1542	·1335	·1141
0·7	·2414	·2407	·2392	·2365	·2320	·2250	·2153	·2025	·1870	·1694	·1506	·1315	·1131
0·8	·2116	·2113	·2106	·2092	·2067	·2024	·1960	·1870	·1753	·1611	·1452	·1283	·1113
0·9	·1840	·1839	·1835	·1829	·1816	·1792	·1753	·1694	·1611	·1505	·1377	·1234	·1084
1·0	·1586	·1586	·1585	·1582	·1576	·1563	·1542	·1506	·1452	·1377	·1281	·1167	·1041
1·1	·1357	·1356	·1356	·1355	·1352	·1346	·1335	·1315	·1283	·1234	·1167	·1082	·0981
1·2	·1151	·1151	·1150	·1150	·1149	·1147	·1141	·1131	·1113	·1084	·1041	·0981	·0906
1·3	·0968	·0968	·0968	·0968	·0967	·0966	·0964	·0959	·0950	·0934	·0908	·0870	·0818
1·4	·0808	·0808	·0808	·0808	·0807	·0807	·0806	·0804	·0800	·0792	·0778	·0755	·0721
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0667	·0665	·0661	·0654	·0642	·0622
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0547	·0545	·0542	·0536	·0525
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0445	·0445	·0445	·0443	·0440	·0435
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0355
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0285
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

r = ·90.

h =	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k = *0·0*	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0965	·0806	·0667	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0962	·0805	·0667	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0958	·0802	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0950	·0798	·0663	·0546	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0939	·0792	·0660	·0544	·0444	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0923	·0782	·0654	·0540	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·8	·0900	·0767	·0645	·0535	·0439	·0356	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·9	·0869	·0747	·0632	·0528	·0435	·0353	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·0	·0830	·0720	·0614	·0516	·0428	·0350	·0282	·0225	·0177	·0138	·0107	·0082	·0062	·0047
1·1	·0783	·0686	·0591	·0501	·0418	·0344	·0279	·0223	·0176	·0138	·0107	·0082	·0062	·0047
1·2	·0727	·0645	·0562	·0481	·0405	·0338	·0274	·0220	·0175	·0137	·0106	·0082	·0062	·0047
1·3	·0664	·0596	·0526	·0456	·0388	·0325	·0267	·0216	·0173	·0136	·0106	·0081	·0062	·0046
1·4	·0596	·0543	·0485	·0426	·0367	·0310	·0258	·0211	·0169	·0134	·0105	·0081	·0062	·0046
1·5	·0526	·0485	·0439	·0391	·0341	·0292	·0246	·0203	·0164	·0131	·0103	·0080	·0061	·0046
1·6	·0456	·0426	·0391	·0353	·0312	·0271	·0231	·0193	·0158	·0127	·0101	·0079	·0060	·0046
1·7	·0388	·0367	·0341	·0312	·0281	·0247	·0214	·0181	·0150	·0122	·0098	·0077	·0059	·0045
1·8	·0325	·0310	·0292	·0271	·0247	·0221	·0194	·0167	·0140	·0116	·0094	·0074	·0058	·0044
1·9	·0267	·0258	·0246	·0231	·0214	·0194	·0173	·0151	·0129	·0108	·0088	·0071	·0056	·0043
2·0	·0216	·0211	·0203	·0193	·0181	·0167	·0151	·0134	·0116	·0099	·0082	·0067	·0053	·0042
2·1	·0173	·0169	·0164	·0158	·0150	·0140	·0129	·0116	·0102	·0088	·0075	·0062	·6050	·0040
2·2	·0136	·0134	·0131	·0127	·0122	·0116	·0108	·0099	·0088	·0078	·0067	·0056	·0046	·0037
2·3	·0106	·0105	·0103	·0101	·0098	·0094	·0088	·0082	·0075	·0067	·0058	·0050	·0042	·0034
2·4	·0081	·0081	·0080	·0079	·0077	·0074	·0071	·0067	·0062	·0056	·0050	·0044	·0037	·0031
2·5	·0062	·0062	·0061	·0060	·0059	·0058	·0056	·0053	·0050	·0046	·0042	·0037	·0032	·0027
2·6	·0046	·0046	·0046	·0046	·0045	·0044	·0043	·0042	·0040	·0037	·0034	·0031	·0027	·0024

r = ·95.

h =	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k = *0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0964	·0806	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0959	·0804	·0667	·0548	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0950	·0800	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0934	·0792	·0661	·0545	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0908	·0778	·0654	·0542	·0443	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0870	·0755	·0642	·0536	·0440	·0357	·0286	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0818	·0721	·0622	·0525	·0435	·0355	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
1·3	·0752	·0676	·0593	·0508	·0426	·0350	·0283	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·4	·0676	·0619	·0554	·0483	·0411	·0342	·0279	·0224	·0177	·0139	·0107	·0082	·0062	·0047
1·5	·0593	·0554	·0505	·0450	·0390	·0330	·0273	·0221	·0176	·0138	·0107	·0082	·0062	·0047
1·6	·0508	·0483	·0450	·0409	·0362	·0312	·0263	·0215	·0173	·0137	·0106	·0082	·0062	·0047
1·7	·0426	·0411	·0390	·0362	·0328	·0289	·0248	·0207	·0169	·0135	·0105	·0081	·0062	·0047
1·8	·0350	·0342	·0330	·0312	·0289	·0261	·0229	·0195	·0162	·0131	·0104	·0080	·0062	·0046
1·9	·0283	·0279	·0273	·0263	·0248	·0229	·0205	·0179	·0152	·0125	·0101	·0079	·0061	·0046
2·0	·0226	·0224	·0221	·0215	·0207	·0195	·0179	·0160	·0139	·0117	·0096	·0077	·0060	·0046
2·1	·0178	·0177	·0176	·0173	·0169	·0162	·0152	·0139	·0124	·0107	·0090	·0073	·0058	·0045
2·2	·0139	·0139	·0138	·0137	·0135	·0131	·0125	·0117	·0107	·0095	·0082	·0068	·0055	·0043
2·3	·0107	·0107	·0107	·0106	·0105	·0104	·0101	·0096	·0090	·0082	·0072	·0062	·0051	·0041
2·4	·0082	·0082	·0082	·0082	·0081	·0080	·0079	·0077	·0073	·0068	·0062	·0054	·0046	·0038
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0060	·0058	·0055	·0051	·0046	·0040	·0034
2·6	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0045	·0043	·0041	·0038	·0034	·0030

Supplementary Tables for determining High

$r = 1{\cdot}00$.

$h=$	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
$k=$*0·0*	·5000	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·1	·4602	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·2	·4207	·4207	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·3	·3821	·3821	·3821	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·4	·3446	·3446	·3446	·3446	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·5	·3085	·3085	·3085	·3085	·3085	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·6	·2743	·2743	·2743	·2743	·2743	·2743	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·7	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2119	·1841	·1587	·1357	·1151
0·8	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·1841	·1587	·1357	·1151
0·9	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1587	·1357	·1151
1·0	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1357	·1151
1·1	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1151
1·2	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151
1·3	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968
1·4	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = 1{\cdot}00.$

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·4	·0808	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·5	·0668	·0668	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·6	·0548	·0548	·0548	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·7	·0446	·0446	·0446	·0446	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0139	·0107	·0082	·0062	·0047
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0107	·0082	·0062	·0047
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0082	·0062	·0047
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0062	·0047
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0047
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Supplementary Tables for determining High Correlations from Tetrachoric Groupings.

$r = 1{\cdot}00.$

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·5000	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·1	·4602	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·2	·4207	·4207	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·3	·3821	·3821	·3821	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·4	·3446	·3446	·3446	·3446	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·5	·3085	·3085	·3085	·3085	·3085	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·6	·2743	·2743	·2743	·2743	·2743	·2743	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·7	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2119	·1841	·1587	·1357	·1151
0·8	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·1841	·1587	·1357	·1151
0·9	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1587	·1357	·1151
1·0	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1357	·1151
1·1	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1151
1·2	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151
1·3	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968
1·4	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

$r = 1{\cdot}00.$

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·4	·0808	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·5	·0668	·0668	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·6	·0548	·0548	·0548	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·7	·0446	·0446	·0446	·0446	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0139	·0107	·0082	·0062	·0047
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0107	·0082	·0062	·0047
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0082	·0062	·0047
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0062	·0047
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0047
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

NOTE ON THE EXTENT TO WHICH THE DISTRIBUTION OF CASES OF DISEASE IN HOUSES IS DETERMINED BY THE LAWS OF CHANCE.

By J McD. TROUP, B.A., M.B., B.C. (Cantab.) and G. D. MAYNARD, F.R.C.S.E.

There is a problem of some importance in connection with medical investigations which we venture to think has not received sufficient attention from the mathematical standpoint. Stated briefly it is this: If n cases of a certain disease occur in a town with m houses, what are the probabilities that a houses will be affected with one case, b houses with two cases, and so on? In actual practice the problem presents itself in this way: a houses are known to have contained one case, b houses two cases, c houses three cases, and so on: what evidence is to be got of house infection or other disturbing factor from such a distribution, or can it be shown that such a distribution may have arisen by chance and be explicable by the laws of probability?

In order to render the problem suitable for mathematical treatment, we assume for the present that each house is equally liable to infection, i.e., that each house contains the same number of inhabitants, all of whom are equally liable to infection, irrespective of age and sex. We also assume that any particular house is equally likely to be infected at any time during the period under consideration; in practice this involves the exclusion from the data of known instances of case-to-case infection.

With these assumptions the problem now becomes equivalent to the following. If n balls are thrown into m equal compartments which are so arranged that it is equally likely that any ball will fall into any compartment, then the probability P that they will distribute themselves so that,

p_0 compartments will contain 0 balls,
p_1 ,, ,, ,, 1 ball,
p_2 ,, ,, ,, 2 balls,
p_3 ,, ,, ,, 3 ,,
....
p_s ,, ,, ,, s ,,
and so on,

is

$$P = \frac{m\,!\,n\,!}{m^n p_0\,!\,p_1\,!\ldots p_s\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s}\ldots} \qquad \ldots\ldots\ldots\ldots\ldots\text{(i)}.$$

In the above expression it should be noted that both series of factors in the denominator can be carried to infinity, for as there are only n balls, p_{n+1}, p_{n+2}, &c., are all zero, and therefore $p_{n+1}\,! = 1$, $(\overline{n+1}\,!)^{p_{n+1}} = 1$, &c. The importance of this will appear later.

Now let
$$\eta = \frac{m\,!}{p_0\,!\,p_1\,!\ldots p_s\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s}\ldots}\ \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}.$$

Then η is the general term in the expansion of
$$\left(1 + x + \frac{x^2}{2\,!} + \ldots + \frac{x^s}{s\,!} + \ldots\right)^m$$
for all positive integral values of p_0, p_1, p_2, &c., subject to the conditions:
$$p_0 + p_1 + p_2 + \ldots + p_s + \ldots = m\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)},$$
$$p_1 + 2p_2 + \ldots + rp_r + \ldots = n\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)},$$
and
$$S(\eta) = \text{coeft. of } x^n \text{ in } \left(1 + x + \frac{x^2}{2\,!} + \ldots + \frac{x^s}{s\,!} + \ldots\right)^m$$
$$= \text{coeft. of } x^n \text{ in } e^{mx} = \frac{m^n}{n\,!}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$
or
$$S\left(\frac{m\,!\,n\,!}{m^n\,p_0\,!\,p_1\,!\ldots p_s\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s}\ldots}\right) = 1\ldots\ldots\ldots\ldots\text{(vi)},$$
where the summation is taken for all values of p_0, p_1, ... subject to the conditions (iii) and (iv). This merely expresses the fact that the sum of all the probabilities in the case is unity.

Now let
$$\bar{p}_0 = \frac{S(\eta p_0)}{S(\eta)},\quad \bar{p}_1 = \frac{S(\eta p_1)}{S(\eta)},\ \ldots\ \bar{p}_s = \frac{S(\eta p_s)}{S(\eta)},\ \ldots,$$
and let
$$\sigma_0^{\,2} = \frac{S[\eta(p_0 - \bar{p}_0)^2]}{S(\eta)},\ \ldots\ \sigma_s^{\,2} = \frac{S[\eta(p_s - \bar{p}_s)^2]}{S(\eta)},\ \ldots;$$
we have
$$\bar{p}_s = S\left(\frac{m\,!\,p_s}{p_0\,!\,p_1\,!\ldots(p_s)\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s}\ldots}\right)\Big/\,S(\eta)$$
$$= S\left(\frac{m\,!}{p_0\,!\,p_1\,!\ldots(p_s-1)\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s}\ldots}\right)\Big/\,\frac{m^n}{n\,!}$$
$$= \frac{m}{s\,!}\,S\left(\frac{(m-1)\,!}{p_0\,!\,p_1\,!\ldots(p_s-1)\,!\ldots(1\,!)^{p_1}(2\,!)^{p_2}\ldots(s\,!)^{p_s-1}\ldots}\right)\Big/\,\frac{m^n}{n\,!}$$
$$= \frac{m}{s\,!}\cdot\frac{(m-1)^{n-s}}{(n-s)\,!}\cdot\frac{n\,!}{m^n}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$
for the final summation is equivalent to (v) with $p_s - 1$ substituted for p_s, $m - 1$ for m, and $n - s$ for n, the conditions becoming
$$p_0 + p_1 + p_2 + \ldots + (p_s - 1) + \ldots = m - 1,$$
$$p_1 + 2p_2 + \ldots + s(p_s - 1) + \ldots = n - s.$$

It is, therefore, the summation in the case when there are $(n-s)$ balls and $(m-1)$ compartments, and it is to be noticed that p_{n-s+1}, p_{n-s+2}, &c., are all zero, so that it is unnecessary to eliminate the factors containing these expressions.

To find σ_s^2, assume

$$(p_s-\bar{p}_s)^2 = Ap_s(p_s-1)+Bp_s+C$$

for all values of p_s. Then, putting p_s successively $=0, 1, 2,$

$$\begin{aligned} \bar{p}_s^2 &= C, \\ (1-\bar{p}_s)^2 &= B+C, \\ (2-\bar{p}_s)^2 &= 2A+2B+C, \\ \therefore\quad A=1,\quad B&=1-2\bar{p}_s,\quad C=\bar{p}_s^2, \end{aligned}$$

and therefore

$$(p_s-\bar{p}_s)^2 \equiv p_s(p_s-1)+(1-2\bar{p}_s)p_s+\bar{p}_s^2 \text{ ....(viii)},$$

$$\begin{aligned} \therefore\quad S[\eta(p_s-\bar{p}_s)^2] &= S[\eta p_s(p_s-1)]+(1-2\bar{p}_s)S(\eta p_s)+\bar{p}_s^2 S(\eta) \\ &= S[\eta p_s(p_s-1)]+(1-2\bar{p}_s)\bar{p}_s S(\eta)+\bar{p}_s^2 S(\eta) \\ &= S[\eta p_s(p_s-1)]+(\bar{p}_s-\bar{p}_s^2).S(\eta). \end{aligned}$$

Now proceeding as in the previous case, we have

$$\begin{aligned} S[\eta p_s(p_s-1)] &= S\left(\frac{m!}{p_0!\,p_1!\dots(p_s-2)!\dots(1!)^{p_1}(2!)^{p_2}\dots(s!)^{p_s}\dots}\right) \\ &= \frac{m(m-1)}{(s!)^2}S\left(\frac{(m-2)!}{p_0!\,p_1!\dots(p_s-2)!\dots(1!)^{p_1}(2!)^{p_2}\dots(s!)^{p_s-2}\dots}\right) \\ &= \frac{m(m-1)}{(s!)^2}\cdot\frac{(m-2)^{n-2s}}{(n-2s)!} \text{ ....(ix)}. \end{aligned}$$

For the final summation is equivalent to (v), with p_s-2 substituted for p_s, $m-2$ for m, and $n-2s$ for n, the conditions becoming

$$\begin{aligned} p_0+p_1+p_2+\dots+(p_s-2)+\dots &= m-2, \\ p_1+2p_2+\dots+s(p_s-2)+\dots &= n-2s. \end{aligned}$$

We have therefore,

$$\sigma_s^2 = \frac{m(m-1)(m-2)^{n-2s}}{(s!)^2(n-2s)!}\,\frac{n!}{m^n}+\bar{p}_s-\bar{p}_s^2 \text{ ....(x)},$$

and

$$\bar{p}_s = \frac{m(m-1)^{n-s}\,n!}{s!\,(n-s)!\,m^n} \text{ ....(vii)}.$$

It is only necessary to calculate the first term and the others are rapidly obtained as follows:

$$\frac{\bar{p}_s}{\bar{p}_{s-1}} = \frac{m(m-1)^{n-s}}{s!\,(n-s)!}\times\frac{(s-1)!\,(n-s+1)!}{m(m-1)^{n-s+1}} = \frac{n-s+1}{s(m-1)},$$

$$\therefore\quad \bar{p}_1=\bar{p}_0\frac{n}{m-1},\quad \bar{p}_2=\bar{p}_1\frac{n-1}{2(m-1)} \text{ ....(xi)},$$

and for successive terms in σ^2 if α_s be the first term in σ_s^2 of equation (x),

$$\frac{\alpha_s}{\alpha_{s-1}} = \frac{(n-2s+2)(n-2s+1)}{s^2(m-2)^2},$$

$$\therefore \quad \alpha_1 = \alpha_0 \times \frac{n(n-1)}{(m-2)^2}, \quad \alpha_2 = \alpha_1 \frac{(n-2)(n-3)}{2^2(m-2)^2} \quad \text{.....................(xii).}$$

We proceed now to deal with an example taken from Newsholme's *Vital Statistics*, p. 344. In a town with an average of 106,721 houses there were in the course of 7 years 3512 cases of Enteric Fever. By means of the above formulae we can calculate the mean number of houses that would be affected once, twice, thrice, on the supposition that the distribution is a purely chance one, and compare them with the observed numbers, viz., 3350, 78 and 2.

The values of $\bar{p}_1$, $\bar{p}_2$, $\bar{p}_3$ to the nearest unit are

$$\bar{p}_1 = 3398, \quad \bar{p}_2 = 56, \quad \bar{p}_3 = 1.$$

If we had a large number of samples of this size and the laws of chance alone were at work, the mean number of houses affected twice would be 56. By means of the Standard Deviation we can determine what variations from this value may reasonably be expected.

Now $\sigma_2^2 = 51$ and therefore $\sigma_2 = 7$ (to the nearest unit). This gives a probable error of 5, and it would therefore be an even chance that in any sample the number of houses affected twice would be as many as 61 or as few as 51.

We now proceed to obtain by a simple method a close approximation to the modal values for p_0, p_1, ... p_s, &c., and as the values thus obtained are very closely the same as those found for the mean, the formula forms a useful method for quickly calculating the approximate values of $\bar{p}_0$, $\bar{p}_1$, ... $\bar{p}_s$, &c.

To find the values of p_0, p_1, ... p_s, ... which give a maximum value to the probability

$$\frac{m!\,n!}{m^n p_0!\,p_1!\dots p_s!\dots(1!)^{p_1}(2!)^{p_2}\dots(s!)^{p_s}\dots}.$$

The variable part of the above expression is

$$p_0!\,p_1!\,p_2!\dots(1!)^{p_1}\,(2!)^{p_2} \text{..........................(xiii).}$$

If this is a minimum it will be nearly equal to each of the expressions got by substituting successively

$$p_0-1,\ p_1+2,\ p_2-1 \ \text{ for } p_0,\ p_1,\ p_2;$$

$$p_0-2,\ p_1+3,\ p_3-1 \ \text{ for } p_0,\ p_1,\ p_3;$$

$$p_0-3,\ p_1+4,\ p_4-1 \ \text{ for } p_0,\ p_1,\ p_4;$$

and so on,

where it is to be observed that the conditions (iii) and (iv) are satisfied by the new values. The expression (xiii) is a little less than each of the following:

$$(p_0 - 1)!\,(p_1 + 2)!\,(p_2 - 1)!\,p_3!\ldots(1!)^{p_1+2}(2!)^{p_2-1}(3!)^{p_3}\ \ldots\ldots\text{(xiv)},$$

$$(p_0 - 2)!\,(p_1 + 3)!\,p_2!\,(p_3 - 1)!\ldots(1!)^{p_1+3}(2!)^{p_2}(3!)^{p_3-1}\ldots,$$

and so on.

Dividing each of these by the original expression (xiii), we get

$$\frac{(p_1 + 2)(p_1 + 1)}{p_0\,p_2\,2!} = 1, \text{ nearly},$$

$$\frac{(p_1 + 3)(p_1 + 2)(p_1 + 1)}{p_0(p_0 - 1)\,p_3\,3!} = 1, \text{ nearly},$$

and so on;

whence

$$p_2 = \frac{(p_1 + 2)(p_1 + 1)}{p_0\,2!}, \text{ nearly},$$

$$p_3 = \frac{(p_1 + 3)(p_1 + 2)(p_1 + 1)}{p_0(p_0 - 1)\,3!}, \text{ nearly},$$

and so on.

Now where m is considerably greater than n, as in most of the problems of this nature with which we have to deal, p_0 will obviously be considerably greater than p_1, and the values of p_2, p_3, &c., found above will rapidly diminish so that we may put for a first approximation,

$$p_0 = p_0,$$

$$p_1 = p_0\frac{p_1}{p_0} \qquad \text{and} \qquad p_1 = p_1,$$

$$p_2 = p_0\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!}, \qquad 2p_2 = p_1\frac{p_1}{p_0},$$

$$p_3 = p_0\left(\frac{p_1}{p_0}\right)^3\frac{1}{3!}, \qquad 3p_3 = p_1\left(\frac{p_1}{p_0}\right)^2\frac{1}{2!},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots\cdots\cdots$$

therefore adding,

$$m = p_0\left[1 + \frac{p_1}{p_0} + \left(\frac{p_1}{p_0}\right)^2\frac{1}{2!} + \left(\frac{p_1}{p_0}\right)^3\frac{1}{3!} + \ldots\right] = p_0 e^{\frac{p_1}{p_0}}\ \ldots\ldots\ldots\ldots\text{(xv)},$$

and

$$n = p_1\left[1 + \frac{p_1}{p_0} + \left(\frac{p_1}{p_0}\right)^2\frac{1}{2!} + \ldots\right] = p_1 e^{\frac{p_1}{p_0}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xvi)}.$$

Dividing (xv) by (xvi), we get

$$\frac{m}{n} = \frac{p_0}{p_1},$$

therefore

$$m = p_0 e^{\frac{n}{m}}, \quad n = p_1 e^{\frac{n}{m}}.$$

Writing p_0', p_1', p_2', &c., for these values of p_0, p_1, p_2, &c.

$$\left.\begin{aligned} p_0' &= me^{-\frac{n}{m}} \\ p_1' &= ne^{-\frac{n}{m}} &&= \frac{n}{m}p_0' \\ p_2' &= \frac{n^2}{m\,2!}e^{-\frac{n}{m}} &&= \frac{n}{2m}p_1' \\ p_3' &= \frac{n^3}{m^2\,3!}e^{-\frac{n}{m}} &&= \frac{n}{3m}p_2' \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xvii}).$$

The close correspondence between (xi) and (xvii) will be noticed. Referring to the example dealt with above, we find the numerical values to the nearest whole number for p_1', p_2', &c., as found by this method to be: $p_1' = 3398$, $p_2' = 56$, $p_3' = 1$, an identical result to that found above for $\bar{p}_1$, $\bar{p}_2$, $\bar{p}_3$.

We thus see that the mean and the approximate modal values closely correspond when m is large.

Turning now to the assumption that all houses have the same number of inhabitants; there are of course very few towns in which the distribution of the population is so homogeneous as to admit of this approximation being used with safety. It is clear that where some houses contain only 2 or 3 inhabitants, while others have from 20 to perhaps 50, a considerable error might be introduced by grouping them together. It is, therefore, necessary to investigate a method of obtaining the values of $\bar{p}_0$, $\bar{p}_1$, &c., when the distribution of the population is not homogeneous.

Keeping to our assumption that all individuals are equally likely to contract the disease, it follows that a house containing (say) six people is three times as likely to be attacked as a house containing only two. Let us suppose then that there are

m_1 houses containing an average of a_1 inhabitants,

m_2 " " " " " a_2 "

and so on; where each house in the group contains approximately the same number of inhabitants.

Then the total population $N = m_1a_1 + m_2a_2 + m_3a_3 + \ldots$,

and the number of houses $m = m_1 + m_2 + m_3 + m_4 + \ldots$.

If n is as before the total number of cases of disease, the m_1 houses will on the average contain $m_1a_1\frac{n}{N}$ cases, the m_2 houses $m_2a_2\frac{n}{N}$ cases, and so on.

Writing $n_1 = m_1 a_1 n/N$, $n_2 = m_2 a_2 n/N$, &c., and dealing separately with each class of house as in the original problem, we get, by adding the results for the various classes together (taking the approximation $\bar{p}_0 = p_0'$ and writing $\bar{q}_0$ for the new value of $\bar{p}_0$),

$$\bar{q}_0 = m_1 e^{-\frac{n_1}{m_1}} + m_2 e^{-\frac{n_2}{m_2}} + m_3 e^{-\frac{n_3}{m_3}} + \ldots$$

$$= m_1 e^{-\frac{a_1 n}{N}} + m_2 e^{-\frac{a_2 n}{N}} + \ldots,$$

$$\bar{q}_1 = n_1 e^{-\frac{a_1 n}{N}} + n_2 e^{-\frac{a_2 n}{N}} + \ldots,$$

$$\bar{q}_2 = \frac{n_1^2}{m_1 2\,!} e^{-\frac{a_1 n}{N}} + \frac{n_2^2}{m_2 2\,!} e^{-\frac{a_2 n}{N}} + \ldots.$$

It is thus a simple matter to calculate the values of $\bar{q}_0$, $\bar{q}_1$, $\bar{q}_2$, when we have given the number of cases of disease, the total population and the number of houses averaging (say) 3, 5, 10, &c., inhabitants. An imaginary example will show that the change may quite reasonably be a 33 °/₀ increase on the value of $\bar{p}_2$.

If in a town with 50,000 inhabitants and 10,000 houses there are 500 cases of a certain disease, then we find $\bar{p}_0 = 9512$, $\bar{p}_1 = 476$, $\bar{p}_2 = 12$, and $\bar{p}_3 = 0$ to the nearest whole number; and $p_0' = 9512$, $p_1' = 476$, $p_2' = 12$, and $p_3' = 0$. If we now assume 3000 houses to have 2 inhabitants, 5000 to have 5, 1000 to have 7, 800 to have 10, and 200 to have 20, then $\bar{q}_0 = 9516$, $\bar{q}_1 = 467$, $\bar{q}_2 = 16$, and $\bar{q}_3 = 1$.

Returning now to our original example we are unfortunately unable to obtain a distribution of houses according to number of inhabitants in the town dealt with (viz., Manchester). We, therefore, assume that the town would show a somewhat similar distribution to that of Baltimore, U.S.A., for which we were able to obtain the necessary figures. The following table shows the values found for $\bar{q}_2$ on this assumption. While it is probably only approximately accurate it will give an idea of the change in value of $\bar{p}_2$ which may be reasonably expected.

Columns in Table.

(1) Number of people to a house.

(2) Number of houses out of 10,000 with 1, 2, 3, &c., persons (Baltimore).

(3) Number of houses with 1, 2, 3, &c., people in Manchester if the distribution had been the same as that in Baltimore.

(4) Number of persons in each group of houses.

(5) Number of cases of Enteric occurring in each group.

(6) Calculated number of houses affected twice in each group.

a	Baltimore	m	am	n	$\bar{p}_2$
(1)	(2)	(3)	(4)	(5)	(6)
1	162	1730	1730	10	0·029
2	875	9340	18680	109	0·629
3	1313	14010	42030	245	2·105
4	1570	16760	67040	391	4·455
5	1564	16690	83450	487	6·901
6	1380	14730	88380	516	8·727
7	1056	11270	78890	460	8·807
8	757	8080	64640	377	8·394
9	510	5440	48960	286	7·133
10	314	3350	33500	196	5·409
13	416	4440	57720	337	11·855
17	52	550	9350	55	2·461
22	31	330	7260	42	2·300
Totals	10000	106720	$601630 = N$	3511	$69 = \bar{q}_2$

The probable error of $\bar{q}_2$ may be assumed to be at least as great as that of $\bar{p}_2$, which we have found to be 5. Hence it is an even chance that the value of $\bar{q}_2$ will lie between 74 and 64. The observed value 78 would occur on the average at least once in four observations on a similar scale. There is, therefore, on the abovementioned assumption, no clear evidence of house infection in this example.

Neither the conclusions drawn from the results, nor the results themselves, as found by the method adopted by Dr Niven, seem to us to be justified. Dr Newsholme while approving of the method does not give any mathematical proof of its validity, and the results differ considerably from the results found by the method described in this paper.

Although we have been able to eliminate the error arising from variations in number of persons to a house, there still remains the disturbing factor introduced by variations in age and sex. It is difficult to see how this can be dealt with statistically, as it is almost impossible to obtain the necessary information. In some diseases it will not be of so much importance as in others, viz. those which are most prone to attack the very young, e.g., scarlet fever, or the aged, as, for instance, cancer. Statistics of "cancer-houses," if suitably collected and obtainable in sufficient quantity, would form a useful and interesting subject for mathematical investigation.

In conclusion, we would observe that while the formulae given in this paper have, we consider, a field of practical utility in connection with some public health problems, yet the results obtained must be interpreted with caution. The unknown factors will tend to increase the value of the probable error, that is to say, observed variations from the calculated values may be greater than would be expected, without justifying the conclusion that house infection, for instance, is a factor in their causation.

PRETORIA, *March* 26, 1911.

ON THE APPEARANCE OF MULTIPLE CASES OF DISEASE IN THE SAME HOUSE.

By KARL PEARSON, F.R.S.

1. *Introductory.*

This problem is one of surpassing interest both from the standpoint of the mathematician and of the public health officer. Contributions to the theory of the problem have recently appeared from Dr Greenwood, Jnr[*], Drs Troup and Maynard[†], and Captain McKendrick[‡]. Each of the papers referred to have thrown light on the subject, but cannot be considered as giving a final solution. The solution I am proposing in the present paper is at best only an approximate one. The fundamental condition requisite for its sufficiency is that the number of deaths should be small as compared with the number of houses among which they are distributed. This condition is amply fulfilled by the data actually used by Troup and Maynard. Hence to such cases my theory applies. It would not apply without including higher terms in the approximation to cases in which there was a heavy epidemic visiting a large percentage of all houses. It is, however, applicable to cancer distributions.

Troup and Maynard have been the first, so far as I am aware, to reach perfectly general mathematical expressions for (i) the probable number, $\bar{p}_s$, of houses with s cases, supposing the same number of inhabitants to each house, and each house to be equally likely to be attacked, and (ii) the standard-deviation, σ_{p_s}, due to random sampling of the frequency p_s under the same conditions.

They do not go further than this and they do not proceed to find the correlation in deviations of p_s and p_t due to the variations of random sampling. This stage seems to me needful, if we are to provide an answer to the general problem: Is the observed distribution of houses with multiple cases compatible with the theoretical distribution on the assumption that the cases are distributed at random? Troup and Maynard confine their attention to the divergence between the expected and observed numbers of houses with *two* cases, and conclude that for enteric fever in

---

[*] See *Journal of Hygiene*, Vol. x. p. 416.

[†] See *Biometrika*, Vol. VIII. p. 396.

[‡] *Ibid.* p. 413.

a case from Newsholme's *Vital Statistics* (p. 344) there was not a very remarkable divergence. I admit the difficulty of finding the correlation of deviations in p_s and p_t in the general case (n commensurable with m). But this difficulty does not exist when m is large as compared with n; and I venture to think that in any case the *whole* distribution of houses must be compared in the observed and theoretical instances. If my analysis be correct—I frankly admit the matter is obscure and difficult—the case of enteric fever when treated from this broader standpoint indicates that the observed distribution of frequency was *not,* on the hypothesis of equivalent houses, compatible with a mere random scattering of the cases.

Greenwood in his paper on plague distribution has seen the importance of comparing the *whole* of the observed and theoretical distributions. He has found the "goodness of fit" of the observed to the theoretical frequency for the whole system of villages (not houses) in his data. I feel confident that the "goodness of fit" method is the correct method to be applied in such cases, but I doubt if it can be applied without weighting the terms of the χ^2, as, I think, Greenwood has omitted to do. In other words, I do not consider that

$$\chi^2 = S\left\{\frac{(p_s' - \bar{p}_s)^2}{\bar{p}_s}\right\} \quad \text{..................................(i)}$$

is the correct expression to use*. This expression is only true when

$$S(p_s) = m,$$

but we have to remember that in our present case

$$S(sp_s) = n,$$

or the number of cases as well as the number of houses is fixed. Accordingly the fundamental relations on which (i) is based, i.e.

$$\left.\begin{aligned}\sigma^2_{p_s} &= p_s\left(1 - \frac{p_s}{m}\right)\\ \sigma_{p_s}\sigma_{p_t}r_{p_sp_t} &= -\frac{p_sp_t}{m}\end{aligned}\right\} \quad \text{................................(ii),}$$

are no longer correct and must be replaced before we can determine the true form of χ^2. Greenwood is correct in his view that we must use χ^2, and I think this has been overlooked by Troup and Maynard. They on the other hand give the full values for p_s and $\sigma^2_{p_s}$, showing that the first equation of (ii) is incorrect, but they do not attempt the second equation of (ii).

The present writer only offers the approximately correct values of (ii) and the correspondingly corrected value of (i). These results apply absolutely to the enteric data, but would only apply as an approximation to houses in plague data. It has been recently half suggested† that the application of mathematics to medicine does

---

[*] Greenwood's case is not exactly the present case, but the need for weighting the contributions to χ^2 still I think remains true.

[†] *British Medical Journal*, June 17, 1911, p. 1431.

not need the highest mathematical powers and the most complete mathematical training. Now here is a case which absolutely confutes such a suggestion—just as it is confuted by almost every medico-biometric problem that arises. None of us, who are biometricians or medical men, have yet succeeded in fully solving the problem—a most vital one in many respects. We are simply pottering round it and nibbling off little corners of it. And why is this? Solely because we have no transcendent mathematical power at our service. Those with such power have not the faintest notion of the existence of these statistico-medical problems, and those without it do not recognise the essential difficulties of the analysis. Thus we go on nibbling off one bit after another of such hard nuts, where the highly trained mathematician—if he could be made to grasp our problems—might reach the kernels in a few hours. Strange as the confession may seem, when I come to these medico-statistical problems, my regret is not for my want of medical training, but for the extreme defectiveness of my powers of mathematical analysis.

This must be my apology for adding still another incomplete solution to those of my colleagues.

2. *Mean Values.*

Let it be required to drop n balls into m compartments. Then fixing all attention on one compartment, the chance that a ball falls into it is $\frac{1}{m}$, and that it fails to fall into it $\frac{m-1}{m}$. Repeating the process n times, we have the binomial

$$\left(\frac{m-1}{m}+\frac{1}{m}\right)^n$$

as giving the theoretically expected frequency or

$$m\left(\frac{m-1}{m}+\frac{1}{m}\right)^n$$

gives by its terms the distribution of compartments with 0, 1, 2, ... s ... balls in them, i.e.

$$\bar{p}_s = m\frac{n!}{s!\,(n-s)!}\left(\frac{m-1}{m}\right)^{n-s}\left(\frac{1}{m}\right)^s \quad \text{.....................(iii).}$$

This is the result reached as equation (vii) of Troup and Maynard's memoir.

Now let us apply Stirling's Theorem

$$x! = x^x e^{-x}\sqrt{2\pi x}\left(1+\frac{1}{12x}+\frac{1}{288x^2}+\dots\right)$$

to (iii) for $n!$ and $(n-s)!$ and also taking logarithms evaluate

$$(n-s)\log_e\left(1-\frac{1}{m}\right)$$

by the well-known formula

$$\log_e(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\dots$$

and neglect terms of the orders n/m^2 (and therefore of course s/m^2) and s^2/n^2. We find

$$\bar{p}_s = \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n-s}{m} - \frac{1}{2}\frac{s(s-1)}{n}} \text{ approximately } \ldots\ldots\ldots\ldots\ldots\text{(iv)}.$$

Since in many cases s is a very small number as compared with n or m, this may frequently be written

$$\bar{p}_s = \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n}{m}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$

which is the approximate form given by Troup and Maynard (see p. 402).

3. *Correlations.*

I now turn to the correlation in deviations of p_s and p_t. I reserve $s\mu$ deaths out of my total of n deaths; I intend to put these deaths into μ houses, s apiece, so that I reduce my number of available houses to $m - \mu$. I have accordingly to distribute $n - s\mu$ deaths among $m - \mu$ houses. The most probable distribution will be

$$(m-\mu)\left(\frac{m-\mu-1}{m-\mu} + \frac{1}{m-\mu}\right)^{n-s\mu},$$

i.e. by (iii): $$\bar{p}_s{}' = (m-\mu)\frac{(n-s\mu)!}{s!\,(n-s\mu-s)!}\left(\frac{m-\mu-1}{m-\mu}\right)^{n-s\mu-s}\left(\frac{1}{m-\mu}\right)^s$$

and $$\bar{p}_t{}' = (m-\mu)\frac{(n-s\mu)!}{t!\,(n-s\mu-t)!}\left(\frac{m-\mu-1}{m-\mu}\right)^{n-s\mu-t}\left(\frac{1}{m-\mu}\right)^t.$$

Clearly we shall now, introducing the $s\mu$ deaths in μ houses, have

$$\left.\begin{aligned}\bar{p}_s + \delta p_s &= \mu + \bar{p}_s{}' \\ \bar{p}_t + \delta p_t &= \bar{p}_t{}'\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)}.$$

Now if we suppose μ small as compared with m or n, we can expand by the same theorems as we have used before and determine $\bar{p}_s{}'$ and $\bar{p}_t{}'$. This will enable us to find the ratio of the *mean* change in p_t to an arbitrary change in p_s, or if the relation turn out *linear*, we have

$$\frac{\delta p_t}{\delta p_s} = \frac{\sigma_{p_t}}{\sigma_{p_s}} R_{p_t p_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$

where $R_{p_t p_s}$ is the correlation between an arbitrary change in p_s and the resulting change in p_t.

It will be simplest to work by making the requisite changes in $\bar{p}_s$ as given by (iv). We have

$$\frac{\bar{p}_t{}'}{\bar{p}_t} = \frac{\left(1 - \frac{s\mu}{n}\right)^t}{\left(1 - \frac{\mu}{m}\right)^{t-1}} e^{-\frac{n\mu}{m^2} + \frac{s\mu}{n} - \frac{1}{2}\frac{s\mu}{n}\frac{t(t-1)}{n}}.$$

But μ being small compared with n and m, and remembering that we have neglected terms of order s^2/n^2 and n/m^2, we have on expanding the exponential

$$\frac{\bar{p}_t'}{\bar{p}_t} = 1 - \mu\left(\frac{st}{n} - \frac{s+t-1}{m}\right).$$

For the special case of $t = s$:

$$\frac{\bar{p}_s'}{\bar{p}_s} = 1 - \mu\left(\frac{s^2}{n} - \frac{2s-1}{m}\right).$$

Accordingly from (vi) we find,—showing a linear relation—

$$\delta p_s = \mu\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\},$$

$$\delta p_t = -\mu\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right).$$

Whence from (vii):

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\frac{\sigma^2_{p_s}\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right)}{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)} \quad\text{..................(viii).}$$

It remains to find $\sigma^2_{p_s}$ to the same order of approximation.

Clearly $\sigma_{p_t}\sigma_{p_s}R_{p_tp_s}$ must be a function perfectly symmetrical in s and t. Hence we must have

$$\sigma_{p_t}\sigma_{p_s}R_{p_tp_s} = -\frac{\sigma^2_{p_t}\bar{p}_s\left(\frac{st}{n} - \frac{s+t-1}{m}\right)}{1 - \bar{p}_t\left(\frac{t^2}{n} - \frac{2t-1}{m}\right)},$$

whence it follows that

$$\sigma^2_{p_s} = \lambda\bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\},$$

and

$$\sigma^2_{p_t} = \lambda\bar{p}_t\left\{1 - \bar{p}_t\left(\frac{t^2}{n} - \frac{2t-1}{m}\right)\right\},$$

where λ is independent of s or t, because σ_{p_s} and σ_{p_t} cannot respectively involve a particular t or s; i.e. λ can only depend on m and n, or be an absolute number.

If $s = 0$, we have

$$\sigma^2_{p_0} = \lambda\bar{p}_0\left(1 - \frac{\bar{p}_0}{m}\right),$$

which is the correct value only if $\lambda = 1$.

Hence generally we have

$$\sigma^2_{p_s} = \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\} \quad\text{........................(ix).}$$

To test this value take Troup and Maynard's more general result (see p. 398):

$$\sigma^2_{p_s} = \frac{m(m-1)(m-2)^{n-2s}}{(s!)^2(n-2s)!}\frac{n!}{m^n} + \bar{p}_s - \bar{p}_s^2 \quad\text{..................(x).}$$

Call this

$$\sigma^2_{p_s} = \epsilon + \bar{p}_s - \bar{p}_s^2.$$

Then

$$\epsilon/\bar{p}_s^2 = \frac{m-1}{m} \cdot \times \frac{m^n (m-2)^{n-2s}}{(m-1)^{2n-2s}} \times \frac{(n-s)!\,(n-s)!}{n!\,(n-2s)!}.$$

We may evaluate the factors separately,

$$u_2 = \frac{(n-s)!\,(n-s)!}{n!\,(n-2s)!} \text{ and by Stirling's Theorem } = \frac{(n-s)^{2n-2s+1}}{n^{n+\frac{1}{2}}(n-2s)^{n-2s+\frac{1}{2}}},$$

$$\log u_2 = -(2n-2s+1)\left(\frac{s}{n} + \frac{s^2}{2n^2} + \dots\right) + (n-2s+\tfrac{1}{2})\left(\frac{2s}{n} + \frac{4s^2}{2n^2} + \dots\right)$$

$$= -\frac{s^2}{n} - \text{etc.},$$

or

$$u_2 = e^{-\frac{s^2}{n}} = 1 - \frac{s^2}{n} \text{ nearly},$$

$$u_1 = \frac{(m-1)}{m} \frac{m^n (m-2)^{n-2s}}{(m-1)^{2n-2s}} = \left(1 - \frac{2}{m}\right)^{n-2s} \left(1 - \frac{1}{m}\right)^{-(2n-2s+1)}$$

$$= 1 - \frac{2(n-2s)}{m} + \frac{2n-2s-1}{m} + \text{terms of order } \left(\frac{n}{m}\right)^2,$$

which we have agreed to neglect.

Hence

$$\epsilon/\bar{p}_s^2 = u_1 u_2 = 1 - \frac{s^2}{n} + \frac{2s-1}{m},$$

and accordingly

$$\sigma^2_{p_s} = \bar{p}_s \left\{1 - \bar{p}_s \left(\frac{s^2}{n} - \frac{2s-1}{m}\right)\right\} \quad \text{........................(xi)},$$

which agrees with our previous value in (ix).

Clearly in the enteric fever case, to which Troup and Maynard apply their results, the term $(2s-1)/m$ is quite negligible, and we may always write when m is large compared with n:

$$\left.\begin{aligned} \bar{p}_s &= \frac{m}{s!}\left(\frac{n}{m}\right)^s e^{-\frac{n}{m}} \\ \sigma^2_{p_s} &= \bar{p}_s \left(1 - \frac{s^2}{n}\bar{p}_s\right) \\ \sigma_{p_s}\sigma_{p_t} R_{p_s p_t} &= -\frac{st\bar{p}_s\bar{p}_t}{n} \end{aligned}\right\} \quad \text{............................(xii)}.$$

To test the accuracy of (xii) the case of enteric fever has been worked out by the long formulae, (iii) and (x), and by the short formulae of (xii). We have

	$\bar{p}_1$	$\sigma^2_{p_1}$	$\bar{p}_2$	$\sigma^2_{p_2}$	$\bar{p}_3$	$\sigma^2_{p_3}$
Long Formulae ...	3398·34	214·49	55·88	52·10	·613	·612
Short Formulae ...	3398·33	214·60*	55·89	52·33	·613	·612

---

* Equation (xi) used to find σ_1^2, but only this first value for which (xii) is not close enough.

The values of m and n are $m = 106{,}721$, $n = 3512$; the observed values are $p_1 = 3350$, $p_2 = 78$, $p_3 = 2$; the higher p's are zero.

It will be seen that, for such an example as enteric fever (and cancer falls into the same category), the approximate formulae are as good as the complete formulae, and there is much less danger of error in using them than in dealing with high powers of large numbers, such as arise in an equation like (x), which is based upon the differences of large numbers*.

4. *Application of Goodness of Fit Formulae.*

I shall now proceed to modify the second and third equations of (xii) or in the first place (viii) and (ix). These may be written:

$$\sigma_{p_t}\sigma_{p_s}R_{p_t p_s} = -\bar{p}_s\bar{p}_t\left(\frac{st}{n} - \frac{s+t-1}{m}\right) \quad \text{..........................(xiii)}$$

$$= -\bar{p}_s\bar{p}_t\left\{st\left(\frac{1}{n} - \frac{1}{m}\right) + \frac{(s-1)(t-1)}{m}\right\}$$

$$= -\bar{p}_s\bar{p}_t\left\{\frac{st}{n\left(1+\frac{n}{m}\right)} + \frac{(s-1)(t-1)}{m}\right\} \quad \text{.........(xiv),}$$

since we are neglecting terms of the order n^2/m^2 compared with unity; and

$$\sigma^2_{p_s} = \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n} - \frac{s^2}{m} + \frac{(s-1)^2}{m}\right)\right\}$$

$$= \bar{p}_s\left\{1 - \bar{p}_s\left(\frac{s^2}{n\left(1+\frac{n}{m}\right)} + \frac{(s-1)^2}{m}\right)\right\} \quad \text{..................(xv).}$$

Now in the cases with which we are dealing s or t is merely a number of the order 1 to 3, and small, while m is very large. Hence such terms as $(s-1)(t-1)/m$ and $(s-1)^2/m$ are negligible as compared with st/N or s^2/N, where $N = n\left(1 + \frac{n}{m}\right)$. We may therefore to the same degree of approximation as before write

$$\sigma_{p_t}\sigma_{p_s}R_{p_t p_s} = -\frac{st\,p_s p_t}{N} \quad \text{..........................(xvi),}$$

$$\sigma^2_{p_s} = \bar{p}_s\left(1 - \frac{\bar{p}_s s^2}{N}\right) \quad \text{..................(xvii)†,}$$

where
$$N = n\left(1 + \frac{n}{m}\right) \text{........................(xviii).}$$

I now propose to find $S(s^2\bar{p}_s)$.

---

* In the case of $\sigma^2_{p_1}$ the results from (x) must be worked correct to 10 figures, if we wish to get the first decimal place correct in its value.

† Equation (xvii) is as accurate as (xi) and it is better to use it than (xii).

To do this we require to know the value of

$$\frac{1^2 x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^2 x^3}{3!} + \frac{4^2 x^4}{4!} + \dots .$$

Now it clearly equals

$$x \frac{d}{dx}\left(x \frac{d}{dx} e^x\right) = x(x+1)e^x.$$

Let $x = \frac{n}{m}$, then

$$me^{-\frac{n}{m}} \left\{\frac{1^2}{1!}\frac{n}{m} + \frac{2^2}{2!}\left(\frac{n}{m}\right)^2 + \frac{3^2}{3!}\left(\frac{n}{m}\right)^3 + \frac{4^2}{4!}\left(\frac{n}{m}\right)^4 + \dots\right\}$$

$$= S(s^2 \bar{p}_s) \text{ by the first equation of (xii)}$$

$$= me^{-\frac{n}{m}} \frac{n}{m}\left(\frac{n}{m} + 1\right) e^{\frac{n}{m}}$$

$$= n\left(1 + \frac{n}{m}\right) = N, \text{ to our degree of approximation.}$$

For example $n\left(1 + \frac{n}{m}\right) = 3627{\cdot}6$ in the enteric data cited above, $= 3627{\cdot}4$ actually, when calculated as $S(s^2 \bar{p}_s)$, a difference of no importance.

Now let us write $x_s = s^2 p_s$; then clearly $\sigma^2_{x_s} = s^2 \sigma^2_{p_s}$, and $R_{x_s x_t} = R_{p_s p_t}$, for the new variables only differ from the old by constant multipliers. Accordingly we have

$$\left.\begin{aligned} \sigma^2_{x_s} &= \bar{x}_s \left(1 - \frac{\bar{x}_s}{N}\right) \\ \sigma_{x_s} \sigma_{x_t} R_{x_s x_t} &= -\frac{\bar{x}_s \bar{x}_t}{N} \\ \text{while} \qquad S(x_s) &= N \end{aligned}\right\} \dots\dots\dots\dots\text{(xix)}.$$

Thus $x_1, x_2, \dots x_s \dots$ may be looked upon as frequencies, which have a constant sum N and for which the standard deviations and correlations are precisely those defined in my original memoir on "Goodness of Fit": see *Phil. Mag.* Vol. L. Eqns (vii) and (viii), p. 161, July, 1900. We are not troubled by p_0 for it does not enter into the system (i.e. $x_0 = 0$), and p_0 is merely an additional quantity to be found ultimately by the relation

$$p_0 = m - \overset{n}{\underset{1}{S}}\left(\frac{x_s}{s^2}\right).$$

Proceeding exactly as in the memoir just referred to, we find

$$\chi^2 = \overset{n}{\underset{1}{S}} \frac{(x_s - \bar{x}_s)^2}{\bar{x}_s} = \overset{n}{\underset{1}{S}} \left\{\frac{s^2 (p_s - \bar{p}_s)^2}{\bar{p}_s}\right\} \dots\dots\dots\dots\text{(xx)}.$$

It will be observed that this result differs from the χ^2 for a series of frequencies $p_0, p_1, \dots p_s \dots$, i.e. from

$$\chi^2 = \overset{n}{\underset{0}{S}} \frac{(p_s - \bar{p}_s)^2}{\bar{p}_s},$$

by the weighting of each contribution to the contingency with the square of the number of cases occurring in houses of the given category. A little thought seems to indicate that this weighting is *a priori* probable, and it is possible that a direct proof of it may be discoverable. At any rate when discovered, it seems so reasonable, that I am inclined to think that (xx) may be safely applied even beyond the range (i.e. m and n large and m large compared to n) for which it has been demonstrated.

In actually applying the method to statistical data of disease occurrence, we must confine our attention to those frequencies which give at least a whole house. Thus we have practically for the case of enteric

$$\bar{p}_1 = 3398, \qquad \bar{p}_2 = 56, \qquad \bar{p}_3 = 1,$$

and the observed frequencies were

$$p_1 = 3350, \qquad p_2 = 78, \qquad p_3 = 2,$$

$$\chi^2 = 1^2 \frac{(3398 - 3350)^2}{3398} + 2^2 \frac{(78 - 56)^2}{56} + 3^2 \frac{(2-1)^2}{1}$$

$$= \cdot 678 + 34\cdot 571 + 9 = 44\cdot 25.$$

I think we must say that this is the χ^2 for *four* groups, although by the nature of the expression for χ^2, the houses with no cases do not enter*. Consulting Palin Elderton's Tables (*Biometrika*, Vol. I. p. 158) we conclude that such an improbability would not occur on the average once in a million random samplings, and therefore with a high degree of probability cases of enteric do tend to repeat in the same houses. If we had judged simply on the group p_2 we should have $(p_2 - \bar{p}_2)/\sigma_2 = 22/7$ roughly $= 3$, and such an excess would occur about once in a 1000 trials. This again would be extremely improbable, but not so improbable as the result based on an examination of the entire frequency. The importance of using the latter is thus indicated. At the same time the caution given by Troup and Maynard as to the influence in houses of age and sex must have full weight, and the problem of the number of inhabitants to the house requires further consideration, which I postpone for the present.

I hope shortly to publish Dr Law Webb's data for "cancer houses," which were sent to me in 1910, discussed by this method: see below, *Miscellanea*, p. 430.

---

* The upper limit to the number of groups may be considered disputable. Experimentally all n balls might be cast in one compartment, or there are $n+1$ frequency classes conceivable. This is impossible in the case of multiple cases of disease, for n persons have not lived in any one house during the period under observation. To fix the upper limit the average number of persons to the house and the average duration of occupation would need to be considered. The fact, however, that χ^2 for those groups which show *theoretically* at least a unit is found *experimentally* to give reasonable values of P, justifies empirically its use to test vital statistics.

MISCELLANEA.

I. A Simplified Method of Calculating Frequencies of Occurrence, from a Large Number of Unequal Probabilities.

By Captain A. G. McKENDRICK, I.M.S., Assistant Director, Pasteur
Institute of Southern India, Coonoor.

In the study of epidemics in particular it is desirable to determine, in certain cases, whether the proneness of a particular community to recurring attack by disease, is regulated by the laws of chance ; or whether other factors are operative.

Such an investigation must be carried out in two steps ; firstly, from the data given, the chance distribution must be calculated, and secondly, the chance distribution must be compared with actual figures.

Problems of this sort occur in many forms, for example :

(1) If a series of epidemics of varying magnitude occurs in a group of communities, do the numbers of communities actually attacked never, once, twice, etc., agree with the numbers as calculated by the calculus of Probabilities ?

(2) If a regiment is attacked by a number of different diseases in a given time, do the numbers of men actually attacked by none, one, two, etc. of these diseases, agree with the probable numbers?

It is only with the first step of the investigation, i.e., the calculation of the probable numbers, that this paper will deal.

M. Greenwood, Jr. (*Journal of Hygiene*, Vol. x. No. 3, p. 416) in an admirable analysis of the statistics of Plague epidemics in the Punjaub, had, amongst others, the following problem to deal with :

In the Amritsar District out of 1,062 villages,

62 were attacked by plague in 1901—2
506 „ „ „ 1902—3
445 „ „ „ 1903—4
669 „ „ „ 1904—5
276 „ „ „ 1905—6
604 „ „ „ 1906—7

He had to find from these data what would be the probable numbers of villages one would expect to be attacked no times, once, twice, etc. He used the ordinary summation method, that is he calculated the most probable number of infections for each possible combination of epidemics. He states that "although the calculation of the probable number of villages to be infected in any combination of epidemics is merely a matter of simple arithmetic, yet the arithmetic becomes extremely laborious, when the number of epidemics under review is even moderately large. Thus in Amritsar District there were six epidemics and a complete evaluation of the various combinations of villages attacked never, once, twice, etc., requires the determination of 64 distinct products each composed of six terms. Without a mechanical calculator this is an impracticable task, and even with such help it is very tedious."

Two years ago Major Lamb of the Plague Commission (India) gave me the problem of the Amritsar District to work out. In order to reduce the labour of the work I devised the following process, which, although it contains nothing of mathematical novelty, still attains the required result with great economy of labour. In place of the determination of 64 products of six terms each, 29 products of two terms each are evaluated, the remainder of the calculation being simple addition and subtraction. A mechanical calculator is unnecessary. The time employed in the calculation is little over an hour.

As a simple case let us consider only two probabilities p and q, then, the total frequency being taken as unity, the frequency F is

$$\begin{array}{llll} \text{for 2 times} & F_2=p\,.\,q & = & pq \\ \text{,, 1 time} & F_1=p(1-q)+q(1-p) & = & (p+q)-2pq \\ \text{,, 0 times} & F_0=(1-p)(1-q) & = & 1-(p+q)+pq. \end{array}$$

Now let P_r=sum of the products of all the probabilities taken r at a time, P_0 being unity.

Then for the n probabilities, $a, b, c, d, e, f, \ldots\ldots$

$$\begin{array}{lrl} (1) & F_0= & P_0-P_1+P_2-P_3+P_4-P_5+P_6-\ldots \\ & F_1= & P_1-2P_2+3P_3-4P_4+5P_5-6P_6+\ldots \\ & F_2= & P_2-3P_3+6P_4-10P_5+15P_6-\ldots \\ & F_3= & P_3-4P_4+10P_5-20P_6+\ldots \\ & F_4= & P_4-5P_5+15P_6-\ldots,\ \text{etc.} \end{array}$$

and obviously

$$\Sigma_0^n F_r=P_0=1,$$

where the coefficients of the P's in the value F_r are the same as those in the expansion in powers of x, of $(1+x)^{-(r+1)}$, namely,

$$(1+x)^{-(r+1)}=1-(r+1)x+\frac{(r+1)(r+2)}{1\,.\,2}x^2-\frac{(r+1)(r+2)(r+3)}{1\,.\,2\,.\,3}x^3+\ldots.$$

The general formula for F_r is thus

$$(2)\qquad F_r=P_r-(r+1)P_{r+1}+\frac{(r+1)(r+2)}{1\,.\,2}P_{r+2}-\frac{(r+1)(r+2)(r+3)}{1\,.\,2\,.\,3}P_{r+3}+\ldots$$

$$\ldots+(-)^{n-r}\frac{(r+1)(r+2)\ldots n}{1\,.\,2\ldots(n-r)}P_n.$$

A concise and economical method of calculating the values of the P's may be exhibited in the following scheme :—

$$
\begin{array}{llllll}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
[a+b+c+d+e+f] & & & & & \\
 & a[b+c+d+e+f] & & & & \\
 & +b[c+d+e+f] & (a)\,b[c+d+e+f] & & & \\
 & +c[d+c+f] & +(a+b)\,c[d+e+f] & (ab)\qquad c[d+e+f] & & \\
 & +d[e+f] & +(a+b+c)\,d[e+f] & +(ab+\overline{a+b}\,.\,c\quad)\,d[e+f] & (abc)\qquad d[e+f] & \\
 & +e[f] & +(a+b+c+d)\,e[f] & +(ab+\overline{a+b}\,.\,c+\overline{a+b+c}\,.\,d)\,e[f] & +\{abc+(ab+\overline{a+b}\,.\,c)\,d\}\,.\,e[f] & abcd\,.\,e[f]
\end{array}
$$

Write this with serial coefficients, thus:

$$
\begin{array}{llllll}
A[a+b+c+d+e+f] & & & & & \\
 & B_2[b+c+d+e+f] & & & & \\
 & +C_2[c+d+e+f] & C_3[c+d+e+f] & & & \\
 & +D_2[d+c+f] & +D_3[d+e+f] & D_4[d+e+f] & & \\
 & +E_2[c+f] & +E_3[e+f] & +E_4[e+f] & E_5[e+f] & \\
 & +F_2[f] & +F_3[f] & +F_4[f] & +F_5[f] & F_6[f]
\end{array}
$$

then

$$
\begin{array}{llll}
C_3=B_2\,.\,b & & & \\
D_3=(B_2+C_2)\,c & D_4=C_3\,.\,c & & \\
E_3=(B_2+C_2+D_2)\,d & E_4=(C_3+D_3)\,d & E_5=D_4\,.\,d & \\
F_3=(B_2+C_2+D_2+E_2)\,e & F_4=(C_3+D_3+E_3)\,e & F_5=(D_4+E_4)\,e & F_6=E_5\,.\,e
\end{array}
$$

A numerical example will bring out the points of the algorithm: take $a\ b\ c\ d\ e\ f$ to be $2\ 3\ 5\ 7\ 11\ 13$

then the calculation follows through these steps,—

Step (I)

$P_1=[2+3+5+7+11+13]$

$$
\begin{aligned}
P_2= &\ \ 2[\ 3+\ 5+\ 7+11+13] \\
&+\ 3[\ 5+\ 7+11+13] \\
&+\ 5[\ 7+11+13] \\
&+\ 7[11+13] \\
&+11[13]
\end{aligned}
\qquad
\begin{aligned}
P_3= &\ \ 6[\ 5+\ 7+11+13] \\
&+\ 25[\ 7+11+13] \\
&+\ 70[11+13] \\
&+187[13]
\end{aligned}
\qquad
\begin{aligned}
P_4= &\ \ 30[\ 7+11+13] \\
&+\ 217[11+13] \\
&+1111[13]
\end{aligned}
\qquad
\begin{aligned}
P_5= &\ \ 210[11+13] \\
&+2717[13]
\end{aligned}
\qquad
P_6=2310[13]
$$

Step (II)

$P_1=41$

$$
\begin{aligned}
P_2= &\ \ 2\times39=\ 78 \\
&+\ 3\times36=108 \\
&+\ 5\times31=155 \\
&+\ 7\times24=168 \\
&+11\times13=143 \\
\hline
&\qquad\qquad 652
\end{aligned}
\qquad
\begin{aligned}
P_3= &\ \ 6\times36=\ 216 \\
&+\ 25\times31=\ 775 \\
&+\ 70\times24=1680 \\
&+187\times13=2431 \\
\hline
&\qquad\qquad 5102
\end{aligned}
\qquad
\begin{aligned}
P_4= &\ \ 30\times31=\ 930 \\
&+\ 217\times24=\ 5208 \\
&+1111\times13=14443 \\
\hline
&\qquad\qquad 20581
\end{aligned}
\qquad
\begin{aligned}
P_5= &\ \ 210\times24=\ 5040 \\
&+2717\times13=35321 \\
\hline
&\qquad\qquad 40361
\end{aligned}
\qquad
\begin{aligned}
P_6= &\ 2310\times13=30030 \\
\hline
&\qquad\qquad 30030
\end{aligned}
$$

$\overline{\ 41\ }$

The whole arithmetic is shown here.

Example. In P_3: The first coefficient is $C_3=B_2 . b$; look in P_2, we see "2 [3...," hence the required coefficient is 6. Place this in front of its [ ] in P_3.

The second coefficient is $D_3=(B_2+C_2)\,c$. Look in P_2 we see "$\begin{matrix}2 \\ 3\,[5\ldots\end{matrix}$" and get $(2+3)\,5=25$. Place this in front of its [ ] in P_3.

The third coefficient is $E_3=(B_2+C_2+D_2)\,d$. Look in P_2, we see "$\begin{matrix}2 \\ 3 \\ 5\ [7\ldots\end{matrix}$" and get $(2+3+5)\,7=70$. Place this in front of its [ ] in P_3.

The fourth coefficient is $F_3=(B_2+C_2+D_2+E_2)\,e$. Look in P_2 we see "$\begin{matrix}2 \\ 3 \\ 5 \\ 7\,[11\ldots\end{matrix}$" and get $(2+3+5+7)\,11=187$. Place this in front of its [ ] in P_3.

Similarly 217 in P_4 comes from "$\begin{matrix}6 \\ 25\,[7\ldots\end{matrix}$," which gives $31\times 7=217$: and so on.

Now note that *as placed* the [ ] factor continues always the same in the *same horizontal line*, and therefore needs only to be calculated *once for all* in step (II).

Perform each calculation in step (II) directly under its place in step (I),

First, do all the parts in ordinary type,
Second, insert the parts in Italics,
Third, make the final additions.

Note. The [ ]'s are all calculated in P_2, and keeping the calculation *lined* as above, the value of each [ ] has merely to be repeated for each successive P, until it runs out. Thus 31 is the third [ ] in P_2, and it reappears automatically in P_3 and P_4 and then runs out.

The process as detailed above, may be shortly described thus:—

Of the series of letters $a, b, c, d, e, f, g, h, i, k, l\ldots$ select any one, say f, and place it so as to have free positions in front of and after it: then place after it as a multiplier, the [ ] containing the sum of all the letters that follow it (that is, formally, the sum of the products of all the letters that follow it *taken one at a time*). This letter f and the square bracket $[g+h+\ldots]$ constitute a sum of products of *two* letters each, hence if we are calculating say P_7, we need each multiplier of $f[g+h+\ldots]$ to be the product of 5 letters: and this multiplier is "the sum of the *products* 5 *at a time*" of all the letters which precede f. Thus one term in P_7 is

$$\begin{pmatrix} abcde+abcdf+abcdg+abcef+abceg+\ldots \\ +bcdef+bcdeg+\ldots \\ +cdefg+\ldots\ \text{21 terms in all}\end{pmatrix} h\,[i+k+\ldots].$$

The algorithm presents a systematic method of calculating the whole series of values of $P_1, P_2, P_3\ldots P_n$, up to any required value of n, so that each calculation not only serves for its own place, but is a step in the calculation of future terms. In short the process is an economical method for the calculation of P_r, where r has all the values $r=1, 2, \ldots n$, and where P_r is the sum of the products r at a time of a series of n quantities.

The calculation of the problem of the Amritsar District is exhibited on the opposite page.

It will be noted in this calculation, in which the whole arithmetic is shown, that in step (I) 10 products, and in step (II) 14 products of two terms each were evaluated by logarithms, and that to obtain the final values of the P's, 5 quotients of two terms each were evaluated. In all 29 logarithmic calculations of two terms each were made. The remainder of the calculation is simple addition. A complete check may be made by comparing the value of F_0 derived as above from *all* the P's with the value of F_0 obtained from $F_0=(1-a)\,(1-b)\ldots(1-n)$.

$$a=\frac{62}{1062}\qquad b=\frac{506}{1062}\qquad c=\frac{445}{1062}\qquad d=\frac{669}{1062}\qquad e=\frac{276}{1062}\qquad f=\frac{604}{1062}$$

$$P_0=1$$

$$P_1=[62+506+445+669+276+604]\div 1062$$

$$P_2=\left.\begin{array}{l}62\,[506+445+669+276+604]\\+506\,[445+669+276+604]\\+445\,[669+276+604]\\+669\,[276+604]\\+276\,[604]\end{array}\right\}\div 1062^2$$

$$P_3=\left.\begin{array}{l}31{,}372\,[445+669+276+604]\\+252{,}760\,[669+276+604]\\+677{,}690\,[276+604]\\+464{,}230\,[604]\end{array}\right\}\div 1062^3$$

$$P_4=\left.\begin{array}{l}13{,}960\times 10^3\,[669+276\times 604]\\+190{,}080\times 10^3\,[276+604]\\+265{,}460\times 10^3\,[604]\end{array}\right\}\div 1064^4$$

$$P_5=\left.\begin{array}{l}9{,}3392\times 10^5\,[276+604]\\+56{,}3150\times 10^5\,[604]\end{array}\right\}\div 1062^5$$

$$P_6=25776\times 10^{10}\,[604]\div 1062^6$$

$$P_0=1$$

$$P_1=2562\div 1062$$

$$P_2=\left.\begin{array}{l}62\times 2500\\+506\times 1994\\+445\times 1549\\+669\times\ \ 880\\+276\times\ \ 604\end{array}\right\}\div 1062^2$$

$$P_3=\left.\begin{array}{l}31{,}372\times 1994\\+252{,}760\times 1549\\+677{,}690\times\ \ 880\\+464{,}230\times\ \ 604\end{array}\right\}\div 1062^3$$

$$P_4=\left.\begin{array}{l}13{,}960\times 10^3\times 1549\\+190{,}080\times 10^3\times\ \ 880\\+265{,}460\times 10^3\times\ \ 604\end{array}\right\}\div 1062^4$$

$$P_5=\left.\begin{array}{l}93392\times 10^5\times 880\\+563150\times 10^5\times 604\end{array}\right\}\div 1062^5$$

$$P_6=25776\times 10^{10}\times 604\div 1062^6$$

Multiply throughout by 1062

$$P_0=1062$$

$$P_1=2562$$

$$P_2=\frac{\begin{array}{r}155{,}000\\+1{,}008{,}960\\+\ \ 689{,}300\\+\ \ 588{,}720\\+\ \ 166{,}700\\\hline 2{,}608{,}680\end{array}}{1062}\qquad P_3=\frac{\begin{array}{r}62{,}555\times 10^3\\+\ \ 391{,}520\times 10^3\\+\ \ 596{,}370\times 10^3\\+\ \ 280{,}390\times 10^3\\\hline 1{,}330{,}835\times 10^3\end{array}}{1062^2}\qquad P_4=\frac{\begin{array}{r}21{,}624\times 10^6\\+167{,}270\times 10^6\\+160{,}330\times 10^6\\\hline 349{,}224\times 10^6\end{array}}{1062^3}\qquad P_5=\frac{\begin{array}{r}82185\times 10^8\\+340140\times 10^8\\\hline 422325\times 10^8\end{array}}{1062^4}\qquad P_6=\frac{15568\times 10^{11}}{1062^5}$$

$$P_0=1062\quad P_1=2562\quad P_2=2456{\cdot}38\qquad P_3=1179{\cdot}95\qquad P_4=291{\cdot}56\qquad P_5=33{\cdot}2008\qquad P_6=1{\cdot}1524$$

$$\begin{array}{lrll}F_0= & P_0-P_1+\ P_2-\ P_3+\ P_4-\ \ P_5+\ \ P_6= & 35{\cdot}9416 & (\ \ 36\ \ )\\F_1= & P_1-2P_2+3P_3-4P_4+\ 5P_5-\ 6P_6= & 181{\cdot}9396 & (\ 181{\cdot}9)\\F_2= & P_2-3P_3+6P_4-10P_5+15P_6= & 351{\cdot}168 & (\ 351{\cdot}1)\\F_3= & P_3-4P_4+10P_5-20P_6= & 322{\cdot}670 & (\ 322{\cdot}6)\\F_4= & P_4-\ 5P_5+15P_6= & 142{\cdot}842 & (\ 142{\cdot}6)\\F_5= & P_5-\ 6P_6= & 26{\cdot}2864 & (\ \ 26{\cdot}4)\\F_6= & P_6= & 1{\cdot}1524 & (\ \ \ 1{\cdot}2)\\\hline & \Sigma_0^6 F_r= & 1062{\cdot}0000 & (1061{\cdot}8)\end{array}$$

The bracketed figures are those calculated by Greenwood.

A comparison of the amount of labour required in the two modes of calculation is shown in the following table (where $\Sigma_0^n F_r = 1$) :—

	Simplified method. Number of products + quotients	Summation method. Number of products
$n = 2$	2 of two terms	4 of 2 terms
$= 3$	6 "	8 " 3 "
$= 4$	12 "	16 " 4 "
$= 5$	20 "	32 " 5 "
$= 6$	30 "	64 " 6 "
$= 7$	42 "	128 " 7 "
$= 8$	56 "	256 " 8 "
$= 9$	72 "	512 " 9 "
$= 10$	90 "	1024 " 10 "
$= n$	$n(n-1)$ of two terms	2^n of n terms

In the case of the Amritsar district the actual frequencies were

$$F_0 = 155$$
$$F_1 = 183$$
$$F_2 = 211$$
$$F_3 = 230$$
$$F_4 = 169$$
$$F_5 = 93$$
$$F_6 = 21$$
$$\Sigma_0^6 F_r = 1062$$

obviously the distribution was not a random one.

As examples of distributions in which accordance with the laws of chance was more marked, I quote two instances from the statistics collected by the Plague Commission.

Name of Village	Total Number of Houses	Number of houses attacked in			
		First Epidemic	Second Epidemic	Third Epidemic	Fourth Epidemic
Dhand	418	101	198	40	26
Kasel	806	308	252	230	67

The comparative agreement between calculated and observed frequencies is shown in the following table :—

Number of Houses Attacked	DHAND		KASEL	
	Calculated	Observed	Calculated	Observed
Never	141·495	136	224·30	209
Once	196·77	208	350·58	383
Twice	71·26	65	189·22	169
Thrice	8·19	9	39·62	43
Four times ...	0·285	0	2·28	2
Totals	418	418	806	806

Formulæ (1) may be written, in reverse order,

$$(3)\qquad \begin{aligned} F_n &= P_n,\\ F_{n-1} &= P_{n-1} - nP_n,\\ F_{n-2} &= P_{n-2} - (n-1)\,P_{n-1} + \frac{n\,(n-1)}{1\,.\,2}\,P_n,\\ &\vdots\\ F_{n-r} &= P_{n-r} - (n-r+1)\,P_{n-r+1} + \frac{(n-r+2)\,(n-r+1)}{1\,.\,2}\,P_{n-r+2} - \dots\\ &\qquad \dots + (-)^r\,\frac{n\,(n-1)\dots(n-r+1)}{1\,.\,2\dots r}\,P_n \end{aligned}$$

In the special case where $a=b=c=\dots=p$, we have

$$P_n = p^n,$$

$$P_{n-1} = {}_nC_{n-1}\,p^{n-1} = np^{n-1},$$

and generally

$$P_{n-r} = {}_nC_{n-r}\,p^{n-r} = \frac{n\,!}{(n-r)\,!\;r\,!}\,p^{n-r}.$$

Thus by (3)

$$F_n = p^n,$$

$$F_{n-1} = np^{n-1} - np^n = np^{n-1}(1-p),$$

$$F_{n-2} = \qquad = \frac{n\,(n-1)}{1\,.\,2}\,p^{n-2}\,(1-p)^2,$$

and so on, giving again the well-known result

$$F_r = \frac{n\,!}{(n-r)\,!\;r\,!}\,p^{n-r}\,(1-p)^r.$$

II. The Application of the Correlation Coefficient to Mendelian Distributions.

By E. C. SNOW, M.A.

In a paper* published in the Proceedings of the Royal Society of Edinburgh, Dr John Brownlee has employed various methods to determine theoretical values of the parental and fraternal correlations under special conditions, on the basis of the Mendelian formulæ. We do not propose to deal with Dr Brownlee's conclusions, but only to draw attention to some of his methods. The importance of criticising them is not diminished by the fact that they may in some cases give correct results. It is, indeed, for this reason the more essential that they should be scrutinised, as their employment in other circumstances, where they may not give correct results, is rendered the more likely.

The methods to be employed in determining correlation depend entirely on the nature of the frequency distributions dealt with. The following general types of distribution can be recognised: (*a*) continuous and quantitative, e.g. head length and many other anthropometric measurements, (*b*) continuous and yet for convenience of classification treated as qualitative, e.g. health and intelligence, (*c*) discontinuous and quantitative proceeding by equal steps, e.g. the number of veins on a leaf, the position of an individual within a family, (*d*) discontinuous and quantitative proceeding by unequal steps, e.g. various botanical distributions, and the frequency within any grade of various salaries in government departments, (*e*) discontinuous and qualitative, e.g. various types of occupations.

The chief methods which have been discovered for the determination of correlation are:

(i) The four-fold table method, which applies only when the table consists of two rows and two columns, and can only legitimately be employed when the distributions are perfectly continuous and at least approximately Gaussian.

(ii) The method of contingency (giving $C_2 = \sqrt{\frac{\phi^2}{1+\phi^2}}$, where ϕ^2 is the mean square contingency). This can be applied whatever the number of cells and the nature of the distributions, but C_2 is only equal to r—the value of the correlation coefficient for the table—when the distribution is Gaussian and the number of cells is large.

(iii) The 'product moment' method. As all the cases with which we are concerned in the present paper are of tables of two rows and two columns or of three rows and three columns we need only discuss this method in relation to those cases. In both of them it is necessary that the observations should be supposed concentrated at points for each row and each column, and in the second of the cases it is necessary that the distances between these points for consecutive rows, and also for consecutive columns, should be equal, and also that the regression of each variable on the other should be linear. In the case of a two by two table, the value of r given by this method is

$$r^2 = \frac{(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)},$$

and this, for the same case, is also the value of ϕ^2†. But this is not the same as the value of C_2 which is usually taken as the measure of relationship when the method of contingency is

---

* *The Significance of the Correlation Coefficient when applied to Mendelian Distributions*, *Proc. Roy. Soc. Edin.* Vol. xxx, Part vi. (No. 34).

† See *Drapers' Research Memoirs*, Biometric Series i. p. 21. r is also the correlation between random deviations in the means of the two variates, when these deviations are expressed in terms of the standard deviations of the variates as units. See Pearson, *Phil. Trans.* Vol. 195, A, pp. 12 and 14.

employed. C_2 is approximately the same as ϕ only when ϕ is small. If $\phi = r = \cdot 5$, its value is $\cdot 45$. Thus in such tables, and for values of r not greater than $\cdot 5$, those values will not greatly differ from the values of C_2.

Now in Mendelism we are dealing with discrete entities known as gametes. Referring first to the problem of gametic correlation and taking for an example that between parent and offspring, each individual in the population considered has a gamete which possesses 0, 1 or 2 protogenic elements of a particular character. Thus this case falls under the head (*c*). Taking parents with, say, 0 protogenic elements we have an array of offspring some with 0, some with 1, and some with 2 protogenic elements, and we can find the mean number of protogenic elements in members of the array. Now the parent's gametic character is supposed to be perfectly correlated with the number of protogenic elements for the character which he possesses, and this is likewise true for the offspring. Assuming then, that if, of four variables u, v, x and y, u is perfectly correlated with x and v perfectly correlated with y, then the correlation between x and y is the same as that between u and v, we find that the gametic correlation between parent and offspring is the same as that between the number of protogenic elements possessed by the parent and the number possessed by the offspring. Thus such a table as

Number of Protogenic elements in Gamete of Parent.

Number of Protogenic elements in Gamete of Offspring.	(2)	(1)	(0)
(2)	a	d	g
(1)	b	e	h
(0)	c	f	k

is equivalent to a table between parent and offspring for the particular gametic character. Since the observations are concentrated at points within each group, these points in any row or column of the above table are equidistant. Thus the legitimate method to employ on this case is (iii). A value of C_2 as by (ii) can be found to give a measure of relationship, but as the conditions referred to in (ii) are not satisfied it will not be the true value of r. We can determine 'regression' lines from a knowledge of the means of the three rows and columns. Here we must note the fundamental disagreement between continuous variation and Mendelism. For a number of characters regression lines have now been determined of offspring on parent. Many of these are perfectly continuous regressions, that is, for any small increase or decrease in the value of the parental character we can observe a corresponding small increase or decrease in the mean value of the character for the offspring. In other cases, e.g. the petals of a flower, the regressions must proceed by steps. But a number of cases which have been shown to give perfectly continuous regressions, the Mendelians claim to depend upon a single unit character, e.g. piebaldism. All that the term 'regression' can denote in a Mendelian instance is that as we jump from 2 to 1 protogenic elements in the parent, the effect on the mean number of protogenic elements in the offspring is the same as when we jump from 1 to 0, for we cannot split up the protogenic element. Thus although we use the term 'regression' in the Mendelian cases we only do so in the signification of the last sentence, viz., that an increase or decrease of one protogenic element in the parent is accompanied by an increase or decrease of, say, m protogenic elements in the mean value for the offspring. This is fundamentally different from the continuous case, in which we can observe an increase or decrease of 1 °/ₒ of the parent's character followed by an increase or decrease of, say, n °/ₒ in the mean of the value for the offspring. The comparisons throughout this paper are to characters which appear to be continuous, but which Mendelians claim to depend upon 'unit characters.'

If for the columns in the above table,

$$\text{Mean of column } (2) - \text{Mean of } (1) = \text{Mean of column } (1) - \text{Mean of } (0),$$

this Mendelian 'regression' is linear, and can be written in the form $r\sigma_y/\sigma_x$. If, in the same way, the other 'regression' is linear, it can be written in the form $r\sigma_x/\sigma_y$. Hence r can be obtained.

In the case of somatic characters, the individuals with 1 and 2 protogenic elements are considered to be somatically the same. Thus we have individuals with 0 protogenic elements, and others with a definite number, say, m. The correlation table (two rows and two columns) formed for this case will refer to individuals some concentrated at a point representing 0 protogenic elements, and others concentrated at a point representing m protogenic elements. The correct method to apply in this case is that of (iii). As before, a value of C_2 can be found, to give a measure of relationship, but this will not be the true coefficient of correlation. The two observations in each column have a mean, and these means can be joined by a line which may be called a 'regression' line. But this is less comparable to the actual regression lines found in practice for continuous distributions than in the case of gametic correlations, and it is doubtful if a correlation determined from it is of the same significance as one ascertained from a table with many rows and columns. But whether the comparison be legitimate or not, no other method of ascertaining correlation for a two-by-two table in which the observations are concentrated at points seems to be as sound as that indicated in (iii).

Dr Brownlee in his paper applies all methods to all cases *indiscriminately* and reaches very divergent results. In a particular case (p. 477) in which the product moment method gave $r = \cdot 33$, the four-fold table method gave ·53, and the mean square contingency (C_2) ·37, the first and third of these not greatly differing. Dr Brownlee believed that "in a Mendelian instance such as this, the four-fold table seems specially applicable," but we can imagine no single case in which that method (assuming as it does a particular form of continuous distribution) is less applicable. The fact that its use leads to a value more in accordance with results found for continuous characters could surely be used as an argument against the use of the hypothesis of the 'unit-character,' but this hypothesis is the essential foundation of Mendelism. In any problem on the determination of correlation it is very necessary to understand clearly the nature of the variation dealt with. When Dr Brownlee applies all methods to a single case, he is assuming that the distributions are at the same time both continuous and discontinuous, and we can get little information of value from his results.

We need not point out in detail the many cases in which Dr Brownlee uses methods which strike at the very root of Mendelism, and which would be rejected with scorn, if they were understood, by the supporters of Mendelian principles. He has throughout stated the method by which he has obtained his many results, so that these results can easily be assessed at their true value.

We may, however, point to one or two other instances of the looseness of thought which permeates the paper. On p. 476 Dr Brownlee uses the gametic distribution $2(AA)+4(Aa)+2(aa)$. He then clubs together the (AA) and (Aa) groups and obtains $6(A)+2$ (not A), and states "in this last case, however, the distribution is markedly skew." He then proceeds to use the four-fold table method, which should never be used if there is a suspicion that the distribution is more than slightly skew. But, in fact, Dr Brownlee knows nothing whatever about the distribution. Assuming that he was dealing with continuous variation, all he has done is to divide a frequency curve which may or may not be symmetrical into two parts, and this process of division does not in the slightest alter the distribution. If the one distribution be symmetrical, the other is also. Even such extreme numbers as $99(A)$ to 1 (not A) may well represent a symmetrical distribution; the actual numbers depend solely upon the point at which the division is made.

Further evidence that Dr Brownlee appears to consider Mendelian distributions as perfectly continuous is given in his § 8, in which he states "if for the moment the distinguishing character of the hybrid and the dominant be assumed somewhat indefinite, we can make several tentative divisions, either bisecting the hybrid or dividing it into such divisions that one-fourth resembles the recessive." He then applies the four-fold table method to these cases and finds the corresponding correlations are ·44 and ·50 and concludes "again, Mendelian principles do not lead to low correlations but to figures approximately equal to those found by observation." But these "tentative divisions," by assuming continuity of distribution, throw over the whole Mendelian case.

Later on Dr Brownlee deals with the cases of two and three pairs of zygotes, looking at the problem solely as one in continuous variation. For he states "it is evident that when two and three pairs of zygotes are condensed* we do not go straight back to the normal distribution. The reason of this is that the normal surface obtained when the elements are considered separately, represents something different from the surface which is condensed into the last tables." It is difficult to find the meaning of this paragraph, but at any rate it shows that Dr Brownlee has overlooked the fact that Mendelism was rediscovered by biologists who were seeking for a theory to explain *discontinuous* variation.

Dr Brownlee devotes part of his paper to an investigation, on the Mendelian basis, into the effects of selection and of assortative mating. His confusion between continuous and discontinuous variation is evident in this part as elsewhere. As regards selection, he concluded that on a Mendelian mechanism it does not follow that the higher the parental selection the lower the correlation coefficients, the result which had been reached by Prof. Pearson by biometric methods. The question is an important one and can by no means be got over so simply as Dr Brownlee appears to have done, and we have attempted to investigate it elsewhere. We may state here, however, that our results do not agree with those of Dr Brownlee, and that we find that the assumption of Mendelian discontinuous variation leads to qualitative (and often quantitative) conclusions similar to those discovered by Prof. Pearson on the assumption of continuity of distribution. There is one point, however, in Dr Brownlee's work on this subject to which we may refer. In certain cases he obtains an approximate value of the correlation coefficient for three-by-three tables and by the product moment method when the regressions are not linear. We take, for example, the table from his p. 487, which we give below.

Parent.

		(AA)	(Aa)	(aa)	Totals
Offspring.	(AA)	$m+1$	$m+1$	—	$2m+2$
	(Aa)	2	$m+3$	$m+1$	$2m+6$
	(aa)	—	2	2	4
	Totals	$m+3$	$2m+6$	$m+3$	$4m+12$

Proceeding by the ordinary product moment method the expression found for r is

$$r=\frac{m+3}{\sqrt{2(m^2+14m+17)}}.$$

If both regressions were linear this expression should be the square root of the product of those regressions. Now the regression of offspring on parent in the above table is linear, and equal to ·5. But the regression of parent on offspring is not linear unless $m=1$.

---

* This phrase is a typical example of Dr Brownlee's looseness of language. The definition of a zygote as "the cell formed by the fusion of a male with a female gamete" seems to render impossible the operation of condensing 'pairs of zygotes.' So far as we can ascertain, Dr Brownlee merely means that the character considered depends upon a number of Mendelian couplets, and not on a single one.

If x_2, x_1 and x_0 denote the positions of the means of the (AA), (Aa) and (aa) rows we find

$$x_2 - x_1 = \frac{m+1}{m+3} \text{ and } x_1 - x_0 = \frac{2}{m+3}.$$

The above value of r will be reasonably accurate only as long as these two expressions do not greatly differ, i.e. in the neighbourhood of $m=1$. Actual values of the expressions are:

m	0	·5	1	2	10	∞
x_2-x_1	·33	·43	·50	·60	·85	1·0
x_1-x_0	·67	·57	·50	·40	·15	0

Thus, except when m is very close to unity, the expressions differ considerably, and the above formula for r cannot be taken as giving a true value of the correlation for such a table. If we approximate by taking the mean of the values of x_2-x_1 and x_1-x_0 weighted according to the frequency on which each is based, we reach results which in the present case are not far different from the values given by Dr Brownlee's formula. In another case, however, which we have worked out the difference is greater. If we do not weight, the difference may be very great.

We have by no means exhausted the possible criticisms of Dr Brownlee's work. Sufficient has been done, however, to show that his results must be interpreted with very great caution. Finally, in all cases of the determination of correlation, we would emphasize the necessity for a clear understanding of the nature of the distributions dealt with.

III. A Mendelian View of racial Heredity.

By H. J. LASKI, New College, Oxford.

The application of Mendelian principles to the problems of inter-racial heredity has not thus far met with any very conspicuous success. The work of Davenport on the mulatto is quite inconclusive in character. The incomplete pedigrees of Mudge merely defeat the end which doubtless they were intended to serve. It is not, indeed, too much to say that the endeavour to make man a complex of sharply-defined unit characters has failed, and failed completely. Even the researches of Hurst, which were received with an almost lyrical enthusiasm by the adherents of Mendelism, are not above suspicion; for post-mortem microscopic examination has revealed the presence of anterior pigment in eyes or parts of eyes which, after most careful examination, were judged free from it in life. Undeterred, however, by previous failures Dr Redcliffe Salaman* has endeavoured to prove the applicability of Mendelian theory to the problems of Jewish heredity. His paper is so characteristic an example of much recent Mendelian work that it is not entirely purposeless to examine it in some detail.

Its conclusions may, at the outset, be briefly summarised. Dr Redcliffe Salaman investigated the transmission of a Jewish facial characteristic which, somewhat mysteriously, he calls Jewishness. His method was to collect cases of intermarriage between Jew and Gentile, and thus to discover whether the resulting offspring were Jewish, non-Jewish, or intermediate in appearance. He was assisted throughout by Jewish observers who, he states, seemed desirous whenever possible of claiming the child as Jewish†. This is, perhaps, a little curious in the light of our knowledge of the Jewish aversion for intermarriage and their known desire to dissociate its consequences from anything Jewish. It is remarkable that the bias of the observers influenced their judgment so little if the personal equation is so high as Dr Salaman

---

* *Journal of Genetics*, Vol. I. pp. 273—290.

† *Loc. cit.* p. 280.

believes. The data, it should be said, relate entirely to the Jews of Northern and Central Europe and the Gentile is either English or North European. This limitation is noteworthy because there does not seem to be a specifically Jewish type among the Sephardim, or Jews of Eastern Europe and Africa. They are rarely to be distinguished from the natives of the country to which they belong, and to distinguish between the Jew and the Armenian is almost impossible.

Dr Salaman provides the following table:

Number of families	Father	Mother	Children's appearance: Gentile	Jewish	Intermediate
50	Gentile	Jewish	88	15	4
86	Jewish	Gentile	240	11	4
Total 136	—	—	328	26	8

His conclusion may be briefly summarised as follows:

(1) The Gentile features, or rather, facial expression act as practically a complete dominant to the Jewish.

(2) In the matings of the hybrids the following results were reached:

(*a*) Where the Jew is father and the mother the hybrid: in nine families there were "thirteen children undoubtedly Gentile and twelve unequivocally Jewish";

(*b*) Where the mother is Jewish and the father is the hybrid: in four families there are seven children of whom two are Gentile and five are Jewish. Dr Salaman adds (*a*) and (*b*) together and obtains a very close approximation to the Mendelian expectation of equality.

(3) He has five cases where the hybrid married a Gentile and in all of these the resulting eight children are Gentile in appearance. This is not perhaps unexpected on any theory of heredity, since three-quarters of the offsprings' blood would be Gentile and the difference between the results of (3) and (2) (*b*) is not a little curious.

(4) When the Jew who lacks the characteristically "Jewish" expression marries the markedly "Jewish" type the latter is dominant. Pedigree 6 is quoted by Dr Salaman in support of this important result.

(5) Finally, "The conclusion to which these results inevitably lead is that the Jewish facial type, whether it be considered to rest on a gross anatomical basis, or whether it be regarded as the reflection in the facial musculature of a peculiar psychical state is a character which is subject to the Mendelian law of Heredity*."

These results seem open in the very highest degree to criticism. Objection must, in the first place, be made to the categories used. I do not think it is possible to understand the nature of the character of which the transmission and behaviour were investigated. One would, I think not unfairly, expect Dr Salaman to provide us with a definition or at least a more or less exact description of the character with which he deals. It is surely a lamentable lack of scientific precision not to do so. Dr Salaman himself is not at all certain what is meant by this "Jewishness." In the earlier part of his paper he is at some pains to demonstrate the fact that the Jews cannot be considered racially pure. He quotes, indeed, with approval the statement of Weissenberg† "the Jew in an anthropological sense forms no specific type, but the facial expression is absolutely characteristic"; but immediately above the opinion of Ripley‡ who favours the existence of a Jewish type is quoted as bringing comfort to the observer. But

---

* *Loc. cit.* p. 285.

† *Archiv f. Anthropol.* Bd. XXIII. S. 347—423 and 531—579.

‡ Ripley, *Races of Europe*, pp. 393—400.

Dr Salaman's conclusion (5) makes it a matter of indifference whether we accept the opinion (it is hardly more) of Ripley or of Weissenberg. We have nothing definite or tangible upon which to seize. The Jewish nose has a prominent place in the essay, but as to its value as a possible index of racial character Dr Salaman is perfectly agnostic. We are told of a face with "rounded features, long sloping jaw, fairly developed chin which is round and not square, a good sized forehead devoid of that angularity in the temples not uncommon among Teutonic peoples," but these highly complex anatomical peculiarities, each of which from its mere description seems to demand quantitative expression, do not constitute "Jewishness" but merely accompany it. For Dr Salaman, in fact, the "Jew is a Jew because he looks like one." This, indeed, seems to be his most characteristic argument. He deals with a character which does not admit even of qualitative expression. While it is necessarily a function of the various organs of the face, and therefore an allelomorph so highly compound as to be scarcely resoluble into its component parts, it does not admit of expression in terms of them. Now, if the facial features are each one and separately a Mendelian unit, is *each* unit in the Jewish face recessive to the corresponding unit in the Gentile face so that the whole Jewish face is recessive to the Gentile? A blend would be produced even if some units were dominant over others in the general appreciation of Jewishness; but if Dr Salaman's view be justified, then every Jewish unit must be recessive to every other Gentile unit; but since this has not been tested it is purely arbitrary to assume its truth. It is not easy to see why the shape of a Gentile nose should be dominant to the shape of a Jewish nose in precisely the same way as a Jewish chin should be recessive to a Gentile one. When we ask what "Jewishness" is, we are told that it is the quality of looking Jewish, and to the question of what it means to look Jewish Dr Salaman would doubtless reply that it is to possess the quality of Jewishness. So vicious a circle of reasoning seems, scientifically, almost ludicrous, and it can hardly have value from the standpoint of anthropology. Dr Salaman appears in some degree troubled by this. "In determining the nature of so complex a character," he writes*, "the personal equation of the observer must play an important part. I have in some cases found that observers not specially acquainted with the subject, although agreeing that a given individual of the first generation is of Gentile appearance have yet felt that there was somewhere lurking in the face an expression which suggested 'Jewishness,' and there is very little doubt that such an opinion may be well founded. I have myself come across a few cases where without doubt the recessive Jewish facial expression has come to the surface as the individual grew older." Stated in other words, the first sentence would seem to imply that individuals classed as dominant may in reality constitute a blend. This has some importance in view of the suggestion made above that not all the Jewish characters may be recessive to the Gentile, and, if this be once conceded, what becomes of the correlated inheritance which is necessary for the establishment of Mendelian theory? The second sentence indicates a very real absence of gametic purity. These facts as stated by Dr Salaman throw the gravest doubt on the value of the categories used. When a definition of the character investigated is lacking because, seemingly, it can only be "felt" by the observer; when his observers are classifying as Gentile offspring with a tinge of Jewishness—surely nothing more than a Mendelian gloss for an intermediate; when Dr Salaman observes the appearance of the supposed recessive with increase of age; the nature of Gentile dominance is very far indeed from being so satisfactory as his table would seem to show. Professor Pearson tells me of a typical case of intermarriage where the Gentile relatives say the children are Jewish, the Jewish relatives Gentile, in appearance. Here we have the dominance of Jewish features when judged by a Gentile, of Gentile when judged by a Jew. Dr Salaman's observers, as he himself tacitly admits, do not feel that his categories are sufficiently precise. The personal equation plays, I think, an even more important part than Dr Salaman imagines. In Weissenberg's experiment the Jew and the Gentile differed in their estimate by 20 per cent. and to the Jew 30 per cent. and to the Gentile 50 per cent. did not seem Jewish,—a highly

---

* *Loc. cit.* p. 282.

significant series and entirely unfavourable to Dr Salaman's view. It seems to show that the Jewish conception of "Jewishness" and that of the Gentile are something very different indeed, and that an independent classification of Dr Salaman's data by a Gentile observer would not lead to the same result. The fact is,—and in research of this kind it is an absolutely fatal objection to such loose and worthless categories,—the Jew catches the Gentile, and the Gentile the Jewish, features in what is probably a blend, and is guided in his judgment by the degree in which he perceives them. It is not until the nose, the eyes, pigmentation and other Gentile-Jewish differentia are reduced to measured features that any real solution of the problem can be reached.

I think therefore, that Dr Salaman's categories are open to grave and even fatal objection. Let us turn now to the results of the investigation itself. Dr Salaman has himself admitted throughout his paper the existence of a large number of Jews who do not possess the character "Jewishness." We do not find that he has given us any facts as to the appearance of those Jews who married Gentiles and whose offspring form Table I. To the ordinary observer it would appear obvious that when a Gentile marries a Jew whose features do not possess "Jewishness" the resulting offspring would be Gentile in appearance. We require a fairly large sample of the Jewish population showing the distribution of its component elements before Dr Salaman's conclusions can be accepted. Now in Weissenberg's* experiment a proportion of his samples consisting either of 30 or 50 per cent. of his population, according to the personal equation of the observer did not appear distinguishably Jewish. It seems therefore reasonable to assume that the non-Jewish looking Jew forms a fairly large proportion of the population, larger, as I shall show, than it is possible for Dr Salaman to admit. Again, it must surely have struck him that the distribution of types in his Table I is not without significance. In the first case, ♂ Gentile × ♀ Jewish, the distribution is 88 Gentile, 15 Jewish and 4 intermediate, i.e. in the sample, classing Jewish and intermediate as Jewish, 18 per cent. were Jewish in appearance. In the second case, ♀ Gentile × ♂ Jewish, gave 240 Gentile, 11 Jewish, and 4 intermediate, i.e. with the same classification as before, 5 per cent. are Jewish. This is surely very anomalous, since on Mendelian theory there is no distinction between the two cases. Dr Salaman ascribes the appearance of the recessive Jewish types and the intermediates to (1) the Jewish bias in the observers and (2) a Jewish permeation of the English people in particular districts. In reference to (1) it is only necessary to say that the bias in favour of distinguishing the offspring as Jewish must be very great when the observer makes an incorrect distinction between two types which Dr Salaman tells us cannot be mistaken. His inconsistency here is destructive of his entire position. If this incorrectness be once admitted, it is sufficient to cast doubt upon the entire investigation. In (2) Dr Salaman, I take it, means that there is a Jewish ancestry even among the Gentile inhabitants. The Gentile inhabitant would therefore be an imperfect dominant on this theory; but if there be this ancestry it is difficult to see why a Jewish looking Jew should always be dominant over a non-Jewish looking Jew, who would be often only a Jew in name and not a gametic Jew. The complete data should, I think, be tabulated before this exceedingly hypothetical explanation can be accepted as satisfactory. The mating of the hybrids with pure types seems to Dr Salaman entirely Mendelian in result, the approximate equality being the Mendelian expectation. I think the evidence I shall now adduce in criticism of this conclusion is sufficient to render it untenable.

When the father is a Jew and the mother the hybrid and conversely, the distribution of types in the resulting offspring is 15 Gentile to 17 Jewish, the Mendelian expectation being, as we have said, equality. But Dr Salaman's analysis is very far from sufficient. What was the appearance of the hybrid parent? This, the really crucial point, is left a mystery. The following scheme indicates the possibilities which might have occurred.

---

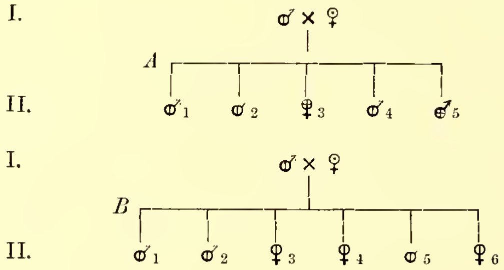
* *Globus*, Vol. XCVII. 6, 9, 10, 1910.

Hybrid Jewish in appearance × Jew Jewish in appearance,
„ „ „ × Jew intermediate in appearance,
„ „ „ × Jew non-Jewish in appearance.

Hybrid intermediate in appearance × Jew Jewish in appearance,
„ „ „ × Jew intermediate in appearance,
„ „ „ × Jew non-Jewish in appearance.

Hybrid Gentile in appearance × Jew Jewish in appearance,
„ „ „ × Jew intermediate in appearance,
„ „ „ × Jew non-Jewish in appearance.

It is evident from this scheme that the problem is immensely more complex than is indicated by Dr Salaman. His belief that the Jew who appears recessive carries an inhibiting factor which prevents the appearance of "Jewishness" works in any case against his view, for when crossed with pure Gentile the offspring could not possibly be Jewish on any theory. But we can go further than this. Is the non-Jewish type so markedly recessive to the Jewish type? While I thoroughly distrust any such vague category as "Jewishness," I have yet endeavoured to the best of my ability to apply it to certain data of my own and see in how far the results are consonant with Dr Salaman's. The pedigrees here given refer in every case to families where intermarriage has not, within knowledge, occurred.

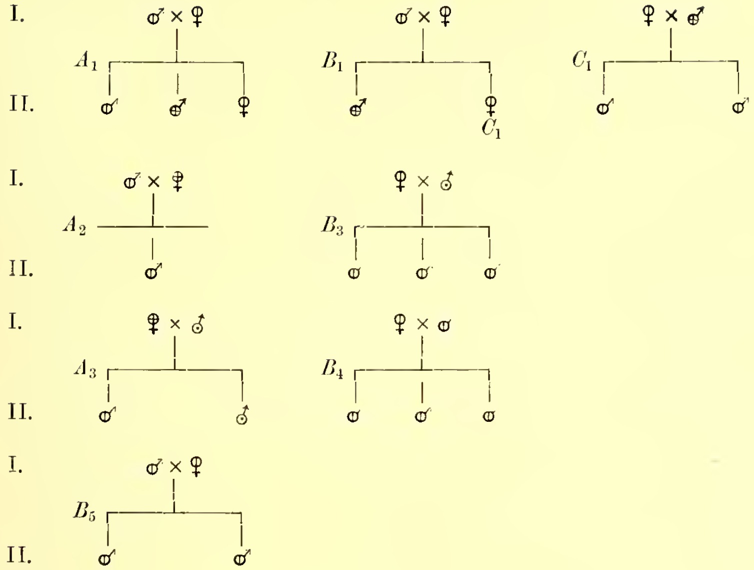


⊙ = Marked Jewish type.
⦶ = Non-Jewish type.
⊕ = Intermediate type.

Fig. I.

These pedigrees indicate, I think, a dominance of the non-Jewish type. I give the results of the marriage of some of the above offspring on the opposite page.

These show absolute divergence from Dr Salaman's conclusions. In every case the cross between the distinctly non-Jewish type and the distinctly Jewish type gives dominance to the former. In the case of the intermediate type no definite conclusion seems possible. The result indeed is what the biometrician is beginning almost to expect in Mendelian work on man, that while one pedigree may admit of a Mendelian interpretation others no less certainly contradict it. If we use Dr Salaman's classification, it is obvious that the non-Jewish looking Jew cannot possibly be regarded as a recessive. If it be regarded as DR, then in the crosses $DR \times DR$ the absence of RR is inexplicable. Now it does not seem to me that this divergence disproves the applicability of Mendelism to the facial expression in man, but it does demonstrate that it is purely idle to use vague categories of this kind, and then claim that any theory can be



I have also

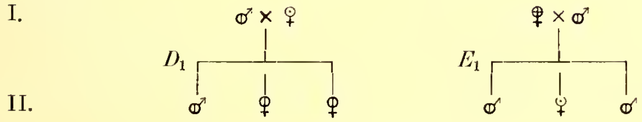
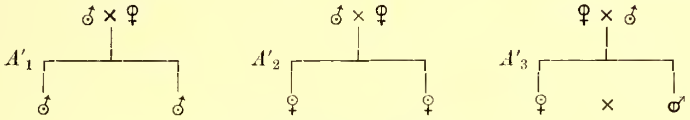


Fig. II.

demonstrated by such use. I have in my possession several pedigrees illustrating the result of crossing pure Jew with pure Gentile. These give



⊙ = Distinctly Jewish.

⦶ = Pure Gentile birth.

⊕ = Intermediate appearance.

Fig. III.

In each case the cross of a distinctly Jewish type with a no less distinctly Gentile type produces offspring either of Jewish or of intermediate appearance. It is, in fact, characteristic of the application of Mendelism to problems such as these, that imperfect categories and incomplete analysis should superficially render soluble a problem which, on a more intimate dissection, does not yield its secret to the Mendelian enthusiast.

In fact, this problem, as every genetic problem, is to be solved only by the method of quantitative analysis, that is, by the methods of biometry. We must know the numerical distribution of the various types of Jew in a population sufficiently random in character to represent the general Jewish population. We must actually measure the "Jewishness" by analysing it into its component factors and measuring them by accepted and specially devised anthropometric methods—such as the nasal index, the cephalic index, the profile angle, etc., together with any number of new facial measurements which it would be quite easy to devise, and also pigmentation determinations. Where, as in man, we cannot be certain of the gametic constitution of an individual, we can only guess at it by an inquiry as to purity of the ancestry and the somatic characters both in Gentile and Jew. This, especially in regard to non-Jewish looking Jews, has been wholly omitted by Dr Salaman in each individual case. From such knowledge we can, by well-known biometric methods, calculate the probable character of the offspring. The result of such an investigation may be to confirm Mendelian theory—as to that I make no assertion. But we stand now, and until this fact has penetrated into the Mendelian consciousness we shall continue to stand, in the position indicated by Weldon ten years ago—"the fundamental mistake which vitiates all work based on Mendel's method is the neglect of ancestry and the attempt to regard the whole effect upon the offspring produced by a particular parent as due to the existence in the parents of particular structural characters; while the contradictory results obtained...show clearly enough that not only the offspring themselves but their race, that is, their ancestry, must be taken into account before the result of pairing them can be predicted."

IV. On "Cancer Houses," from the Data of the late Th. Law Webb, M.D.

By KARL PEARSON, F.R.S.

THE data dealt with in the present paper contain a record of the house distribution of 377 cases of cancer occurring between 1837 and 1910 in the Madeley registration subdistrict. They include both cases of sarcoma and carcinoma. It has not been possible to separate the two, for in many cases the entry is merely cancer. The data include the exact position of the house, its water supply; the age, occupation, and date of death of subject, and the organ affected by the growth. The data consist, besides the record of cases, of large scale maps of the district upon which every inhabited house is marked in with blue wash and every house in which a case of cancer followed by death has occurred in the given period with red wash. A red cross marks existing cases of cancer in the living. The collection was formed by Th. Law Webb, Esq., M.D.

The deaths from cancer are as follows:

1831—1840: [20?]*
1841—1850: 14
1851—1860: 20
1861—1870: 48
1871—1880: 41
1881—1890: 77
1891—1900: 81
1901—1910: 88.

The number of cancer deaths has thus increased very considerably, but the population has increased† and its average age has no doubt very considerably increased, so that it is not possible to say whether this increase of cancer deaths marks an actual increase of cancer.

* Based on four years of the decade only.

† Not very markedly, and lately it has fallen again.

There appear to be very few large houses in the district, most being by their descriptions of the cottage type. Certainly those in which multiple cases of cancer have occurred are often quite small. It is not very easy to determine from the maps the exact number of inhabited houses, but it appears to be about 2865*. Twelve cases of cancer appeared to be living in the district at the date of the return; these are not included in our number of cases. Taking 377 death cases in all and supposing these to be distributed at random among the 2865 houses we should anticipate according to the formulæ of my article (*Biometrika*, Vol. VIII. p. 410).

330·6 houses with one case,	in round numbers		331
21·7 houses with two cases,	,,	,,	22
·95 houses with three cases,	,,	,,	1
·03 houses with four cases,	,,	,,	0

A total is thus obtained of 354 houses with 377 cases and there will be 2511 houses with no cases.

The actually recorded numbers appear to be as follows: 2523 houses with no cases, 315 houses with one case; 20 houses with two cases; six houses with three cases; and one house with four cases. The identification of the houses is not always quite clear, but I think I have erred, if at all, on the side of reducing the multiple houses, e.g. I have supposed nine cases in which the house was not known to have occurred in non-multiple houses. On the other hand, when the sufferer lived in a house up to death, going perhaps to a general hospital or the workhouse just before death, I have reckoned the cancer as developed in that house.

If we now substitute in the general formula

$$\chi^2 = S\left\{s^2 \frac{(p_s-\bar{p}_s)^2}{\bar{p}_s}\right\},$$

we find

$$\chi^2 = \frac{(16)^2}{331} + \frac{4\times(2)^2}{22} + \frac{9\times(5)^2}{1} + \frac{16\times(\cdot 97)^2}{\cdot 03}$$

$$= \text{about } 727.$$

The probability P is therefore infinitesimal, being widely outside any existing tables for χ^2 and P. We may therefore say that if these numbers be correct the distribution cannot possibly be a random one. Six houses with three cases each and one with four are wholly beyond the bounds of the possible, assuming cancer to be distributed at random among the houses.

I have made a second estimate of this improbability. I have assumed that Dr Law Webb's district actually coincides with the Madeley subregistration district and that a count of houses on the Ordnance Map is likely to be defective †.

The 1841 Census gives the number of houses as 1802 and the population as 8732. The 1851 Census gives 2006 houses and a population of 9848; the 1861 Census provides 2154 houses and a population of 10,733. In 1871 we have 2291 houses and a population of 10,535. In 1881 the numbers are 2359 houses and 10,026 persons. In 1891 we have 2228 houses and 8825 persons. In 1901, 9129 persons and 2196 houses. But it is not quite certain that the boundaries remained absolutely the same. In 1911 the population was 8859 and the *inhabited* houses 2037. I take 2000 as an average number of houses, and if 9000 to 10,000 be the average population then 4·5 to 5·0 are the average number of inhabitants per house.

---

* Taking about eight cancer deaths a year for the last three decades, this suggests an *average* recognised duration of the disease of about eighteen months.

† The estimate of houses must be very elastic, many houses in the period have come into being, large numbers have ceased to be: the 2865 is a maximum limit.

For $n = 377$ $m = 2000$, we find:

$$\left.\begin{aligned} p_1 &= 312{\cdot}4 \\ p_2 &= 29{\cdot}4 \\ p_3 &= 1{\cdot}84 \\ p_4 &= 0{\cdot}086 \end{aligned}\right\} \text{giving a total number of cases} = 377{\cdot}06,$$

$$\begin{aligned} \chi^2 &= \frac{(2{\cdot}6)^2}{312{\cdot}4} + \frac{4\,(9{\cdot}4)^2}{20} + \frac{9\,(4{\cdot}16)^2}{1{\cdot}84} + \frac{16\,({\cdot}914)^2}{{\cdot}086} \\ &= {\cdot}01 + 17{\cdot}67 + 84{\cdot}65 + 155{\cdot}42 \\ &= 257{\cdot}75. \end{aligned}$$

P lies therefore outside any table, i.e. the probability that such a distribution could arise from random sampling is only one in many many millions. It seems therefore clear that the reduction of our number of houses by 800 makes no substantial difference in the improbability of our result. The houses with three cases are quite sufficient in themselves—even if we neglected the four-case house—to make the distribution indefinitely improbable.

We have next to consider the sources of this improbability.

The possibilities are:

(*a*) that these "cancer houses" are larger and contain more inhabitants than the others, thus they would be more likely to have multiple cases.

(*b*) that constitutions liable to cancer are hereditary and so the "cancer house" marks merely the presence of a "cancer family."

(*c*) that certain houses have been inhabited by persons following the same occupation, and that "cancer houses" are those inhabited by persons with a bad occupational mortality for cancer.

(*d*) that some houses by their environment, or by the presence of some organism render their occupiers more liable to cancer.

I propose to examine the seven instances of three or more cases from the problem of these possibilities.

(A.) No. 1. G—— V——. The first case that occurred in this house was that of a station-master, aged 28, cancer of rectum. He died in 1855. The second case was that of a schoolmistress, who died, aged 64, in 1889 from cancer of the uterus. The third case was that of the servant to a nonconformist minister and his wife who were living next door. This servant attended the second case, that of the schoolmistress, in her last illness, and died in 1890, aged 57, of malignant ulceration of the intestines with perforation. The fourth case lived to her 78th year in this house in association with the third, but removed before her death elsewhere. She died at 88 years in 1899 of cancer of the liver.

(B.) No. 2. G—— V——. This is the *next* house to A above. The first recorded case in this house was that of the nonconformist minister referred to in A. He died about 1870, aged about 60, of cancer of the stomach. The second case was that of his wife who died in 1889, aged 71, of cancer of the rectum. The third case was the wife of the man who followed the widow of the nonconformist minister. She died of cancer of the breast in 1881, barely a year after the minister's wife.

We have thus in 26 years seven cases associated with two small houses, the servant to the occupants of one living in the other. It is clear that neither the size of these houses, the blood relationship of their occupants, nor any similarity of their occupations will account for these multiple cancer cases.

(C.) S—— R——. In this house the servant died in 1862, aged 69, of cancer of the breast. The second case was that of a bricklayer who died in 1885, aged 73, of carcinoma of first part of sigmoid flexure of colon. The third case was that of a woman, whose husband had left her, who died here aged 55, in 1906, of secondary carcinoma.

The adjacent house to this was also a multiple cancer house, being credited with two cases, one of carcinoma of the stomach and one of cancer of the womb.

(D.) C—— C——. The first case is that of a man aged 56, who died in 1869 of cancer of the breast. His brother had also died, but not in this house, of cancer of the rectum. On his death he was succeeded in his official position by his clerk, and the latter also retained the same housekeeper. She died in 1885, aged 57, of cancer of the stomach, and the clerk just mentioned died in the same house in 1894, aged 52, of epithelioma of the tongue.

(E.) I—— B——, F—— M——. The first case in this house was that of a pattern maker who died of cancer of rectum in 1875, aged 53. His father had died of cancer before him. The second case was that of his widow who died in 1885, aged 69, of cancer of the rectum. The third case was that of their daughter in 1888, aged 39, after three years' suffering from cancer of the left breast.

This case may show an hereditary constitutional tendency, but this does not account for *both* husband and wife dying of it. That must either be looked upon as fortuitous, or else it is evidence in favour of (*d*).

(F.) G—— B——, T—— O——. The first case was that of a woman occupant who died in 1877, aged 52, of carcinoma of the large intestine. The second case was that of a man of superior social class who died in 1895, aged 73, of carcinoma of the stomach. The third case was that of the mother-in-law of the second case, who lived in the house with her daughter; she died in 1902, aged 84, of carcinoma of the liver*.

(G.) M—— W——, B—— B——. There is a group of five cases here, all in very close proximity. It is not clear how far they are in separate parts of the same original building, but, I think, we may assume three to have been. The first case is one of cancer of the breast in a woman aged 44, in 1862; the second of malignant disease of the stomach in the wife of a nail-maker, she died aged 77 in 1894, and the last one in a woman, who had suffered from epithelioma of hip in 1887, but died of bronchitis. The other two cases were those of a woman who died in 1897, aged 62, of malignant disease of the intestines, and of a blacking-maker, who died, aged 52, of cancer not further defined, in 1865. Undoubtedly this group (G.) is less definite than the others, because the information is less easy to interpret. It did not seem desirable to omit it, for we have clearly a considerable number of cases within a very circumscribed area, but the exact division of the tenements is not clear. If it be omitted, it will not substantially alter the extreme improbability of the general distribution.

From the above specification of the individual "cancer houses," it seems improbable that occupation or heredity constitution had anything to do with the multiple cases; in only one instance do we find parents and offspring; and sex and occupations are most varied. Nor do we believe that the size of house can be concerned in the matter, as it is quite clear that we are dealing with small tenements. The cancer maps of the district show roughly a fairly uniform distribution of the cancer cases, in a district which is largely industrial. But the fact remains that the frequency of multiple case-houses is exceedingly improbable on the basis of a random

---

* A case of cancer of the breast occurred in the next house in 1876, a woman aged 74. Dr Law Webb writes of the three-case house: "Water, *pump*, often flooded with surface water, when stormy weather; soon dry in summer, drains in close proximity."

distribution of cases. Dr Law Webb's data provide sufficient evidence to justify a demand for a thorough investigation of the subject, such as is not feasible in the case of the individual medical man. They do not finally demonstrate that cancer is more frequent in one house than a second, but they do justify a complete inquiry into the possibility that "cancer-houses" are not wholly a myth, in other words, that immediate environment is in the long run a factor of the frequency of cancer. What is needed is a record of the houses in which cancer has occurred, say for the last 50 or 60 years in (i) a practically fully developed urban district, (ii) a completely agricultural district, and (iii) an industrial area such as occurs frequently in Lancashire or Yorkshire, etc., with relatively small factories, mines or works spread out over a rural district. The examination of the certificates of death of such districts, the careful preparation of "spot maps," and the record of occupations and relationships ought to be a perfectly straight-forward matter, and if it be carried out,—whether it justifies the inference to be drawn from the present data, or does not,—I think Dr Law Webb must be considered as a pioneer in the inquiry.

Experiment. It occurred to me on reading this paper through after completion, that the improbability of multiple cases as measured by the P derived from χ^2, however much it may appeal to the statistician, might not sufficiently impress the medical mind as demonstrating the non-random character of this cancer distribution. Above all a medical man thinking of cancer as a common disease might fail to appreciate, on reading the individual details of the multiple cases, their extreme improbability taken as a whole.

Accordingly, at the suggestion of my colleague, Dr David Heron, I arranged for the drawing at random of 377 cases out of 2000 possibilities. What we want is something equivalent to drawing 377 times at random a ball out of 2000 balls numbered 1 to 2000 in a bag, each ball being replaced after drawing.

The experiment was arranged in the following manner. A series of numbers of four figures having no exact square root, cube root or reciprocal was taken, and the figures in the seventh decimal place of the first and second and in the tenth decimal place of the third were written down. These formed the last three figures of the numbers. To obtain the first figure in the number, in one case the last figure of the cube, i.e. the twelfth was taken, and in three other cases, the tenth figure of the cube. The whole series of 377 numbers were thus taken directly from tables of cubes, square roots, cube roots and reciprocals (Barlow's). Out of the numbers thus obtained, those beginning with 0, 1, 2, 3, 4 were reckoned as belonging to the first thousand, and those beginning with 5, 6, 7, 8 and 9 to the second thousand; 0000 counted however as 2000. Thus: 3004=0004=4, but 5004=1004. We thus had equal chances for every number from 1 to 2000, provided there be no bias in taking numbers consecutively* out of such tables.

The results were as follows:

	Theory	1st Experiment	2nd Experiment	3rd Experiment	4th Experiment
p_1 ...	313	315	321	333	321
p_2 ...	29	31	28	22	28
p_3 ...	2	0	0	0	0
p_4 ...	0	0	0	0	0

The absence of triplets led me to suppose some bias in the tables tending in favour of more uniform distribution than a mere random drawing provides. Accordingly a fifth experiment

---

* Subject to the omission of numbers with perfect square or cube roots etc., as stated above.

was made in the more lengthy manner of drawing 1508 times a card out of a pack containing 20 cards, numbered in duplicate 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The card was returned, and the pack shuffled and the 1508 draws recorded. Each successive four gave a number which was treated exactly as in the previous four experiments. The results were:

	Theory	Experiment	Cancer
p_1 ...	313	316	315
p_2 ...	29	26	20
p_3 ...	2	3	6
p_4 ...	0	0	1
χ^2	—	approx. 6	approx. 258

It will be seen that the χ^2 of experiment is essentially within the bounds of probability ($P = \cdot 2$, say), while the cancer-data give a value of χ^2 which is 43 times as large. For the previous set of experiments the χ^2 for the third is about 25, and for the other three about 18—larger values than occur in the shuffling experiment—but of a quite different order to the cancer value. Many arguments—all houses not being of same age, some houses pulled down, and so forth—might be used to account for the multiple cancer cases, but I think these data certainly justify a fuller inquiry into the whole question. They provide some evidence, of more value than mere impression, that the hypothesis of "cancer" houses is worthy of a fuller consideration.

V. Hybridisation of Canaries.

Note on the Communication by C. L. W. Noorduyn *to the Members of the Genetics Congress held in Paris from September* 18—23, 1911.

By A. RUDOLF GALLOWAY, M.B., C.M., M.A.

At the Genetics Congress held at Paris in September, 1911, Mr C. L. W. Noorduyn, of Groningen, quoting *Biometrika*, Vol. VII., Nos. 1 and 2, gave from my paper on "Canary breeding" the "Table of Canary Hybrids bred since 1891, arranged to show Plumage Colour."

This table indicates the rarity of lightly variegated and "clear" hybrids, and Mr Noorduyn regretted that I did not specify the "five or six hybrids" out of the total of 526 that were not bred from ♂ wild bird × ♀ canary, but from the reverse mating of ♂ canary × ♀ wild bird.

He believed the proportion of lightly variegated hybrids to be greater from ♂ canary × ♀ wild bird than from the reciprocal mating.

In view of the prominence so kindly given to this table by Mr Noorduyn, it is necessary for me to say that the information desired is really included in the paper, but refers to five or six exceptional *matings*, not to five or six individual birds as stated by Mr Noorduyn.

The exceptional matings will be found at the following pages of my paper:

Mating (1), page 36, I. 1904. 1.
„ (2), „ 39, IV 3. 1905.
„ (3), „ 39, IV 4. 1904–07.
„ (4), „ 39, IV 4. 1906.
„ (5), „ 39, IV 4. 1908. 1.
„ (6), „ 39, IV 4. 1908. 2.

Mr Noorduyn will also find the plumage-colour of the canaries and, in all known cases, of their parents mentioned—information the absence of which he also regretted. The eye-colour of the canaries is also mentioned in every case, and of their parents in every case except one which was unknown.

The matings cited above were for the purpose of testing the inheritance of "pink" as against "dark" eye, a difference which can be seen as soon as the egg hatches. A large number of chicks die before the plumage appears, so that only those that survived and became fully fledged birds are mentioned in the table of hybrids quoted at the Genetics Congress by Mr Noorduyn. Of these none showed any appreciable amount of variegation, two only being *slightly* variegated.

As a matter of fact all my light hybrids, including the "clear" siskin-canary, figured in my paper have been bred from ♂ wild bird × ♀ canary.

Mr Noorduyn also reported at the Genetics Congress the breeding of a cinnamon hybrid by me, and evidently considers it to be a "reversion" to the cinnamon grandmother. With this idea, he bred canary hens from cinnamon × yellow parents, supposing that these hens, on being mated to wild birds would produce "clear" hybrids. In this he was bitterly disappointed, for he bred 27 hybrids all quite dark.

This experience is fully in accordance with the results given in my table of canary hybrids. I am afraid the production of clear hybrids is not quite such a simple matter as a reversion of this sort would explain. Otherwise clear hybrids would not be the great and valuable rarities that they are at the present day, after the labours of generations of the keenest light hybrid breeders. Indeed Mr Noorduyn's belief, that the mating of ♂ yellow canary to ♀ goldfinch results in most of the hybrids being variegated or lightly variegated, is an old one, as may be seen by reference to the earliest—1709—edition of Hervieux's "Nouveau Traité des Serins de Canarie," p. 241:—

"On met ordinairement la femelle de Serin, et le mâle Chardonnet, Linote, ou autres; mais pour moy je suis pour qu'on mette le contraire; sçavoir, le mâle Serin, et la femelle Chardonnet, Linote, ou autres, parce que le mâle ordinairement race plus que la femelle; ainsi les mulets qui sortiront d'un mâle Serin seront plus beaux, et chanteront mieux, que s'ils sortoient seulement de la femelle."

Also at page 246 we read:—

"Vous voyez par cette maniere, que vous avez de beaux mulets à esperer; car le mâle Serin, qui race pour l'ordinaire plus que la femelle, donera beaucoup de Blancs, et la femelle Chardonn donant un peu de ces differentes couleurs feront des mulets d'un prix inestimable."

VI. An attempt to ascertain the Prevalence of Syphilis in a large Urban Population.

Notice of FRITZ LENZ: Über die Verbreitung der Lues, speziell in Berlin, und ihre Bedeutung als Faktor des Rassentodes. *Archiv für Rassen- und Gesellschafts-Biologie.* May and June, 1910. Leipzig. B. G. Teubner. pp. 306 *et seq.*

The underlying ideas of the memoir are to use (1) the relative statistics in two places, and (2) the death rates from certain causes in order to estimate the number of persons attacked by the diseases which end in death from those causes. These ideas are excellent, and ultimately many valuable results may be reached, but they are extremely difficult to apply without making assumptions so wide that the conclusions become too rough to afford definite information. The particular case dealt with by Lenz is that of syphilis, which results in many cases in general paralysis of the insane, locomotor ataxia, etc., and the statistical problem involved in his work may be set out as follows: Given that syphilis is notified in Copenhagen, and that the deaths from general paralysis in Copenhagen and Berlin are known, find the proportion of males in Berlin who have at one time or another had syphilis. To solve such a problem one requires to know the age incidence of the deaths from paralysis, the age incidence of the syphilis notifications, the total populations in age groups (and the births) in both cities for several years, and some information as to the average after lifetime of syphilitics. Lenz neglects these preliminaries and boldly takes a short cut which assumes that if syphilis were notified in Berlin the notifications would bear the same proportion to the deaths from paralysis as the notifications in Copenhagen bear to the deaths from paralysis there. The syphilitic population is found by multiplying this number of notifications by the expectation of life at age 15. We have cut down Lenz's problem and have merely tried to indicate his method; he adjusts some details on the way, but the errors in the method we have just indicated exist, we think, in his work, though at times they are obscured. The weakness is that the proportionate method will not hold because the populations vary and the age incidence in the two cities can hardly be the same, while the use of the expectation at age 15 is incorrect, because this would be the youngest age at attack, and if expectation is used at all it should be for the average age of attack. Besides this the expectation of life of a syphilitic is probably less than that of the population as a whole.

These criticisms appear to us to dispose of his applications, but although the problem is an actuarial one of great difficulty it is certainly worth examination, and even though we do not agree with all his work we feel that much credit is due to Lenz for calling attention to the possibility of solving the problem of the extent of syphilis in this manner.

W. P. E.

VII. On the General Theory of the Influence of Selection on Correlation and Variation.

BY KARL PEARSON, F.R.S.

(1) In 1901 a paper of mine was read before the Royal Society and shortly afterwards issued in the *Philosophical Transactions** dealing with this matter. Very shortly afterwards I found out that the formulae therein developed did not depend for their accuracy on the frequencies being Gaussian in character. All the main conclusions were deducible without this limitation,

---

* Vol. 200 A, pp. 1—66.

and proofs of the formulae concerned have been given by me for many years in college lectures without making this assumption. I have, however, stated the fact once or twice in print that the formulae are general, and it seems desirable to reproduce my proofs (from lecture notes) at the present time as some doubt seems to have been cast on the generality of the formulae.

(2) I begin with a preliminary proposition, which is fairly familiar. Let x_1, $x_2 \ldots x_n$ be n variables, continuous or discrete but quantitatively measurable; $\bar{x}_1$, $\bar{x}_2 \ldots \bar{x}_n$ their means, σ_1, σ_2, ... σ_n their standard deviations, r_{pq} the correlation coefficient of the pth and qth variables supposed found by the product moment formula. Then: What is the best linear function u of x_1, $x_2 \ldots x_n$ such that an $(n+1)$th variable x_{n+1} will have a maximum correlation $r_{ux_{n+1}} = \rho$, say, with u?

Let
$$u = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$
$$= S(cx).$$

Clearly
$$\bar{u} = S(c\bar{x}),$$
$$N\rho\sigma_u\sigma_{n+1} = \Sigma(u-\bar{u})(x_{n+1}-\bar{x}_{n+1}) = S\Sigma c_p(x_p-\bar{x}_p)(x_{n+1}-\bar{x}_{n+1})$$
$$= S\Sigma(c_p\sigma_p\sigma_{n+1}r_{p,n+1})$$
$$\rho = \frac{1}{\sigma_u} S(c_p\sigma_p r_{p,n+1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{i}).$$
$$N\sigma_u^2 = \Sigma(u-\bar{u})^2 = S\Sigma\{c_p^2(x_p-\bar{x}_p)^2\} + 2S\Sigma c_p c_q (x_p-\bar{x}_p)(x_q-\bar{x}_q)$$
$$= NS(c_p^2\sigma_p^2) + 2NS(c_p c_q r_{pq}\sigma_p\sigma_q),$$

or:
$$\sigma_u^2 = S(c_p^2\sigma_p^2) + 2S(c_p c_q \sigma_p \sigma_q r_{pq}).$$

But if ρ be a maximum, we must have:
$$\frac{d\rho}{dc_p} = 0,$$
for all values of p.

Or, we reach type-equations of form:
$$\frac{1}{\sigma_u}\sigma_p r_{p,n+1} - \frac{\rho\sigma_u}{\sigma_u^2}\frac{d\sigma_u}{dc_p} = 0,$$

i.e.
$$r_{p,n+1} = \frac{\rho}{\sigma_p\sigma_u}\{c_p\sigma_p^2 + S(c_q\sigma_p\sigma_q r_{pq})\}$$
$$= \frac{\sigma\rho}{\sigma_u}(c_1\sigma_1 r_{1p} + c_2\sigma_2 r_{2p} + \ldots + c_p\sigma_p + \ldots + c_n\sigma_n r_{np})$$
$$= b_1 r_{1p} + b_2 r_{2p} + \ldots + b_p + \ldots + b_n r_{np} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{ii}).$$

Where:
$$b_q = \rho\frac{c_q}{\sigma_u}\sigma_q.$$

Now equations of type (ii) are easily solved by aid of the determinant
$$R = \begin{vmatrix} 1, & r_{1,2}, & r_{1,3}, & \ldots r_{1,n+1} \\ r_{2,1}, & 1, & r_{2,3}, & \ldots r_{2,n+1} \\ \ldots & \ldots & \ldots & \ldots \\ r_{n+1,1}, & r_{n+1,2}, & r_{n+1,3}, & \ldots 1 \end{vmatrix}.$$

We have
$$b_q = -R_{q,n+1}/R_{n+1,n+1}$$
where $R_{q,n+1}$ is the minor of qth row and $n+1$th column.

Hence it follows that:
$$u = -\frac{\sigma_u}{\rho}\left\{\frac{R_{1,n+1}}{R_{n+1,n+1}}\frac{x_1}{\sigma_1} + \frac{R_{2,n+1}}{R_{n+1,n+1}}\frac{x_2}{\sigma_2} + \ldots + \frac{R_{n,n+1}}{R_{n+1,n+1}}\frac{x_n}{\sigma_n}\right\} \quad \ldots\ldots\ldots (\text{iii}).$$

Substitute also in (i):

$$\rho = \frac{1}{\rho} \overset{n}{\underset{1}{S}} (b_p r_{p,n+1}),$$

or:

$$\rho^2 = -\overset{n}{\underset{1}{S}} \left(\frac{R_{p,n+1}}{R_{n+1,n+1}} r_{p,n+1} \right),$$

but:

$$R = r_{1,n+1} R_{1,n+1} + r_{2,n+1} R_{2,n+1} + \ldots + R_{n+1,n+1}.$$

Hence:

$$\rho^2 = 1 - \frac{R}{R_{n+1,n+1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)},$$

$$1 - \rho^2 = 1 + \overset{n}{\underset{1}{S}} \frac{(R_{p,n+1} r_{p,n+1})}{R_{n+1,n+1}} = \frac{R}{R_{n+1,n+1}}.$$

Hence we have the following results:

$$\rho = \pm \sqrt{1 - \frac{R}{R_{n+1,n+1}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$

$$u = -\frac{\sigma_u}{\rho} \left\{ \overset{n}{\underset{1}{S}} \frac{R_{p,n+1}}{R_{n+1,n+1}} \frac{x_p}{\sigma_p} \right),$$

and

$$\sigma_{n+1} \sqrt{1 - \rho^2} = \sigma_{n+1} \sqrt{\frac{R}{R_{n+1,n+1}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)},$$

and is the reduced average variability of x_{n+1} for given values of $x_1, x_2, \ldots x_n$.

The probable value of x_{n+1} is given by

$$x_{n+1} - \bar{x}_{n+1} = \frac{\sigma_{n+1}}{\sigma_u} \rho (u - \bar{u})$$

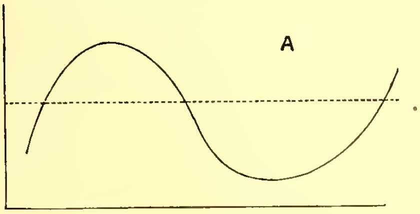
$$= -\overset{n}{\underset{1}{S}} \frac{R_{p,n+1}}{R_{n+1,n+1}} \frac{\sigma_{n+1}}{\sigma_p} (x_p - \bar{x}_p) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$

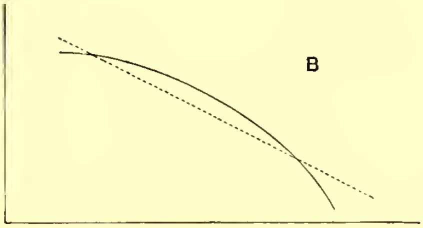
i.e. the ordinary multiple regression formula. It is the "best value," i.e. the mean value of x_{n+1}, for given $x_1 \ldots x_n$, on the assumption that we correlate x_{n+1} with that *linear* function of the n variables, which gives the highest degree of relationship as measured by the correlation coefficient. The method is absolutely independent (i) of Gaussian theory, (ii) of the continuity or discreteness of the variables, but it does assume that linearity applies within the degree of useful approximation*.

Another point deserves re-emphasising here. Equation (iv) gives ρ^2, hence whether ρ be plus or minus, the errors of random sampling will always give a positive ρ^2. It follows therefore that even if ρ be zero, we should find on making a number of trials in each case a positive value of ρ^2; let the mean value of this be $\bar{\rho}^2$, then unless the actual value of ρ^2 is significant not as compared with zero, but with $\bar{\rho}^2$, no value ought to be laid on the actual value of ρ^2. The

---

* The general linearity ought to be tested in all such cases. Nothing can be learnt of association by assuming linearity in a case with a regression line (plane, etc.) like *A*, much in a case like *B*. To *A* we must apply multiple correlation-ratios, the theory of which is being developed at the present time and will shortly be published.





probable error of ρ is $\cdot 67449\,(1-\rho^2)/\sqrt{N}=\cdot 67449/\sqrt{N}$ if ρ be really zero; then if $\bar{\rho}$ be the mean value of ρ we should expect ρ to be

$$\bar{\rho}\pm\cdot 67449/\sqrt{N}$$

if ρ be truly zero. In other words we must consider the question of whether the observed ρ is significant compared with this.

I have found the value of $\overline{\delta\rho^2}$, i.e. the mean increment of ρ^2 due to errors of random sampling, but I postpone its consideration in the hope of still further reducing its determinantal expression in the general case.

Let us now apply these results to the general theory of selection. Suppose we have m variates $x_1, x_2, \ldots x_m$, with means $\bar{x}_1, \ldots \bar{x}_m$, standard deviations $\sigma_1, \sigma_2, \ldots \sigma_m$ and correlations given by R the determinant

$$\begin{vmatrix} 1\,, & r_{12}\,, & r_{13}\,, \ldots r_{1m} \\ r_{21}\,, & 1\,, & r_{23}\,, \ldots r_{2m} \\ \cdots & \cdots & \cdots\cdots \\ r_{m1}\,, & r_{m2}\,, & r_{m3}\,, \ldots\ 1 \end{vmatrix}$$

in the usual way.

Now we may suppose a selection to be made out of this variate complex of a subpopulation $x_1, x_2 \ldots x_n$, $n<m$, given by the means:

$$h_1,\ h_2,\ \ldots\ h_n,$$

the standard deviations

$$s_1,\ s_2,\ \ldots\ s_n$$

and the correlations:

$$\begin{vmatrix} 1\,, & \rho_{12}\,, & \rho_{13}\,, \ldots \rho_{1n} \\ \rho_{21}\,, & 1\,, & \rho_{23}\,, \ldots \rho_{2n} \\ \cdots & \cdots & \cdots\cdots \\ \rho_{n1}\,, & \rho_{n2}\,, & \rho_{n3}\,, \ldots\ 1 \end{vmatrix},$$

the selected population having values consistent with those of the unselected population.

We can then ask:

(i) How will this modify the mean and standard deviation of a non-selected variate x_p, $p>n<=m$?

(ii) How will this modify the correlation r_{pq} between two non-selected variates x_p and x_q, p and $q>n$ and $<=m$?

(iii) How will this modify the correlation r_{pt} of a non-selected and a selected variate $p>n$ and $<=m$, while $t<=n$? These are the fundamental problems of the influence of selection on variation and correlation.

(i) Let us take x_{n+1} as the non-selected organ and let the characters of one of the selected group be given by $x_t=\bar{h}_t+\xi_t$.

Then x_{n+1} will differ from its probable mean value by some quantity η_{n+1} and by (vii) we have

$$x_{n+1}=\bar{x}_{n+1}+\eta_{n+1}-\overset{n}{\underset{1}{S}}\left\{\frac{R_{t,\,n+1}}{R_{n+1,\,n+1}}\,\frac{\sigma_{n+1}}{\sigma_t}\,(\bar{h}_t+\xi_t-\bar{x}_p)\right\}.$$

Or taking the mean value, $\tilde{x}_{n+1}$, $S(\eta_{n+1})=0$ and $S(\xi_t)=0$, and

$$\tilde{x}_{n+1}=\bar{x}_{n+1}-\overset{n}{\underset{1}{S}}\left\{\frac{R_{t,\,n+1}}{R_{n+1,\,n+1}}\,\frac{\sigma_{n+1}}{\sigma_t}\,(\bar{h}_t-\bar{x}_p)\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)}.$$

This establishes the first proposition* of my *Phil. Trans.* memoir, namely: that selection about the means with any variabilities gives the same mean value for a non-selected but correlated variate as if all the selected variates had been taken at their mean selected values.

We have clearly:

$$x_{n+1} - \tilde{x}_{n+1} = \eta_{n+1} - \overset{n}{\underset{1}{S}} \left(\frac{R_{t,n+1}}{R_{n+1,n+1}} \frac{\sigma_{t+1}}{\sigma_t} \xi_t \right) \text{ ....(ix).}$$

Now if we are dealing with N manifolds of variates:

$S(\eta^2_{n+1})/N =$ a standard deviation indicated by $\bar{\sigma}_{n+1}$,

$S(\xi_t^2)/N =$ standard deviation of selected tth organ $= s_t^2$,

$S(\xi_t \xi_{t'})/N = s_t s_{t'} \rho_{tt'}$,

$S\{\eta_{n+1}\xi_t) = 0$ because the tth variate is not selected in reference to the $(n+1)$th variate. Hence if we square (ix) and call Σ_{n+1} the resulting variability of x_{n+1} due to the selection of the n-variates, we have

$$\Sigma^2_{n+1} = \sigma^2_{n+1} \left\{ \frac{\bar{\sigma}^2_{n+1}}{\sigma^2_{n+1}} + \overset{n}{\underset{1}{S}} \left(\frac{R^2_{t,n+1}}{R^2_{n+1,n+1}} \frac{s_t^2}{\sigma_t^2} \right) + 2S \left(\frac{R_{t,n+1} R_{t',n+1}}{R^2_{n+1,n+1}} \frac{s_t s_{t'}}{\sigma_t^2} \rho_{tt'} \right) \right\}.$$

But as we have already seen η_{n+1} is not correlated with ξ_t. Hence we shall find the value of $\bar{\sigma}^2_{n+1}$ by putting all the s_t's zero, or by concentrating the selection at a single value of a manifold. It is therefore the value of Σ^2_{n+1} for an array of x_{n+1} for definite values of $x_1, x_2 \ldots x_n$, i.e. by (vi) $\bar{\sigma}_{n+1}$ equals $\sigma_{n+1} \sqrt{\dfrac{R_{(n+1)}}{R_{n+1,n+1}}}$, where $R_{(n+1)}$ is the determinant of $n+1$ rows and columns. Thus finally:

$$\Sigma^2_{n+1} = \sigma^2_{n+1} \left\{ \frac{R_{(n+1)}}{R_{n+1,n+1}} + \overset{n}{\underset{1}{S}} \left(\frac{R^2_{t,n+1}}{R^2_{n+1,n+1}} \frac{s_t^2}{\sigma_t^2} \right) + 2S \left(\frac{R_{t,n+1} R_{t',n+1}}{R^2_{n+1,n+1}} \frac{s_t s_{t'}}{\sigma_t^2} \rho_{tt'} \right) \right\} \text{......(x).}$$

This is in complete agreement with the value given as Equation (xlv) of my *Phil. Trans.* memoir†, and deduced there on the assumption of a Gaussian frequency distribution.

(ii) I now turn to the second of my problems the correlation between the $(n+1)$th and $(n+2)$th variables. In this work $R'_{n+2,n+2}$ denotes the determinant of n rows and columns bordered by the $(n+2)$th variate correlations, those of the $(n+1)$th being omitted. Clearly as in (ix)

$$x_{n+2} - \tilde{x}_{n+2} = \eta_{n+2} - \overset{n}{\underset{1}{S}} \left(\frac{R'_{t,n+2}}{R'_{n+2,n+2}} \frac{\sigma_{n+2}}{\sigma_t} \xi_t \right) \text{ ....(xi).}$$

Multiply (ix) and (xi) sum and divide by the number of the manifolds, N; then if $\rho_{n+1,n+2}$ be the correlation after selection of the $(n+1)$th and $(n+2)$th variates, we have:

$$\Sigma_{n+1} \Sigma_{n+2} \rho_{n+1,n+2} = \frac{S(\eta_{n+1}\eta_{n+2})}{N} + \sigma_{n+1}\sigma_{n+2} \left\{ \overset{n}{\underset{1}{S}} \frac{R_{t,n+1} R'_{t,n+2}}{R_{n+1,n+1}, R'_{n+2,n+2}} \frac{s_t^2}{\sigma_t^2} \right.$$

$$\left. + S \left(\frac{R_{t,n+1}}{R_{n+1,n+1}} \frac{R'_{t'n+2}}{R'_{n+2,n+2}} \frac{s_t s_{t'}}{\sigma_t \sigma_{t'}} \rho_{tt'} \right) + S \left(\frac{R_{t',n+1}}{R_{n+1,n+1}} \frac{R'_{t,n+2}}{R'_{n+2,n+2}} \frac{s_t s_{t'}}{\sigma_t \sigma_{t'}} \rho_{tt'} \right) \right\} \text{......(xii).}$$

As before $\dfrac{S(\eta_{n+1}\eta_{n+2})}{N}$ will be given by the mean partial product moment of the $(n+1)$th and $(n+2)$th variates for *constant* values of the n variates concentrated at their selected means. This can be found without appeal to the Gaussian frequency surface by extending the formula (vii) to $n+1$ variates.

---

* Vol. 200, A, p. 13.

† *Phil. Trans.* Vol. 200, A, p. 17.

Let Δ be the determinant of $(n+2)$ rows and columns, Δ_{pq} the minor corresponding to the pth column and qth row component. Then the regression equations for x_{n+1} and x_{n+2} on the remaining variates of the $(n+2)$ group are:

$$x_{n+1}-\bar{x}_{n+1}=-\frac{\Delta_{n+2,n+1}}{\Delta_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_{n+2}}(x_{n+2}-\bar{x}_{n+2})-\overset{n}{\underset{1}{S}}\left\{\frac{\Delta_{t,n+1}}{\Delta_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_t}(x_t-\bar{x}_t)\right\}$$

and

$$x_{n+2}-\bar{x}_{n+2}=-\frac{\Delta_{n+1,n+2}}{\Delta_{n+2,n+2}}\frac{\sigma_{n+2}}{\sigma_{n+1}}(x_{n+1}-\bar{x}_{n+1})-\overset{n}{\underset{1}{S}}\left\{\frac{\Delta_{t,n+2}}{\Delta_{n+2,n+2}}\frac{\sigma_{n+2}}{\sigma_t}(x_t-\bar{x}_t)\right\}.$$

Now, when we put $x_1 \dots x_n$ constant, the coefficients of $x_{n+2}-\bar{x}_{n+2}$ and $x_{n+1}-\bar{x}_{n+1}$ and the partial regression coefficients of x_{n+1} on x_{n+2} and x_{n+2} on x_{n+1} for constant 1 to n variates, or the square root of their product is the partial correlation coefficient, i.e.

$$_{1,2,3\dots n}\rho_{n+1,n+2}=\bar{\rho}_{n+1,n+2},$$

say for brevity; therefore

$$\bar{\rho}_{n+1,n+2}=\sqrt{\frac{\Delta^2_{n+1,n+2}}{\Delta_{n+1,n+1}\Delta_{n+2,n+2}}}=-\frac{\Delta_{n+1,n+2}}{\sqrt{\Delta_{n+1,n+1}\Delta_{n+2,n+2}}}\quad\dots\dots\dots\dots\dots\dots(\text{xiv}),$$

a well-known and familiar form[*].

Now let us look at the standard deviations of the arrays of the $(n+1)$th and $(n+2)$th variates for absolutely selected values of the n variates.

The variability of the array of the $(n+1)$ variate is given by (vi), i.e.

$$\bar{\sigma}_{n+1}=\sigma_{n+1}\sqrt{\frac{R}{R_{n+1,n+1}}}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(\text{xv}),$$

and of the $\overline{n+2}$th variate

$$\bar{\sigma}_{n+2}=\sigma_{n+2}\sqrt{\frac{R'}{R'_{n+2,n+2}}}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(\text{xvi}).$$

But

$$R=\Delta_{n+2,n+2},$$

$$R'=\Delta_{n+1,n+1}$$

while clearly $R_{n+1,n+1}=R'_{n+2,n+2}=$ the second minor of Δ obtained by leaving out both $(n+1)$th and $(n+2)$th rows and columns. Hence we have:

$$R_{n+1,n+1}=R'_{n+2,n+2}=\Delta_{n+1,n+1,n+2,n+2},$$

and

$$\frac{\Sigma(\eta_{n+1}\eta_{n+2})}{N}=\bar{\sigma}_{n+1}\bar{\sigma}_{n+2}\bar{\rho}_{n+1,n+2}=-\sigma_{n+1}\sigma_{n+2}\frac{\Delta_{n+1,n+2}}{\Delta_{n+1,n+1,n+2,n+2}}\dots\dots\dots(\text{xvii}).$$

Thus finally we have from (xii):

$$\Sigma_{n+1}\Sigma_{n+2}\rho_{n+1,n+2}=\sigma_{n+1}\sigma_{n+2}\left\{\frac{-\Delta_{n+1,n+2}}{\Delta_{n+1,n+1,n+2,n+2}}+\overset{n}{\underset{1}{S}}\left(\frac{R_{t,n+1}R'_{t,n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}\frac{s_t^2}{\sigma_t^2}\right)\right.$$
$$\left.+S\left(\frac{R_{t,n+1}R'_{t',n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}+\frac{R_{t',n+1}R'_{t,n+2}}{R_{n+1,n+1}R'_{n+2,n+2}}\right)\frac{s_ts_{t'}}{\sigma_t\sigma_{t'}}\rho_{tt'}\right\}\dots\dots\dots\dots(\text{xviii}),$$

which is in complete agreement with the value found from the Gaussian hypothesis[†].

(iii) Lastly we require the correlation $\rho_{t,n+1}$ between a selected and a non-selected variate, $t <= n$. Turning back to (ix) multiply by ξ_t, sum and divide by N, then:

$$s_t\Sigma_{n+1}\rho_{t,n+1}=\frac{S(\eta_{n+1}\xi_t)}{N}-\left(\frac{R_{t,n+1}}{R_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_t}s_t^2\right)-S\left(\frac{R_{t',n+1}}{R_{n+1,n+1}}\frac{\sigma_{n+1}}{\sigma_{t'}}s_ts_{t'}\rho_{tt'}\right).$$

[*] Pearson, *Phil. Trans.* Vol. 200, A, p. 10, Equation (xxvii).

[†] See *Phil. Trans.* Vol. 200, A, p. 17, Equation (xlvi).

The first summation on the right is zero; hence

$$\Sigma_{n+1}\,\rho_{t,\,n+1} = -\sigma_{n+1}\left\{\frac{R_{t,\,n+1}}{R_{n+1,\,n+1}}\,\frac{s_t}{\sigma_t} + S\left(\frac{R_{t'n+1}}{R_{n+1,\,n+1}}\right)\frac{s_{t'}}{\sigma_{t'}}\,\rho_{tt'}\right\} \text{ ....(xix).}$$

This with a slight difference of notation is the result obtained on the Gaussian hypothesis*.

The above proofs justify the statement that the general selection formulae given by me are independent of any Gaussian assumption. They are really peculiar to the general idea of the manifold linear variate u which gives the maximum correlation coefficient of an $(n+1)$th variate with n other variates. They do not involve any idea of continuity or any hypothesis as to the nature of the selected means, standard deviations and correlations beyond the fundamental assumption that the selected population really exists inside the unselected population. There need be no hesitation therefore in applying these formulae to any cases whatever in which the correlation coefficients have valid application at all.

---

* *Phil. Trans.* Vol. 200, A, p. 17, Equation (xlvii). S in our present notation is a summation in (xix) of every value, but t, of t'. In the *Phil. Trans.* paper S_1 is a summation for *all* values of t': see p. 18.

VIII. On a Fallacious Proof of Sheppard's Correction.

The ordinary proofs of Sheppard's corrections for the moments are somewhat lengthy and depend entirely on the principle of high contact at the terminals. Mr G. U. Yule in his recent *Theory of Statistics*, p. 208, has given a proof in a few lines which is absolutely independent of this principle, and which from its very simplicity is likely, if not criticised, to be generally adopted. Unfortunately it is wholly fallacious. The error lies in the words "the correlation between X and δ is zero, for the mean value of δ is zero for every interval." What Mr Yule should have said is that the correlation between his Z and δ is zero, and he should have reached the conclusion

$$\sigma_1^2 = \sigma^2 + \tfrac{1}{12}c^2,$$

and not

$$\sigma_1^2 = \sigma^2 - \tfrac{1}{12}c^2,$$

for he is really working out the mean square for the histogram and not the true figure. He would thus have failed to obtain the correct value, which he does not appear to recognise arises solely from the fact that the 'trapezettes' *cannot* be treated as rectangles. In the case of curves of frequency without terminal contact, Sheppard's corrections are *not* the proper ones, and their general adoption without regard to their limitations is to be deprecated. Such adoption is directly encouraged by a fallacious proof of the above character.

K. P.

GROWTH, VARIABILITY AND CORRELATION IN YOUNG TROUT.

By J. W. JENKINSON, M.A., D.Sc.

THE growth of the organism has been made the subject of numerous quantitative investigations, and amongst those who have dealt with the growth of animals, including the human being, the names of Quetelet, Roberts, Bowditch, Minot and Boas are well known.

From their investigations it appears that there are certain features which are characteristic of growth in general.

The first of these is the fact that the growth-rate is not uniform but suffers a gradual diminution during the life of the organism. This is well established by many researches [Minot (Rabbit), Quetelet, Bowditch, Roberts, Boas (the human being), Potts and Minot (Chickens), Semper (*Limnaea*), Vernon (*Strongylocentrotus*)]. This is also known to be true of the various parts separately, though in the parts the rate of growth and the change of that rate differ a good deal. Moreover there may be more than one period of rapid growth followed by first a rapid, then a slow decline in the growth-rate, during the lifetime of one and the same organism. In the human being for example there is one such period or cycle before birth, another cycle during the first few years after birth, while the growth-rate rises once more, only once more to diminish, about the time of puberty. Robertson has recently suggested that, on the assumption that some autocatalytic chemical process underlies the phenomenon of growth, the maximum growth-rate in each such cycle should occur when the total growth occurring in that cycle is half accomplished. It may be added that results calculated on this theory are in fairly close agreement with those obtained by actual measurement.

A second feature which characterises growth is the relation between change in growth-rate and change in the variability of the organism or of its parts, the latter rising and falling with the former. While it is true that the observed decrease of variability has been in some cases ascribed to a selective death-rate, yet Boas has urged that the relation referred to must necessarily exist, however much it may be obscured by changes in variability due to other causes.

And thirdly Boas has urged and brought forward evidence to support his contention that the value of the coefficient of correlation between various organs diminishes with the diminution of the growth-rate. The data are taken from human growth and it appears that the correlation coefficient, like the variability, falls after birth, but rises again at about the time of puberty, thereafter decreasing once more.

All these points are of so much general interest and importance that it appeared to me to be worth while to endeavour to secure further data for an enquiry into the same three problems, and the animal that soon suggested itself for the purpose was the trout.

The trout is eminently suitable since a large number of parts can be readily and fairly accurately measured. Moreover there is fortunately, at Bibury in the Cotswolds, not more than thirty miles from Oxford, an excellent hatchery owned and superintended by Mr A. Severn. Here the young trout live under very favourable conditions. The temperature, a most important matter of course, is practically constant, for the hatchery is supplied directly from a spring that rises on the spot, and the temperature of this spring does not vary more than a degree the whole year round. The animals are kept free from disease, are abundantly supplied with food and their death-rate is very low; 75 °/₀, often more, come to maturity; 80 °/₀ of the eggs of the American Rainbow trout hatch out, 90 °/₀ of those of the English brown trout. While of those Rainbow trout that hatch out less than 10 °/₀ as a rule die within three months after hatching. There is however far more loss at this period amongst the English fish.

I accordingly requested Mr Severn to be good enough to supply me with some young trout at different stages after hatching, and he has very kindly provided me with the necessary material for my work. I should like to take this opportunity of thanking him for the trouble he has taken in the matter. I have used the American Rainbow trout. All the young whose measurements are given in this paper were the offspring of one pair of parents. The fish were spawned and the eggs fertilized on February 14th, 1910. In the hatchery they were kept in a separate box. The young alevins, with the yolk-sac still large, hatched out about the 19th of March; all did not hatch on the same day. The young were sent in to Oxford as follows:

I.	April 4th . .	200
	(These still had the yolk-sac.)	
II.	April 18th . .	209
III.	May 2nd . .	198
IV.	May 17th . .	200
V.	May 31st . .	161

Immediately on their arrival the fish were killed in formol.

I have thus had at my disposal nearly one thousand young trout, all born of the same parents, and killed at five successive fortnightly stages after hatching, the period of growth examined thus comprising ten weeks. Mr Severn has been kind enough to furnish me with the following particulars of the mortality that occurred in this particular lot. Five per cent. died after hatching during the first fortnight, that is before any were sent to me. When the fish were about three weeks old (that is, between Stages I and II) a trifling number died of dropsy and at the same time about 5°/₀ died which were deformed. Otherwise there was scarcely any loss at all.

The dead individuals were not sent to me and I only measured the survivors. Whether the death-rate is high enough of itself to bring about the diminution of variability which, as we shall see, actually occurs, is a question to be discussed later. But whether that is so or not I think it may be in any case worth while to describe briefly the growth and the change that occurs in the growth-rate, and in the variability both of the whole body and of different parts, both absolute and relative, as well as the correlations that exist between these parts.

I have measured the following—total length from the tip of the nose to the extremity of the tail, diameter of the eye, length of head as determined by the posterior margin of the operculum, length of the bases (properly speaking the projections of these bases on the longitudinal axis of the body) of the median fins —anterior dorsal, posterior dorsal, ventral and caudal—breadth of the body at the level of the anterior end of the anterior dorsal fin, breadth of the tail, and position of the pectoral, pelvic, anterior dorsal, posterior dorsal and ventral fins as determined by the anterior end of the base of attachment in each case. The ventral fin lies just behind the anus as a rule, so that the position of the former can be taken usually as marking the position of the latter, but there may be a gap between the two. In these cases the posterior border of the anus has been separately measured. Lastly the number of bars or spots in the middle row on the animal's side has been counted. In all cases the measurements were made on the right-hand side of the animal.

I begin with the *absolute measurements*. The mean values are given in millimetres in Table I, and in the same table the growth-rate at each successive stage, as expressed by the percentage increment of growth.

The general increase of all parts needs of course no comment. The only exception is the number of spots which is established at once in Stage II and undergoes practically no increase. What is of interest is the diminution of the growth-rate. This is evident everywhere. Generally speaking the decrease is more rapid in the early stages than in the later—which is in harmony with the results obtained by other observers—but in some cases there is irregularity. In the eye the growth-rate sinks at Stage III, to rise again at Stage IV and sink once more in the final stage. In the posterior dorsal fin there is a marked increase at Stage III, followed by a great decrease, and this by a slight rise. A slight, perhaps hardly significant rise, can be detected in the final stage of the total length.

TABLE I. *Giving in tenths of millimetres mean values (M) of total length and of the absolute dimensions of various parts in the five Stages (I—V), the differences (d) between the successive mean values and these differences expressed as percentages (°/₀) of the mean value at the beginning of each interval. The intervals are all of the same magnitude (14 days) with the exception of that between Stages III and IV which was 15 days. The corresponding percentage differences have been reduced accordingly and the corrected values inserted in brackets.*

	Total Length			Diameter of Eye			Length of Head			Length of Anterior Dorsal Fin			Length of Ventral Fin			Length of Posterior Dorsal Fin		
	M	*d*	°/₀	*M*	*d*	°/₀	*M*	*d*	°/₀	*M*	*d*	°/₀	*M*	*d*	°/₀	*M*	*d*	°/₀
I	184·03 ± 1·15	—	—	14·58 ± ·12	—	—	36·05 ± ·34	—	—	22·04 ± ·24	—	—	17·32 ± ·28	—	—	14·27 ± ·31	—	—
II	238·75 ± ·85	54·72	29·73	18·95 ± ·06	4·37	29·97	53 38 ± ·23	17·33	38·07	32·72 ± ·25	10·68	48·46	25·70 ± ·31	8·38	48·38	15·50 ± ·18	1 23	8·62
III	284·65 ± ·70	45·90	19·23	21·51 ± ·06	2·56	13·51	68·14 ± ·21	14·76	27·65	*44·84 ± ·34	12·12	37·04	*34·10 ± ·32	8·40	32·68	19·88 ± ·14	4·38	28·26
IV	319·23 ± ·77	34·58	12·15 (11·34)	26·21 ± ·05	4·70	21·85 (20·39)	79·25 ± ·20	11·11	16·30 (15·21)	*42·45 ± ·43	− 2·39	− 5·33	*32·51 ± ·37	− 1·59	− 4·66	22·06 ± ·16	2·18	10·97 (10·24)
V	358 00 ± 1·47	38·77	12·14	28·08 ± ·08	1·87	7·13	87·34 ± ·30	8·09	10·21	38·14 ± ·13	− 4·31	−10·15	29·58 ± ·23	−2·93	− 9·01	24·78 ± ·23	2·72	12·33

	Length of Caudal Fin			Breadth of Caudal Fin			Breadth of Body			Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin	Number of Spots
	M	*d*	°/₀	*M*	*d*	°/₀	*M*	*d*	°/₀	*M*	*M*	*M*	*M*	*M*	*M*	*M*
I	14·52 ± ·27	—	—	21·98 ± ·31	—	—	†	—	—	37·16 ± ·30	95·96 ± ·41	86 42 ± ·44	126·84 ± ·58	126·00 ± ·60	145·01 ± ·60	‡
II	25·16 ± ·30	10·64	73·28	34·31 ± ·43	12·33	56·10	43·08 ± ·16	—	—	52·13 ± ·20	119·54 ± ·33	109·76 ± ·34	158·47 ± ·45	157·76 ± ·47	172·34 ± ·45	9·12 ± ·06
III	37·44 ± ·21	12·28	48·81	48·50 ± ·36	14·19	41·36	51·62 ± ·22	8·54	19·82	62·18 ± ·19	139·60 ± ·33	127·01 ± ·33	183·31 ± ·43	182·62 ± ·43	196·55 ± ·45	9·75 ± ·06
IV	44·56 ± ·19	7·12	19·02 (17·75)	59·90 ± ·36	11·40	23·51 (21·94)	63·03 ± ·23	11·41	22·10 (20·62)	70·02 ± ·19	152·28 ± ·40	144·29 ± ·31	203·30 ± ·49	203·20 ± ·48	220·14 ± ·53	9·53 ± ·05
V	48·85 ± ·38	4·29	9·63	63·25 ± 58	3·35	5·59	67·20 ± ·35	4·17	6·62	76·18 ± ·28	172·85 ± ·64	161·60 ± ·59	231·24 ± ·81	230·49 ± ·85	250·40 ± ·92	9·55 ± ·05

* In Stages III and IV an error was made in many cases in the determination of the posterior end of the anterior dorsal and of the ventral fin bases. The mean values are consequently too large and hence the differences negative. The error was discovered and avoided in measuring Stage V.

† The presence of the yolk sac prevents the accurate measnrement of the breadth of the body in Stage I.

‡ The spots do not appear till Stage II.

TABLE II.

Variabilities of Absolute Values.

σ (in tenths of Millimetres).

	Total Length	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin	Number of Spots
I	24·13 ± ·81	2·60 ± ·08	7·09 ± ·24	4·90 ± ·17	5·56 ± ·20	4·38 ± ·22	5·73 ± ·19	6·40 ± ·22	—	6·26 ± ·21	8·58 ± ·29	9·21 ± ·31	11·74 ± ·41	12·32 ± ·42	8·54 ± ·42	—
II	18·26 ± ·60	1·32 ± ·04	5·03 ± ·16	5·33 ± ·18	6·58 ± ·22	3·65 ± ·13	6·49 ± ·21	9·25 ± ·30	3·42 ± ·11	4·37 ± ·14	6·98 ± ·23	7·22 ± ·24	9·46 ± ·32	10·07 ± ·33	9·08 ± ·32	1·15 ± ·04
III	14·55 ± ·49	1·21 ± ·04	4·48 ± ·15	*7·10 ± ·24	*6·64 ± ·23	2·95 ± ·10	4·46 ± ·15	7·45 ± ·25	4·56 ± ·16	3·92 ± ·13	6 89 ± ·23	6·80 ± ·23	8·80 ± ·30	9·00 ± ·30	9·26 ± ·32	1 12 ± ·04
IV	16·13 ± 54	1·10 ± ·04	4·09 ± ·14	*8·97 ± ·30	*7·76 ± ·26	3·44 ± ·12	3·88 ± ·13	7·60 ± ·25	4·81 ± ·16	4·01 ± ·13	8·30 ± ·28	6·60 ± ·22	10·03 ± ·35	10·01 ± ·34	11·13 ± ·37	1·03 ± ·04
V	27·70 ± 1·04	1·45 ± ·06	5·72 ± ·21	2·54 ± ·09	4·36 ± ·16	4·18 ± ·16	7·05 ± ·27	11·00 ± ·41	6·55 ± ·25	5·30 ± ·20	12·15 ± ·45	11·19 ± ·42	15 23 ± ·57	16·06 ± ·60	17·02 ± ·65	1·02 ± ·04

C. of V.

I	13·11 ± ·44	17·80 ± ·56	19·68 ± ·69	22·22 ± ·81	32·08 ± 1·27	30·69 ± 1·68	39·44 ± 1·49	29·10 ± 1·08	—	16·86 ± ·59	8·95 ± ·30	10·66 ± ·36	9·25 ± ·32	9·78 ± ·33	5·89 ± ·29	—
II	7·65 ± ·25	6·96 ± ·21	9·43 ± ·30	16·30 ± ·56	25·62 ± ·92	23·52 ± ·89	25·81 ± ·89	26·96 ± ·93	7·94 ± ·26	8·23 ± ·27	5·86 ± ·19	6·58 ± ·22	5·97 ± ·20	6·38 ± ·21	5·27 ± ·19	12·56 ± ·43
III	5·11 ± ·17	5·63 ± ·19	6·57 ± ·22	*15·83 ± ·55	*19·47 ± ·70	14·79 ± ·51	11·96 ± ·40	15·36 ± ·53	8·83 ± ·31	6·30 ± ·21	4·94 ± ·16	5·35 ± ·18	4·80 ± ·16	4·91 ± ·16	4·71 ± ·16	11·51 ± ·41
IV	5·05 ± ·17	4·21 ± ·15	5·16 ± ·18	*21·12 ± ·74	*23·88 ± ·85	15·59 ± ·55	8·71 ± ·29	12·69 ± ·42	7·63 ± ·25	5·73 ± ·19	5·45 ± ·18	4·58 ± ·15	4·93 ± ·17	4·93 ± ·17	5·05 ± ·17	10·87 ± ·42
V	7·74 ± ·29	5·16 ± ·21	6·55 ± ·24	6·65 ± ·23	14·74 ± ·55	16·87 ± ·67	14·43 ± ·56	17·39 ± ·67	9·75 ± ·37	6·96 ± ·26	7·03 ± ·26	6·92 ± ·26	6·58 ± ·25	6·97 ± ·26	6·80 ± ·25	10·68 ± ·42

* For the reasons stated in the note to Table I. these values are untrustworthy.

In the second place the growth-rate differs considerably—both initially and finally—in the various parts. The maximum observed is 73·28 for the second stage of the length of the caudal fin, the minimum 5·59 for the last stage of the breadth of the caudal fin (the negative values obtained for the two last stages of the anterior dorsal, and of the ventral fins, are due to the excessive value given to the means, owing to an error in measurement, in the two previous stages). And lastly, the rate of decline of the growth-rate differs. Thus between Stages II and III there may be a decrease of 10 (total length), 11 (head, anterior dorsal fin, ventral fin), 15 (breadth of caudal fin) or 25 (length of caudal fin). Similarly the decrease in the final stage varies from 5 to 16.

Apart from the irregularities exhibited in the cases alluded to there is no evidence that in the period under observation more than one growth cycle is involved.

We turn next to the *variabilities* of these various dimensions (Table II, in which both standard deviations and coefficients of variability are given). The coefficient of variability (C. V.) may be taken as giving the best measure. Two things are at once evident. First there is a regular decrease during the first four stages (length of anterior dorsal fin and length of ventral fin must be ruled out for the reasons already given). Like the decrease in the growth-rate this decrease of variability is also more rapid at first than later on, at least this is generally so. There are however exceptions. In the posterior dorsal fin (length), caudal fin (length) and caudal fin (breadth) there is nearly as much or even more difference between the variabilities in the third and second stages as there is between the variabilities in the second and first. It is not easy to associate these anomalies with anomalies in the change of the growth-rate of the corresponding parts. It is true that the growth-rate changes irregularly in the posterior dorsal fin: but in the caudal fin length and breadth the difference between the growth-rate from the first to the second stages and that from the second to the third stages is very great. Again, the growth-rate in the eye increases temporarily between Stages III and IV, but there is no corresponding increase in the variability.

Secondly, in respect of all characters except the number of spots there is a slight increase of variability in the final stage, an increase which is not, as a rule, associated with any increase in the growth-rate. This slightly increased variability never however becomes as large as the variability in the first stage and is usually a good deal smaller. The question is whether this general decrease of variability in the *whole* period is to be ascribed in whole or in part to the decrease in the growth-rate, for, since there was a certain mortality, it may be reasonably contended that this was selective and caused the diminished variability.

The 5 °/。 that, as stated, died in the first fortnight, need not be considered, but some attempt must be made to make an allowance for the 5 °/。 that died between Stages I and II.

In Table III are given the corrected values for the mean, standard deviation and coefficient of variability of the total length in the last four stages on the assumption that this 5 °/₀ (say 40 individuals) should have been uniformly spread over these stages. The extra individuals have been placed (α) in the lowest class only of the distribution, and (β) equally in the lowest and the highest class. The variabilities are of course increased, and more by the former than by the latter method. The former perhaps is fairer since Mr Severn says those that died were deformed. But even so, the series shows the same decrease up to Stage IV and a rise in Stage V, and, what is more important, the variability in no case becomes equal to that found in Stage I. Only by including all 40 in the lowest class of Stage II can the variability be made even to approximate to that seen in Stage I. The values are given at the foot of the tables. In this case of course the values for the coefficient of variability in Stages III and IV remain unaltered.

TABLE III.

Total Lengths.

	Values as given in the tables			CORRECTED VALUES					
				(α) By the inclusion of 10 extra individuals in the lowest class			(β) By the inclusion of 5 extra individuals in the lowest and 5 in the highest class		
	M	σ	$C.\ V.$	M	σ	$C.\ V.$	M	σ	$C.\ V.$
I	184·03	24·13	13·11	—	—	—	—	—	—
II	238·75	18·25	7·65	236·22	21·29	9·01	238·16	20·20	8·48
III	284·65	14·55	5·11	282·19	18·92	6·70	284·11	16·88	5·94
IV	319·23	16·13	5·05	316·26	19·39	6·13	319·11	18·88	5·92
V	358·00	27·70	7·74	351·99	36·21	10·29	356·66	33·55	9·41

If all 40 extra individuals are put in the lowest class in Stage II, then

$M=228{\cdot}36, \quad \sigma=26{\cdot}44, \quad C.\ V.=11{\cdot}58.$

I admit that the method is unsatisfactory, but in the absence of any actual knowledge of the size of those that died, it is the best that can be done. The decrease of variability is then perhaps not wholly due to a selective death-rate.

Is it then to be attributed to decrease in growth-rate? This seems possible, but the absence of any marked increase in growth-rate in the last stage suggests that still some other factor may be concerned; and again there is the apparent absence of any connection between the irregularities in the change of growth-rate and anomalies in the alteration of variability.

I had thought at one time that the increased variability in Stage V might be due to imperfect sampling in the collection of the material, smaller, weaker individuals having been perhaps left at the bottom of the box in earlier stages, and only gathered in with the last lot. But I have been assured by the keeper of the hatchery, who himself collected them, that he made no such selection; the fish were taken quite at random.

I can therefore offer no explanation. The change is certainly not to be attributed to any alteration in the environment.

We pass now to the *relative dimensions*. These have been expressed as percentages of the total length.

The means of these indices and their standard deviations are given in Table IV.

First we may notice that the proportions of many organs remain constant throughout or nearly so—the eye, the three median fins, body-breadth, position of pectoral and pelvic fins. There is, however, a slight decrease in the proportion (except in body-breadth), which means, in the case of the position of the fins, that they move a little nearer the head, and this is true also of the positions of the median fins and of the anus. The hinder end of the operculum, on the other hand, moves further away from the head end: in Stage I it is in front of the pectoral fin, but subsequently more and more behind it. But these changes in the proportions of the parts are not very serious.

It is only when we look at the length and breadth of the caudal fin that we see how great may be the changes that occur in their index value. The length increases from 4 °/₀ to nearly 14 °/₀ of the total body-length, the breadth from 12 °/₀ to 18 °/₀ (with a subsequent slight decrease). As has already been shown these two dimensions grow very rapidly indeed in the first period, twice or more than twice as rapidly as the whole body, and this excess in the growth-rate is maintained up to the fourth stage. By the time Stage V is reached the growth-rate of these parts is smaller than that of the total length, and their index value has also decreased.

The figures in Table I also show that the head grows faster than the whole body, although in this case the excess is not so great. And again in the first two stages (the figures for the others are unreliable) growth-rate is high in the anterior dorsal and ventral fins (Table I) and the indices are correspondingly increasing.

The variabilities (σ) of the indices are usually small. Generally speaking they exhibit a progressive decrease. So in the eye, head-length, length of median fins, breadth of body, position of paired and median fins, and of anus. In some of these cases (position of pelvic fin, and of median fins and of anus) there is an appreciable rise of variability in the final stage, though the value of the variability in the first stage is never reached. The two exceptions to this regular decrease of variability are seen in the length and breadth of the caudal fin. Here the variability first increases, then decreases only to rise again to more than its original value in passing from the fourth to the fifth stage.

As we know already these two dimensions are anomalous in other respects. The mean index value rises at first rapidly then more slowly, but finally decreases a little; the growth-rate is very high at first but experiences finally a sharp fall.

TABLE IV.

Mean Values (M) of Indices in per cents.

	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin
I	7·91 ± ·04	19·40 ± ·08	11·91 ± ·08	9·09 ± ·11	7·01 ± ·14	4·33 ± ·06	11·64 ± ·11	—	20·23 ± ·06	52·22 ± ·19	47·01 ± ·11	68·46 ± ·17	68·29 ± ·16	71·87 ± ·19
II	7·97 ± ·02	22·35 ± ·06	13·63 ± ·08	10·22 ± ·11	6·38 ± ·07	10·00 ± ·10	13·73 ± ·15	17·72 ± ·10	21·81 ± ·06	50·13 ± ·09	46·04 ± ·07	66·36 ± ·12	66·19 ± ·10	71·19 ± ·11
III	7·56 ± ·02	23·93 ± ·05	*15·76 ± ·11	*11·92 ± ·10	6·95 ± ·05	12·68 ± ·06	16·59 ± ·10	18·16 ± ·05	21·88 ± ·05	49·09 ± ·06	44·65 ± ·05	64·38 ± ·07	64·32 ± ·07	68·94 ± ·06
IV	7·43 ± ·02	24·86 ± ·05	*13·28 ± ·13	*10·15 ± ·11	6·93 ± ·05	13·93 ± ·04	18·28 ± ·09	19·39 ± ·06	21·95 ± ·05	47·64 ± ·05	45·27 ± ·05	63·63 ± ·06	63·62 ± ·06	69·07 ± ·04
V	7·85 ± ·03	24·67 ± ·05	10·58 ± ·06	8·19 ± ·05	6·83 ± ·05	13·56 ± ·07	17·01 ± ·13	18·76 ± ·05	21·24 ± ·05	48·18 ± ·07	44·98 ± ·06	64·18 ± ·09	64·19 ± ·09	69·30 ± ·07

Differences between successive Mean Index Values.

II	+ ·06	+ 2·95	+ 1·72	+ 1·13	− ·63	+ 5·67	+ 2·09	—	+ 1·58	− 2·09	− ·97	− 2·10	− 2·10	− ·68
III	− ·41	+ 1·58	—	—	+ ·57	+ 2·68	+ 2·86	+ ·44	+ ·07	− 1·54	− 1·39	− 1·98	− 1·87	− 2·25
IV	− ·13	+ ·93	—	—	− ·02	+ 1·25	+ 1·69	+ 1·23	+ ·07	− 1·45	+ ·62	− ·75	− ·70	+ ·13
V	+ ·42	− ·19	—	—	− ·10	− ·67	− 1·27	− ·63	− ·71	+ ·54	− ·25	+ ·55	+ ·57	+ ·23

Variabilities (σ) of Indices in per cents.

	Eye	Length of Head	Length of Anterior Dorsal Fin	Length of Ventral Fin	Length of Posterior Dorsal Fin	Length of Caudal Fin	Breadth of Caudal Fin	Breadth of Body	Position of Pectoral Fin	Position of Pelvic Fin	Position of Anterior Dorsal Fin	Position of Ventral Fin	Position of Anus	Position of Posterior Dorsal Fin
I	·92 ± ·03	1·68 ± ·06	1·77 ± ·06	2·30 ± ·08	1·97 ± ·10	1·15 ± ·04	2·27 ± ·08	—	1·25 ± ·04	3·87 ± ·13	2·32 ± ·08	3·56 ± ·12	3·33 ± ·11	2·68 ± ·13
II	·50 ± ·01	1·27 ± ·04	1·75 ± ·06	2·32 ± ·08	1·34 ± ·05	2·19 ± ·07	3·10 ± ·11	2·13 ± ·07	1·19 ± ·04	1·95 ± ·06	1·54 ± ·05	2·54 ± ·08	2·24 ± ·07	2·15 ± ·08
III	·39 ± ·01	1·11 ± 04	*2·21 ± ·08	*2·16 ± ·07	·95 ± ·04	1·25 ± ·04	2·07 ± ·07	1·07 ± ·04	1·07 ± ·04	1·34 ± ·04	1·11 ± ·04	1·53 ± ·05	1·56 ± ·05	1·20 ± ·04
IV	·43 ± ·01	·98 ± ·04	*2·66 ± ·09	*2·28 ± ·08	·95 ± ·04	·85 ± ·03	1·85 ± ·06	1·15 ± ·04	1·01 ± .04	·96 ± ·04	·98 ± ·04	1·23 ± ·04	1·20 ± ·04	·99 ± ·03
V	·45 ± ·02	·97 ± ·04	1·09 ± ·04	·99 ± ·04	·99 ± ·04	1·36 ± ·05	2·48 ± ·09	·86 ± ·04	1·01 ± ·04	1·23 ± ·05	1·20 ± ·04	1·64 ± ·06	1·62 ± ·06	1·38 ± ·05

* For the reason stated in the note to Table I these values are untrustworthy.

It might be urged therefore that where the part is growing fast relatively to the whole its variability is high, while as its relative growth-rate diminishes its variability decreases too. Thus, between Stages I and II the index value of the caudal fin length passes from 4·33 to 10·00, but between Stages II and III from 10·00 only to 12·68, and between Stages III and IV from 12·68 to 13·93, that is there are successively diminishing increases of 5·67, 2·68, and 1·25, and all this time the variability is also decreasing (2·19, 1·25 and ·85). A similar relation can be seen between rate of change of index value and variability in the breadth of the caudal fin between Stages III and IV.

That the growth-rate is not however the only factor concerned is seen from the fact that in the fifth stage of these two dimensions (length and breadth of caudal fin) and between Stages II and III of the latter, a diminution in index value is accompanied by an increase of variability.

A similar relation between the relative growth-rate—as measured by the differences between successive index values—and the variability obtains in other cases. Thus, although the index value of head-length increases it does so at a decreasing rate, and the variability decreases at the same time, so with the eye-diameter, the body-breadth, the position of the pectoral and posterior dorsal fins, and, but in the final stage only, the positions of the pelvic and ventral fins and of the anus. In others the relation does not hold good. It does however appear to be true of the majority that the variability depends more closely on the relative than we have seen it to do on the absolute growth-rate.

Correlations.

Table V gives the values of such correlation coefficients as have been worked out in Stages I, III and V.

The value of the coefficient varies, but between the same pair of correlates is of the same sort of magnitude throughout.

TABLE V. ρ.

	Total Length and Breadth of Caudal Fin	Total Length and Breadth of Body	Total Length and Length of Anterior Dorsal Fin	Total Length and Length of Ventral Fin	Total Length and Length of Posterior Dorsal Fin	Total Length and Length of Head	Length of Head and Diameter of Eye	Length of Anterior Dorsal and of Posterior Dorsal Fin	Length of Ventral and of Posterior Dorsal Fin	Position of Pectoral and of Pelvic Fin
I	·888 ± ·01	—	·764 ± ·02	·836 ± ·01	·495 ± ·05	·945 ± ·00	·766 ± ·02 † (·943 ± ·01)	·432 ± ·06	·515 ± ·05	·872 ± ·01
III	·736 ± ·02	·790 ± ·02	·493* ± ·04	·584* ± ·03	·452 ± ·04	·729 ± ·02	·646 ± ·03	·441* ± ·04	·646* ± ·03	·691 ± ·02
V	·727 ± ·03	·869 ± ·01	·602 ± ·03	·607 ± ·03	·523 ± ·04	·848 ± ·01	·647 ± ·03	·391 ± ·05	·581 ± ·04	·825 ± ·02

* Unreliable for the reason already given.

† The second and higher value is obtained by the omission of one very aberrant individual.

With two exceptions (total length correlated with length of posterior dorsal fin and length of ventral correlated with length of posterior dorsal fin) there is a sensible diminution in the value of the coefficient. This may depend on the fall in the growth-rate, as Boas has suggested.

On the other hand in passing from the third to the fifth stage there is, except in the case of total length and breadth of caudal fin, a rise in the value of the coefficient, and, as we have seen, this is not accompanied by any rise in the absolute growth-rate.

Nor does there appear to be any uniform connection between a rise or fall in the correlation coefficient and a rise or fall in the relative growth-rate.

Thus between Stages III and V the index values of head and of body-breadth increase, and the coefficient of correlation between these and the total length increases too. But between the same two stages the index value of the position of the pectoral and pelvic fins decreases, while the coefficient increases. In the case of the breadth of the caudal fin on the other hand the coefficient diminishes steadily while the index value increases.

Boas has found that the correlation is greater between axial organs—stature and height sitting, or stature and head-length—than between longitudinal and transverse parts.

In the Trout there is—so far—no indication of this. The coefficient value often is high between longitudinal parts, e.g. total length and length of anterior dorsal or ventral fins, or total length and length of head, but it may be low between such parts, e.g. total length and length of posterior dorsal fin, or length of posterior dorsal fin and that of anterior dorsal or of ventral fin. On the other hand it is high between the total length and the breadth of the body, and of the caudal fin.

Between the positions of one pair of metameric organs—the pectoral and pelvic fins—it is high: between the dimensions of other pairs of such organs (the median fins) it is lower.

Lastly, constancy through the series in the index value is not necessarily a sign of high correlation (e.g. the length of the posterior dorsal fin and total length).

The main results of the investigation may be briefly summarised as follows:

(1) During the first ten weeks after hatching there is a decline in the absolute growth-rate. This is more rapid in earlier than in later stages.

This is true not only of the whole body but of its parts. The growth-rate and the rate of its decline are not the same in the different parts.

(2) There is diminution in the absolute variability accompanying and possibly caused by this general diminution of growth-rate. At the same time there is a sensible increase in variability in the final stage which is not accompanied by any rise in the growth-rate. Also at certain other stages in certain parts the growth-rate may change while the variability does not alter correspondingly.

(3) The indices or proportions of the parts relatively to the whole may exhibit a gradual increase, or an increase at first rapid, later less so, followed eventually by a decrease, or a gradual diminution from the beginning, followed sometimes by a slight final increase or may be nearly constant.

There seems to be some reason to suppose that the changes in the variabilities of the indices, which exhibit on the whole a gradual decrease, though in some cases an increase is shown, depend on changes in the relative growth-rate.

(4) Correlations between parts are generally high. They exhibit a general decrease, with one or two exceptions. This may depend on the observed decrease in the growth-rate.

In the last, as compared with the middle stage, there is however an increase as a rule.

It is difficult to connect this last change with any corresponding change in either the absolute or the relative growth-rate.

It is hoped that these results, though somewhat meagre, may prove useful in the future, as it is proposed if possible to compare the growth-rate, variability and correlation under normal conditions and under conditions which have been made unfavourable experimentally.

LITERATURE.

(1) F. Boas. The growth of Toronto children. *U.S.A. Report of the Commissioner of Education*, II, 1897.

(2) F. Boas and C. Wissler. Statistics of Growth. *United States Education Commission*, I, 1904.

(3) H. P. Bowditch. The growth of children. *Massachusetts State Board of Health*, 1877.

(4) C. S. Minot. Senescence and rejuvenation. *Journ. Phys.* XII, 1891.

(5) ——. The problem of age, growth and death. *Pop. Sci. Monthly*, 1907.

(6) A. Quetelet. *Anthropométrie.* Bruxelles, 1870.

(7) T. B. Robertson. On the normal rate of growth of an individual and its biochemical significance. *Arch. Ent. Mech.* XXV, XXVI, 1908.

(8) C. Roberts. *Manual of Anthropometry.* London, 1878.

(9) K. Semper. *Animal Life*, 5th ed. London, 1906.

(10) H. M. Vernon. The effect of environment on the development of Echinoderm larvae, an experimental enquiry into the causes of variation. *Phil. Trans. Roy. Soc.* CLXXXVI B, 1895.

Biometric Workers and Statistical Reviewers.

By E. C. SNOW, M.A.

The letter which follows this note was sent to the Editors of the *Journal of the Royal Statistical Society* in reply to an editorial criticism in the December number of the Journal of my recent memoir "The Intensity of Natural Selection in Man*." The remarks made in the course of that criticism seemed to me to be based on an inadequate reading of the memoir and a lack of appreciation of the work given to the reduction of the statistical data with which it deals. I thought that an immediate reply was desirable, and wrote pointing out the errors fact, quotation and opinion into which I considered the writer had fallen.

As the Editors have seen fit to extract and publish only certain statements out of my letter without consulting me in the matter, I take this opportunity of issuing my detailed reply.

Sirs,

The writer of the notes on my recent memoir on "The Intensity of Natural Selection in Man" in the December number of the *Journal of the Royal Statistical Society* asks that discussion should be given to a certain point involved in that memoir. I gladly take this opportunity of initiating that discussion since, in addition to the specific point which it is desired to clear up, there is in those notes one flagrant misquotation from the memoir which requires correction, and also two or three other matters which it seems eminently desirable to place in their proper perspective in order that attention may be directed to the kernel of the problem.

In the first of the three paragraphs dealing with the subject, and before the memoir is mentioned, a quotation is given from Mr Yule's Appendix to the Report of the Medical Officer of the Local Government Board in which he states that the data he employed were such that his conclusions could not be put forward "as anything but a very tentative hypothesis." This coming before the title of the memoir or even any connection of mine with the subject is announced would lead the casual reader to conclude that I had given more weight to Mr Yule's work than he himself would desire. Yet the identical quotation used in the note is given in full in the memoir (p. 8) and reiterated on p. 10. As the charge of discourtesy is afterwards made against me, I think it might in common fairness have been made clear that I gave no greater emphasis to Mr Yule's work than he himself desired.

The point on which the writer of the note specifically asks for discussion admits of a very simple investigation. Before stating this I may point out that in giving an example of my method he uses the phrase "the remaining deaths under 5 years of age during 1903—7." In the original the word "remaining" is italicised (p. 15). This may seem at first sight to be

---

* *Drapers' Company Research Memoirs.* No. VII. Dulau and Co.

a small matter, but his failure to notice the emphasis placed on that word is, I think, the basis of the whole of the writer's attack on the method of measuring environment adopted in the memoir. He appears to think that by making those *remaining* deaths constant throughout the districts, I have also made the deaths of the particular cohorts I dealt with constant during their first five years of life. He asks, "Has Mr Snow found anything more than a somewhat obscured consequence of the obvious result that, in a group of districts with the same total mortality for the first five years of life, the districts with the higher mortality for the first three years will tend to have the lower mortality for the last two. It seems to us exceedingly doubtful whether the method of 'correction for environment' used has not led to a series of correlations which are largely fallacious and of no significance as regards selection." It is remarkable that the reviewer did not take the trouble to test the value of his suggestion, for I gave complete data in the table on p. 33 of the memoir for that to be done, and the time required was only a few moments at the outside. In fact an approximation sufficient for his purpose could be worked out mentally in the space of a few seconds. In the table referred to I gave values of $_{03}\sigma_1$ and $_{03}\sigma_2$ as well as of $_{03}r_{12}$ for five sets of data (three from English figures and two from Prussian—the latter, however, dealing with the first ten years of life, x_1 referring to the first two, and x_2 to the next eight years of life) distinguishing males from females. Thus there were ten distinct sets of values given, on any one of which he could have tested his suggestion. If we let x_5 denote x_1+x_2, the phrase "the same total mortality for the first five years of life" obviously requires the values of $_{03}\sigma_5$ to be appreciably zero in each case. Values of this can be found from a well-known elementary formula

$$_{03}\sigma_5{}^2 = {}_{03}\sigma_1{}^2 + {}_{03}\sigma_2{}^2 + 2\,{}_{03}r_{12}\cdot{}_{03}\sigma_1\cdot{}_{03}\sigma_2\,,$$

a proof of which is given in Mr Yule's *Introduction to the Theory of Statistics.* The values of $_{03}\sigma_5$ as found from this formula are given in the following table together with such of the other figures from the memoir as are necessary for an understanding of the matter in its present bearings.

Data		$\bar{x}_0$	$\bar{x}_1$	$\bar{x}_2$	$_{03}\sigma_1$	$_{03}\sigma_2$	$_{03}r_{12}$	$_{03}\sigma_5$
Males:	1870	3227	644	51	44·582	10·634	– ·4483	40·93
English Rural	1871	3226	602	60	42·247	9·589	– ·3574	39·84
Districts	1872	3291	588	62	41·220	12·108	– ·2271	40·24
Prussian Rural	1881	9407	2270	729	198·042	210·803	– ·9278	78·68
Districts	1882	9297	2424	732	148·377	206·413	– ·6050	166·02
Females:	1870	3090	531	48	34·340	11·126	– ·4666	30·77
English Rural	1871	3114	491	59	28·468	12·736	– ·2857	27·67
Districts	1872	3150	494	61	24·011	11·175	– ·5089	20·54
Prussian Rural	1881	8917	1914	711	169·182	183·986	– ·8483	98·30
Districts	1882	8793	2004	729	117·355	178·700	– ·6078	142·17

The figures in the last column at once proclaim how groundless is the assertion that the negative values of $_{03}r_{12}$ are due to the fact that the method I used for correcting for environment was equivalent to assuming all the districts had the same total mortality for the first five years of life. Taking the first case on the table—the cohort of English males born in rural districts in 1870:—*when correction is made for environment* the mean total mortality in the first five years of life was 695, *with a standard deviation of* 41; the mean mortality in the first three years was 644 with a standard deviation of 45; and in the next two the figures were 51 and 11 respectively, the partial correlation for a constant environment between the mortality in the first three years of life and the next two being – ·45. With such a range of values as is indicated by a mean value of 695 and a standard deviation of 41 it cannot be maintained for a moment that the reviewer had any ground whatever for the opinion he stated. (For the reader who has not referred to the original memoir I may say that I have found there the magnitude of the (negative) correlation which should be expected if there were no selection ; this arising solely from the fact that the districts with a high mortality in the first period would have a smaller population to be aimed at in the second, and therefore, without selection, a certain amount of negative correlation should be expected. But in every case except the Prussian one for the 1881 cohort—where the (negative) value of ${}_{03}r_{12}$ is very high—this correlation is less than the corresponding probable error of ${}_{03}r_{12}$.)

The complaint is also made of my "dangerous measure of environment." I admit that the method of measuring environment in the memoir is rather subtle, and that its use requires great care. I believe, however, that the real danger lies in the ease with which it may be misunderstood by those who have not the time or inclination to ascertain what it really means. I hope soon to complete the development of a criterion which shall give a measure of the goodness of any particular correction for environment, etc., but in the meantime I may point out that the danger is considerably less than that involved in the correction which necessitates the correlation of rates. There is often a likelihood of spurious correlation arising when this is done, and there is evidence accumulating that a number of earlier investigations in the field of correlation may be invalidated on account of the use of the correlation between rates. The measurement of environment has always been a difficulty in statistical researches requiring its use and, for the purposes of future work, I shall be grateful if the reviewer can produce any *facts* invalidating the one employed by me, or can make suggestions for a more valuable one. The only improved method I have been able to think of is fundamentally similar to the one actually adopted, but involves the introduction of more variables.

I must confess to considerable amazement that the *Journal* critic should write "It seems to us also that on reconsideration he is not likely to adhere to his present opinion that any investigation on the selectivity of the death-rate must be made on a homogeneous group of districts; heterogeneity is really of the essence of the case in the most important form of the problem." He can hardly have appreciated the facts I stated on p. 10 and the tables on pp. 19 and 21 of the memoir. Briefly, I found that when treating of a mixture of districts I obtained practically zero correlations. When I took out from these the rural ones and also the urban ones (there was a residue of areas coming completely under neither head) I found very significant negative values for the former, but zero correlations for the latter. On this problem and an earlier one (*Journal*, May 1911) I have spent many weeks in analysing heterogeneous material into more or less homogeneous groups, and no amount of verbal disquisition as apart from statistical analysis is likely to overthrow the experience I gained in those weeks. As I perceive the problem of the selectivity of the death-rate, we have statistically to get the populations in a large number of districts under uniform environmental conditions. Differences in the mortality in a given period will then arise solely from *random* causes. We then want to investigate to what extent the mortality in a district which is below the mean of all districts in that period is followed by a mortality which is above the mean of all districts in a following period. The difficulty is the statistical one of the measurement of environment. I spent considerable time in trying to devise a method of attacking the problem which could be applied to a mixture of urban and rural districts, but failed, chiefly on account of the migratory habits of urban populations. I claim, then, that the measurement of selection investigated by the method I adopted essentially requires the analysis of the material into groups of similar characteristics.

In the third of the *Current Notes* dealing with the subject there is a serious misquotation from the memoir. I quote the whole of the sentence in which it occurs. "The aims of the Report seem fairly evident to any reasonably careful reader, but Mr Snow judges that 'the investigation was specially intended to discover whether selection exists under modern conditions,' and makes consequently some criticisms which are entirely wide of the mark and might have been couched in more courteous terms." The inference here is that I carelessly read the Report and assumed that its object was *specially* to investigate the existence or non-existence of selection. I did no such thing, for in my memoir the word written is not "specially" but "specifically" (p. 8), and the charge of careless reading must recoil on the reviewer. I never for one moment thought that the Report was *specially* concerned with that problem; its chief object was the exceedingly important one of the determination of the distribution and causes of excessive infantile mortality. That one of the *specific* objects of the Report was to determine if selection existed cannot be denied. The first of the three objects stated on p. 1 of the Report reads "to determine, on the basis of our national statistics, whether reduction of infant mortality implies any untoward influence on the health of survivors to later years." Again, on p. 9, "attempts to reduce infant mortality are regarded by many as an interference with natural selection, which must be inimical to the average health of those surviving.... The statistics in the following pages do not support this view." In the memoir I have given reasons for the opinion that the statistics referred to do nothing of the kind, though here I am only concerned in showing that the author of the Report intended part of it to refer to the problem of selection. Other quotations to the same end could be given from the Report, though it is true that on p. 17 there is another statement which contradicts them all. I need not quote from the many newspaper accounts of the Report in support of my contention that I did not carelessly assume it to deal with the problem of selection, but I will cull a statement from the review in our own *Journal* (Jan. 1911, p. 208), "Perhaps the most striking conclusion of the report—which is admittedly of a somewhat provisional character in the absence of an extended series of annual statistics of the kind given—is the proof that the alleged selective effect of infantile mortality is not supported by the data available: there may be a slight selective influence extending to the second or third year of life, but it is exceedingly slight at best."

I regret that the charge of discourtesy should be made against me, as I attempted throughout to be studiously courteous, and the *Journal* critic is the first one who in speaking of my attitude to the Report has expressed the opinion that I have erred on the side of *under*-courtesy. On re-reading my words the only sentence I can find which, I think, remotely admits of the charge made is one in which reference is made to the fact that, in the Report, those who have spent months in the laborious collection of *facts* and *figures* relating to the problem, and have come to the conclusion that selection *is* operative, are pilloried with the offensive label a "school of thought." I shall be grateful if the reviewer will point out to me the passages to which he refers, for other critics have expressed the opinion that I was too favourable in my attitude to the Report.

Finally I should like to summarise the evidence we have at present as to the existence of a selective death-rate in man. Full references to the originals are given both in the *Current Note* and in the memoir. Fuller details are given in the latter.

(*a*) The work of Miss Beeton and Prof. Pearson. This was indirect, but its results have never, so far as I am aware, been called into doubt, and have long been accepted by the scientific world. The authors showed that at least from 50 °/ₒ to 70 °/ₒ of all deaths were due to selective action.

(*b*) The investigation of Prof. Ploetz on German data. The result of this was to show that at least 60 °/ₒ of the mortality at ages less than five was due to selective causes.

(*c*) The recent report of the Medical Officer of the Local Government Board and Mr Yule's Appendix to it. This is the only work, whose existence I am aware of, which, since the publication of (*a*), has appeared to call into doubt the operation of a selective death-rate. I have elsewhere given reasons against the validity of that work, and Mr Yule's statement as to the tentative nature of its results has often been quoted.

(*d*) The memoir frequently referred to in this letter. Not every result found there pointed to the existence of selection, but, in my opinion, those which were founded on the best data conclusively showed the prevalence of some degree of selection in the mortality of a general population. I do not believe that the criticisms brought against that work by the *Journal* reviewer in any way affect the legitimacy of its results.

I am, Sirs, etc.,

E. C. SNOW.

January 5th, 1912.

CAMBRIDGE: PRINTED BY JOHN CLAY, M.A. AT THE UNIVERSITY PRESS

Journal of Anatomy and Physiology

CONDUCTED BY

SIR WILLIAM TURNER, K.C.B. ARTHUR THOMSON, University of Oxford
ALEX. MACALISTER, University of Cambridge ARTHUR KEITH, Royal College of Surgeons
ARTHUR ROBINSON, University of Edinburgh

VOL. XLVI

THIRD SERIES—VOLUME VII *PART I NOW READY—PRICE* 6/-

With Illustrations in the Text

ANNUAL SUBSCRIPTION 21/- POST FREE

CONTENTS OF PART I.—OCTOBER 1911

Professor R. W. Reid, M.D. Remarks on Fifteen Years of Anthropometric Work in the University of Aberdeen. (With Folding Schedule.) Professor Paterson, M.D. The Subscapularis Muscle. Arthur Keith, M.D., LL.D., and Francis H. S. Knowles, B.A. A Description of Teeth of Palæolithic Man from Jersey. Professor C. J. Patten, M.A., M.D., Sc.D. Cranium of a Young Orang, showing Bilateral and Symmetrical Complete Bipartite Division of the Parietals. Professor C. J. Patten, M.A., M.D., Sc.D. Cranium of a Chimpanzee, showing Metopic Suture; also Fontanelle and Sutural Bone-Plates. S. E. Whitnall, M.A., M.B. An Instance of the Retractor Bulbi Muscle in Man. Frederic Wood Jones, D.Sc. Some Markings on the Cervical Vertebræ. John Cameron, M.D., D.Sc., F.R.S.E. Further Researches on the Rods and Cones of Vertebrate Retinæ. Sydney J. Cole, M.A., M.D. Oxon. Remarks on Some Points in the Fissuration of the Cerebrum (illustrated by three Chinese Brains). Professor A. C. Geddes, M.D. Notes on the Technical Difficulties of Wax-Plate Reconstruction. W. L. H. Duckworth, M.D., Sc.D. Report on a Skull from British East Africa. W. L. H. Duckworth, M.D., Sc.D., M.A. On the Natural Repair of Fractures, as seen in the Skeletons of Anthropoid Apes. D. G. Reid, M.B., B.A. A Case of Multiple Heart Anomalies. A. B. Appleton, B.A. A Case of Abnormal Distribution of the N. Musculo-Cutaneus, with Complete Absence of the Ramus Cutaneus N. Radialis.

LONDON: CHARLES GRIFFIN and COMPANY, Ltd., Exeter Street, Strand

---

JOURNAL OF THE ROYAL ANTHROPOLOGICAL INSTITUTE

Vol. XLI. January—June, 1911

Contents:—

President's Address. Sir Herbert Risley. Notes on the Customs and Beliefs of the Wa-Giriama, etc., British East Africa. Capt. W. E. H. Barrett, with Introductory Notes by C. W. Hobley. On certain Physical Characters of the Negroes of the Congo Free State and Nigeria. Arthur Keith. Some Technological Notes from the Pomeroon District, British Guiana. (Part III.) Walter E. Roth. Cup and Ring-Markings: Their Origin and Significance. H. J. Dukinfield Astley. On Some Saxon Bones from Folkestone. F. G. Parsons. Report on the Ethnology of the Okanak En of British Columbia, an Interior Division of the Salish Stock. C. Hill Tout. Notes on Some Nigerian Tribal Marks. Major A. J. N. Tremearne. A Note on the Possibility of Analysing Race Mixtures into their Original Elements by the Mendelian Formula. John Brownlee. Notes on the Yezidis. W. B. Heard.

WITH TWENTY-THREE PLATES AND MANY ILLUSTRATIONS IN THE TEXT

PRICE 15s. NET

LONDON: THE ROYAL ANTHROPOLOGICAL INSTITUTE, 50, Great Russell Street, W.C.
or through any Bookseller

MAN

A MONTHLY RECORD OF ANTHROPOLOGICAL SCIENCE

Published under the direction of the Royal Anthropological Institute of Great Britain and Ireland. Each number of **MAN** consists of 16 Imp. 8vo. pages, with illustrations in the text together with one full-page plate; and includes Original Articles, Notes, and Correspondence; Reviews and Summaries; Reports of Meetings; and Descriptive Notices of the Acquisitions of Museums and Private Collections. Price, 1s. Monthly or 10s. per Annum prepaid.

TO BE OBTAINED FROM THE
ROYAL ANTHROPOLOGICAL INSTITUTE, 50, Great Russell Street, W.C.
AND THROUGH ALL BOOKSELLERS

CONTENTS

(All Rights reserved)

The publication of a paper in *Biometrika* marks that in the Editor's opinion it contains either in method or material something of interest to biometricians. But the Editor desires it to be distinctly understood that such publication does not mark assent to the arguments used or to the conclusions drawn in the paper.

Biometrika appears about four times a year. A volume containing about 500 pages, with plates and tables, is issued annually.

Papers for publication and books and offprints for notice should be sent to Professor Karl Pearson, University College, London. It is very desirable that a copy of all measurements made, not necessarily for publication, should accompany each manuscript. In all cases the papers themselves should contain not only the calculated constants, but the distributions from which they have been deduced. Diagrams and drawings should be sent in a state suitable for direct photographic reproduction, and if on decimal paper it should be *blue* ruled, and the lettering only pencilled.

Papers will be accepted in German, French or Italian. In the first case the manuscript should be in Roman not German characters.

Contributors receive 25 copies of their papers free. Twenty-five or fifty additional copies may be had on payment of 3/6 or 6/- respectively per sheet of eight pages; these should be ordered when the final proof is returned.

The subscription price, payable in advance, is 30*s.* *net* per volume (post free); single numbers 10*s.* *net.* Volumes I, II, III, IV, V, VI, VII and VIII (1902—11) complete, 30*s.* *net* per volume. Bound in Buckram 34/6 *net* per volume. Index to Volumes I to V, 2*s.* *net.* Subscriptions may be sent to C. F. Clay, Cambridge University Press, Fetter Lane, London, E.C., either direct or through any bookseller, and communications respecting advertisements should also be addressed to C. F. Clay.

Till further notice, new subscribers to *Biometrika* may obtain Vols. I—VIII together for £9 *net*—or bound in Buckram for £11 *net.*

CAMBRIDGE: PRINTED BY JOHN CLAY, M.A. AT THE UNIVERSITY PRESS.